Symmetry Restoration at Finite Temperature

QFT III Final Presentation

Evan Frangipane[1]

[1]UCSC

June 2020

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FTFT I: Imaginary Time Formalism

- The idea of the imaginary time formalism is to define $\tau = i\ t$ and exchange Minkowski time for $0 < \tau < \beta$ where $\beta = 1/T$
- Before defining the partition function we consider the transition amplitude from $|\varphi_0\rangle$ at $t = 0$ to $|\varphi_1\rangle$ at $t = t_1$

$$\langle\varphi_1|\, e^{-iHt_1}\, |\varphi_0\rangle \tag{1}$$

- Converting to the Euclidean picture, we can write the partition function for a quantum system

$$\begin{aligned} Z = \mathrm{tr}\ e^{-\beta H} &= \sum_{\varphi} \langle\varphi|\, e^{-\beta H}\, |\varphi\rangle \\ &= N(\beta) \int_{\text{periodic}} \mathcal{D}\varphi \exp\{-\int_0^\beta d\tau \int d^3x\ \mathcal{L}\} \end{aligned} \tag{2}$$

- The periodicity of φ is enforced by the trace over states (anti-periodic for fermions) [1]

FTFT II: Feynman Rules

- We have periodicity of $\varphi(\vec{x},\tau)$ in the interval $0 < \tau < \beta$, so decompose "time" into Fourier modes
- As we have seen in many applications, periodic boundary conditions quantize our system, giving $\omega_n = 2\pi n/\beta$ (bosons and ghosts), the Matsubara frequency
- We learned in QFT II how to get Feynman rules through functional methods and we can use the same method at finite temperature by making the following substitutions

$$\int \frac{d^4k}{(2\pi)^4} \rightarrow \frac{i}{\beta}\sum_n \int \frac{d^3k}{(2\pi)^3}$$

$$k_0 \rightarrow i\omega_n$$

$$(2\pi)^4\delta^4(k_1+k_2+\cdots) \rightarrow \frac{1}{i}(2\pi)^3\beta\delta_{\omega_{n_1}+\omega_{n_2}+\cdots} \times \delta^3(\vec{k}_1+\vec{k}_2+\cdots) \quad (3)$$

Effective Potential I

- The finite temperature effective potential is obtained identically to the zero temperature analogue

$$V^\beta(\hat{\varphi}) = -(\text{space-time volume})^{-1}\, \Gamma^\beta(\bar{\varphi})|_{\bar{\varphi}=\hat{\varphi}} \tag{4}$$

- $\bar{\varphi}$ is the average classical field and $\hat{\varphi}$ is the nonzero vacuum expectation value that induces the symmetry breaking
- As we have learned, symmetry breaking occurs when $\partial V^\beta(\hat{\varphi})/\partial\hat{\varphi} = 0$ for nonzero $\hat{\varphi}$
- Now we can break up the potential into zero temperature piece and finite temperature piece

$$V^\beta(\hat{\varphi}) = V^0(\hat{\varphi}) + \bar{V}^\beta(\hat{\varphi}) \tag{5}$$

Effective Potential II: Symmetry Restoration

- For potentials bounded from below, the requirement for symmetry persistence is

$$\frac{\partial V^{\beta}(\hat{\varphi}^2)}{\partial \hat{\varphi}^2}|_{\hat{\varphi}\neq 0} > 0 \tag{6}$$

- A necessary condition for symmetry persistence is now be written

$$\frac{\partial V^{0}(\hat{\varphi}^2)}{\partial \hat{\varphi}^2}|_{\hat{\varphi}=0} + \frac{\partial \bar{V}^{\beta}(\hat{\varphi}^2)}{\partial \hat{\varphi}^2}|_{\hat{\varphi}=0} \geq 0 \tag{7}$$

- The first term in the above equation is the derivative of the classical potential, $m^2/2 < 0$ for our purposes
- This inequality tells us that determining if the symmetry is broken depends on the relative strengths of the $\hat{\varphi}^2$ terms

Effective Potential III: Critical Temperature

- Now we define the critical temperature of symmetry restoration

$$\frac{\partial \bar{V}^{\beta_c}(\hat{\varphi}^2)}{\partial \hat{\varphi}^2}|_{\hat{\varphi}=0} = -\frac{m^2}{2} \tag{8}$$

