QCD and the Parton Model

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Outline

- Motivation and classical history
- Parton model
- Field theory parton model
- DGLAP evolution equations

Motivation

- Although the proton's properties and existence can be explained via QCD its existence cannot be proven
- We have only been able to apply QCD to colored particles not hadrons
- The two other methods presented in Schwartz, chiral perturbation theory and lattice QCD, have problems as well
- The Chiral Lagrangian is not renormalizable and lattice QCD is computationally expensive and ill fit for calculations such as scattering amplitudes

Motivation

- Intuitively we should be able to use QCD for high energy proton scattering as the strong force is weak at short distance scales

- At very high energies the scattering interactions are mainly those involving free quarks and gluons

- In order to study the proton, we will use $e^{-}p^{+}$ scattering to probe the proton's properties

Classical Experiment

- The proton was discovered by Rutherford, Geiger and Marsden after they fired α particles at gold and later aluminum foil

- To there surprise they measured scattering angles greater than 90°

- Assuming a central coulomb potential, Rutherford calculated
$$\frac{d\sigma}{d\Omega} = \left(\frac{Ze^2}{4\pi m v^2}\right)\frac{1}{\sin\frac{\theta}{2}}$$
which agreed with the data

Classical Experiment

- using conservation of energy at zero impact parameter Rutherford derived an r_{max} given by the formula

$$\frac{1}{2}mv^2 = \frac{2Ze^2}{4\pi r_{max}}$$

which gives $r_{max} = 4.8 \times 10^{15} m$ for the proton

Elastic Scattering

- Called coulomb scattering at low energies
- Analogous to $e^-\mu^+$ scattering
- In the lab frame (proton rest frame) the Relativistic cross section for two spin 1/2 particles is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{lab} = \frac{\alpha_e^2}{4E^2\sin^4\frac{\theta}{2}}\frac{E'}{E}\left(\cos^2\frac{\theta}{2} - \frac{q^2}{2m_p^2}\sin^2\frac{\theta}{2}\right)$$

- Where we let $m_e \to 0$ and $q^\mu = k^\mu - k'^\mu$
- The relation $q^2 = -2k\cdot k' = -\left(4E'E\sin^2\frac{\theta}{2}\right)_{lab} = 2m_p\left(E - E'\right)$ is also useful

Elastic Scattering

- When the electron is not considered massless it is easy to see that our result reduces to the classical one in the low energy limit

- Our result depends only on the electron's initial and final state properties

- If we were ignorant to QCD we would expect our result to hold up to arbitrarily short distances

Elastic Scattering

- Similar to in QED we remove the electron and consider the interaction of the proton with an off shell photon of spacelike momentum q^μ

- Again as in QED the most general vertex can be written in the form $\overline{u}(p')(ie\Gamma^\mu)u(p)$ where as before

$$\Gamma^\mu = F_1(q^2)\gamma^\mu + \frac{i\sigma^{\mu\nu}}{2m_p} q_\nu F_2(q^2)$$

- This confirms the proton charge $Q = +1$ at large distances

- It is experimentally known that $g_p = 5.58 \implies$ the proton is not a pointlike particle

Elastic Scattering

- Repeating the elastic scattering calculation with the general form of the vertex gives

$$\left(\frac{d\sigma}{d\Omega}\right)_{lab} = \frac{\alpha_e^2}{4E^2\sin^4\frac{\theta}{2}}\frac{E'}{E}$$
$$\times\left[\left(F_1^2 - \frac{q^2}{4m_p^2}F_2^2\right)\cos^2\frac{\theta}{2} - \frac{q^2}{2m_p^2}\left(F_1+F_2\right)^2\sin^2\frac{\theta}{2}\right]$$

- Rosenbluth formula

Elastic Scattering

- Because $m_\tau = 1.7\text{GeV} \sim m_p$ consider $e^-\tau^+$ scattering as an example

- $|q^2| \gg m_\tau^2 \implies F_2 \to 0$ and $F_1 \sim \log(Energy)$

- This contradicts the protons observed behavior

$$F_1 \sim \frac{1}{\left(1 - \frac{q^2}{0.71\text{GeV}^2}\right)^2}$$

  where a definite scale has appeared

- Up to multiplicative factors F_1 is just the fourier transform of the Born scattering potentials yielding $V(r) = \frac{m^3}{4\pi}e^{-mr} \sim e^{-r/r_0}$ where $r_0 \sim (0.84\text{GeV})^{-1} \sim 1\text{fm}$

Inelastic Scattering

- Slightly inelastic: $e^- p^+ \to e^- p^+ \pi^0$

- Deep inelastic scattering (DIS):$e^- p^+ \to e^- X$ where X represents anything the proton can break into

- Instead of parameterizing the vertex in terms of form factors, we now the $\gamma p^+ \to X$ interactions $\implies$ we parameterize the cross section

- Before integrating over the electrons final state energy the cross section can be written in the form

$$\left(\frac{d\sigma}{d\Omega dE'}\right)_{lab} = \frac{\alpha_e^2}{4\pi m_p q^4} L^{\mu\nu} W_{\mu\nu}$$

Inelastic Scattering

- $L_{\mu\nu}$ is the leptonic tensor and $W_{\mu\nu}$ is the hadronic tensor

- $L^{\mu\nu} = \frac{1}{2} tr\left[\not{k}'\gamma^\mu \not{k}\gamma^\nu\right] = 2\left(k'^\mu k^\nu - k'^\nu k^\mu - k \cdot k' g^{\mu\nu}\right)$

$$e^2 \epsilon_\mu \epsilon_\nu^* W^{\mu\nu} = \frac{1}{2} \sum_{X,spin} \int d\Pi_X (2\pi)^4 \delta^4(q+P-p_X) |\mathcal{M}(\gamma p^+ \to X)|^2$$

- Because final states are integrated over we know $W^{\mu\nu}$ can only depend on P^μ and q^μ

- The ward identity and the fact that unpolarized scattering must be symmetric further constrains the form of $W^{\mu\nu}$

Inelastic Scattering

- The most general form of $W^{\mu\nu}$ we can write is

$$W^{\mu\nu} = W_1\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + W_2\left(P^\mu - \frac{P\cdot q}{q^2}q^\mu\right)\left(P^\nu - \frac{P\cdot q}{q^2}q^\nu\right)$$

- W_1 and W_2 only depend on the scalars P^2, q^2 and $P\cdot q$
- We now define $Q \equiv \sqrt{-q^2} > 0$, $\nu \equiv \frac{P\cdot q}{m_p} = (E-E')_{lab}$ and $x = \frac{Q^2}{2P\cdot q}$ called the Bjorken x

Inelastic Scattering

- Contracting $L^{\mu\nu}$ and $W_{\mu\nu}$ gives

$$\left(\frac{d\sigma}{d\Omega dE'}\right)_{lab} = \frac{\alpha_e^2}{8\pi E^2 \sin^4 \frac{\theta}{2}} \times \left[\frac{m_p}{2} W_2(x,Q) \cos^2 \frac{\theta}{2} + \frac{1}{m_p} W_1(x,Q) \sin^2 \frac{\theta}{2}\right]$$

- As we had before, W_1 and W_2 can be completely determined by measuring the electron's properties

- The cross section's approximate independence of Q for fixed x is called Bjorken scaling

Bjorken Scaling

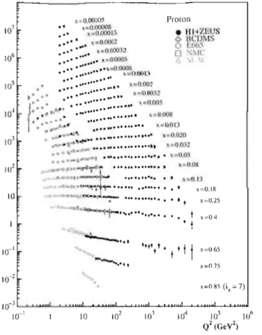

Figure 1: Data confirming Bjorken scaling

The Parton Model

- Assume objects within the proton called "partons" are free
- Parton refers to quarks, gluons and less formally antiquarks, photons and the rest of the SM particles
- Assume some of the partons are charged
- We will determine W_1 and W_2 via elastic scattering off of a parton of mass m_q
- $p_i^\mu + q^\mu = p_f^\mu \implies \frac{Q^2}{2p_i \cdot q} = 1$ is a relation we will use

The Parton Model

- Assume $p_i^\mu = \xi P^\mu$ where ξ is called the momentum fraction
- $x = \frac{Q^2}{2P \cdot q} = \xi$
- Measuring x is measuring the fraction of P involved in the parton scattering

Parton Distribution Functions (PDF's)

- $f_i(\xi)d\xi$ is the probability of photon hitting parton i with momentum fraction ξ
- Intuitively this is allowed because momentum is exchanged between partons on time scales $\sim m_p^{-1}$ which are much slower than Q^{-1} which is relevant to the photon
- Rigorously $Q \gg \Lambda_{QCD} \implies$ de-coherence of the parton wave functions allowing for a probabilistic interpretation

The Parton Model

- We can now write the cross section in terms of the partonic cross sections

$$\sigma(e^- P^+ \to e^- X) = \sum_i \int_0^1 d\xi f_i(\xi)\hat{\sigma}(e^- p_i \to e^- X)$$

- At lowest order we can use the Rosenbluth formula with $F_1 = 1$ and $F_2 = 0$ for the partonic cross section

- Plugging into the formula above gives

$$\left(\frac{d\sigma(e^- P^+ \to e^- X)}{d\Omega dE'}\right) = \sum_i f_i(x)\frac{\alpha_e^2 e_i^2}{4E^2 \sin^4 \frac{\theta}{2}}$$
$$\times \left[\frac{2m_p x^2}{Q^2}\cos^2\frac{\theta}{2} + \frac{1}{m_p}\sin^2\frac{\theta}{2}\right]$$