- The zero temperature piece of the effective potential contains symmetry breaking and thus a negative m^2
- The finite temperature piece contributes positively to m^2
- At critical temperature β_c the two terms balance out and the symmetry breaking is removed [2]

One Loop I: Interacting Scalar Field φ^4

- A precursor to the effective potential is the finite temperature Green's function

$$D_\beta(x-y) = \frac{\text{tr } e^{-\beta H} \mathrm{T}\varphi(x)\varphi(y)}{\text{tr } e^{-\beta H}} \tag{9}$$

- Using the machinery of functional determinants we learned in QFT II we know that we will encounter

$$\log \text{Det } iD_\beta^{-1}(x-y) = (\text{space-time volume}) \text{ tr} \log iD_\beta^{-1}(k) \tag{10}$$

- The tree-level piece of the effective potential is easily computed and temperature independent

$$V^0(\hat{\varphi}^2) = \frac{1}{2}m^2\hat{\varphi}^2 + \frac{\lambda}{4!}\hat{\varphi}^4 \tag{11}$$

One Loop II: Effective Potential

– Now to calculate the one loop temperature dependent effective potential

$$V_1^{\beta}(\hat{\varphi}^2) = \frac{-i}{2}\,\mathrm{tr}\log(i\,D^{-1}(\hat{\varphi};k))$$
$$= \frac{1}{2\beta}\sum_n \int \frac{d^3k}{(2\pi)^3}\log(k^2 - M^2)$$
$$= \frac{1}{2\beta}\sum_n \int \frac{d^3k}{(2\pi)^3}\log(-\omega_n^2 - \omega_k^2) \quad (12)$$

– $M^2 = m^2 + \frac{1}{2}\lambda\hat{\varphi}^2$, ω_n is the Matsubara frequency, $\omega_k = \sqrt{\vec{k}^2 + M^2}$ contributes to the zero point energy of the vacuum

One Loop III: High Temperature Limit

- Computing the sum requires some clever manipulation, see appendix for details, the result is

$$V_1^\beta(\hat{\varphi}^2) = V_1^0(\hat{\varphi}^2) + \bar{V}_1^\beta(\hat{\varphi}^2)$$
$$= \int \frac{d^3k}{(2\pi)^3}\frac{\omega_k}{2} + \frac{1}{\beta}\int \frac{d^3k}{(2\pi)^3}\log(1 - e^{-\beta\omega_k}) \quad (13)$$

- The second term is the free energy of an ideal bose gas!
- Now we approximate the second term by expanding in the limit $\beta \to 0$, the high temperature limit

$$\bar{V}_1^\beta(\hat{\varphi}^2) = -\frac{\pi^2}{90\beta^4} + \frac{M^2}{24\beta^2} - \frac{1}{12\pi}\frac{M^3}{\beta}$$
$$- \frac{1}{64\pi^2}M^4\log M^2\beta^2 + \frac{c}{64\pi^2}M^4 + \mathcal{O}(M^6\beta^2) \quad (14)$$

One Loop IV: Critical Temperature

- Using the definition of β_c defined previously, we calculate the critical temperature for the symmetry restoration

$$\frac{1}{\beta_c^2} = \frac{-12m^2}{\frac{1}{2}\lambda} \tag{15}$$

- This result only depends on the T^2 term in the series
- As a consistency check, take λ to be small and we see that the critical temperature is indeed large
- So we now have a phase transition, but what order is it?

Phase Transition I

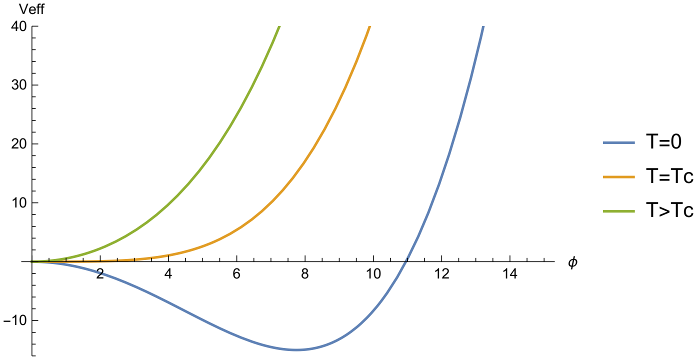


Figure 1: V_{eff} at varying temperatures

– This plot shows a second order phase transition, but the one loop analysis cannot unambiguously tell us what order transition occurs