- We can now read off

$$W_1(x, Q) = 2\pi \sum_i e_i^2 f_i(x)$$

$$W_2(x, Q) = 8\pi \frac{x^2}{Q^2} \sum_i e_i^2 f_i(x)$$

- We have also derived the Callan-Gross relation

$$W_1(x, Q) = \frac{Q^2}{4x^2} W_2(x, Q)$$

Sum Rules

- Because PDF's are probabilities they must obey certain properties

- For example down quark number is conserved within a proton so

$$\int d\xi (f_d(\xi) - f_{\overline{d}}(\xi)) = 1$$

- Each rule corresponds to a classically conserved current

- Numerically evaluating $\int d\xi \xi (f_u(\xi) + f_d(\xi)) \approx 0.38$

- The valence quarks only contribute 38% of the protons momentum

- the rest is comprised of gluons and sea quarks

DGLAP Equations

- Structure functions should have weak logarithmic on Q^2
- Want to combine parton model with perturbative QCD
- Assume parton model holds
- Define the partonic version of the hadronic tensor in terms of $|\mathcal{M}(\gamma q \to X)|^2$ and also define $z \equiv \frac{Q^2}{2p_i \cdot q} \implies x = z\xi$

DGLAP Equations

- Integrating over ξ we get

$$W^{\mu\nu}(x,Q) = \sum_i \int_0^1 dz \int_0^1 d\xi f_i(\xi) \hat{W}^{\mu\nu}(z,Q)\delta(x - z\xi)$$
$$= \sum_i \int_x^1 \frac{d\xi}{\xi} f_i(\xi) \hat{W}^{\mu\nu}\left(\frac{x}{\xi}, Q\right)$$

- At leading order only $\gamma q \to q$ contributes and we have

$$\hat{W}^{\mu\nu}(z,Q) = 2\pi e_i^2 \delta(1-z)$$
$$\times \left[\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + \frac{4z}{Q^2}\left(p_i^\mu - \frac{p_i \cdot q}{q} q^\mu\right)\left(p_i^\nu - \frac{p_i \cdot q}{q} q^\nu\right)\right]$$

DGLAP Equations

- We can now read off

$$\hat{W}_1 = 2\pi e_i^2 \delta(1-z) = \frac{Q^2}{4z}\hat{W}_2$$

  confirming the Callan-Gross relation at leading order

- Plugging this into our formula for the hadronic tensor gives

$$W^{\mu\nu}(x,Q) = 2\pi \sum_i e_i^2 f_i(x)$$
$$\times\left[\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + \frac{4x^2}{Q^2}\left(P^\mu - \frac{P\cdot q}{q}q^\mu\right)\left(P^\nu - \frac{P\cdot q}{q}q^\nu\right)\right]$$

DGLAP Equations

- Now consider $W_0 \equiv -g^{\mu\nu} W_{\mu\nu}$
- When $Q \gg m_p$,

$$W_0 = 2W_1 = 4\pi \sum_i e_i^2 f_i(x)$$

- We will use W_0 to define PDF's at higher orders
- Similarly for the partonic version we get

$$\hat{W}_0^{LO} = 4\pi e_i^2 \delta(1-z)$$

NLO

- At NLO there are three diagrams, one corresponding to the virtual correction to $\gamma q \to q$ and two for the process $\gamma q \to qg$

- All have divergences and require renormalization or dimensional regulation (Too long to display in one slide, the full expression for $\hat{W}_0$ can be found on Schwartz p. 679)

$$P_{qq}(z) = C_F \left[(1+z^2) \left[\frac{1}{1-z} \right]_+ \frac{3}{2} \delta(1-z) \right]$$

- $P_{qq}(z)$ is called DGLAP splitting function

NLO

- Taking a difference at two different scale Q and Q_0 gives

$$W_0(x,Q) - W_0(x,Q_0) = 4\pi \sum_i \int_x^1 \frac{d\xi}{\xi} f_i(\xi) \left[\frac{\alpha_s}{2\pi} P_{qq}(\frac{x}{\xi}) \log \frac{Q^2}{Q_0} \right]$$

- It is clear that Bjorken scaling is violated by the logarithmic dependence on Q^2

DGLAP Evolution equation

- Now if we define

$$W_0(x, Q) \equiv 4\pi \sum_i e_i^2 f_i(x, \mu = Q)$$

for any scale Q and plug into the previous equation we arrive at the result

$$\mu \frac{d}{d\mu} f_i(x, \mu) = \frac{\alpha_s}{\pi} \int_x^1 \frac{d\xi}{\xi} f_i(\xi, \mu) P_{qq}(\frac{x}{\xi})$$

- DGLAP evolution equation

References

[1] M.D. Schwartz, Quantum Field Theory and the Standard Model.