Phase Transition II

- In the calculation of the one loop effective potential, we may worry about the strength of $-\frac{1}{12\pi}\frac{M^3}{\beta}$
- This term includes $\hat{\varphi}^3$ and if relevant can cause a first order phase transition

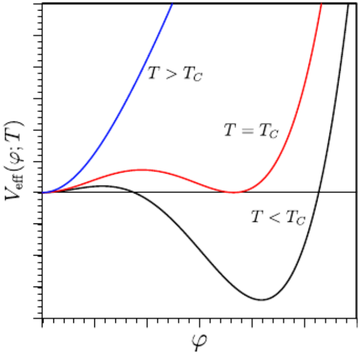
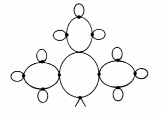


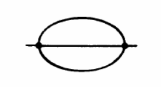
Figure 2: First order phase transition (Senaha)

Phase Transition III

- To determine the order we must resum the series but this also contains ambiguity
- If one only includes a resummation of daisy or superdaisy diagrams, the transition is first order
- If along with daisy/superdaisy one includes sunset diagrams the phase transition is determined to be second order
- Peskin and Schroeder show that φ^4 has second order phase transition at zero temperature so the latter resummation is considered correct [3] [4]



(a) Superdaisy



(b) Sunset

Figure 3: Borrowed from L. Dolan and R. Jackiw [2]

Gauge Theory

– Extension to gauge theory is nontrivial because the partition function is gauge dependent

– Only physical gauges will give meaningful results, for example consider the Feynman gauge

$$\log Z = 3\int \frac{d^3k}{(2\pi)^3}[\frac{-\beta\omega_k}{2} - \log(1 - e^{-\beta\omega_k})] + \int \frac{d^3k}{(2\pi)^3}[\frac{-\beta\omega_k}{2} - \log(1 + e^{-\beta\omega_k})] \quad (16)$$

– There are two extra unphysical states, the longitudinal and timelike photons [1]

– Bernard solves this problem with Fadeev Popov ghosts and defines a gauge-invariant partition function that is equal to tr $e^{-\beta H}$ only in physical gauges

EWPT

- The SM case of symmetry restoration is more complicated because the previously real scalar is now a complex doublet and we have additional interactions with SM particles not included in φ^4
- Following Senaha, the effective potential for the SM case considering first order phase transition is

$$V_{\text{eff}}(\varphi, T) \simeq D(T^2 - T_0^2)\varphi^2 - ET|\varphi|^3 + \frac{\lambda_T}{4}\varphi^4 + \cdots \quad (17)$$

- The cubic term in φ is indicative of a first order PT
- Numerical calculation gives $T_C \simeq 163.4$ GeV

Conclusion

- The difficulty of doing finite temperature studies of phase transitions is that they are generically non-perturbative
- Senaha suggests that lattice calculations are a better approach for numerical results
- Finite temperature field theory has interesting consequences for the SM and cosmology and hopefully this presentation has piqued your interest
- First order phase transitions can create gravitational waves which recently has become a very interesting observable (ask Anthony)

References

[1] Claude W. Bernard. Feynman rules for gauge theories at finite temperature. *Phys. Rev. D*, 9:3312–3320, Jun 1974.

[2] L. Dolan and R. Jackiw. Symmetry behavior at finite temperature. *Phys. Rev. D*, 9:3320–3341, Jun 1974.

[3] Michael E. Peskin and Dan V. Schroeder. *An Introduction To Quantum Field Theory (Frontiers in Physics)*. Westview Press, 1995.

[4] Eibun Senaha. Symmetry restoration and breaking at finite temperature: An introductory review. *Symmetry*, 12(5):733, May 2020.

Appendix: Computing Sum

– Simplest method following Jackiw

$$\nu(E) = \sum_n \log(\frac{4\pi^2 n^2}{\beta^2} + E^2) \tag{18}$$

$$\frac{\partial \nu(E)}{\partial E} = \sum_n \frac{2E}{4\pi^2 n^2/\beta^2 + E^2} \tag{19}$$

$$\sum_{n=1}^{\infty} \frac{y}{y^2 + n^2} = -\frac{1}{2y} + \frac{1}{2}\pi \coth \pi y \tag{20}$$

$$\nu(E) = 2\beta[\frac{E}{2} + \frac{1}{\beta}\log(1 - e^{-\beta E})] + \text{indep. E} \tag{21}$$