Introduction to SU(5) and SO(10) Grand Unification

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Overview

1. Motivation for Grand Unification
2. SM introduction (first generation)
3. SU(5) Unification
4. SO(10) Unification (briefly)

Motivation

- Can the large number of parameters inserted into the standard model be reconciled?
- Is it possible to unify SM forces, account for seemingly arbitrary coupling constants?
- Likewise can the fermions be united?
- Why are the hypercharge values of multiplets the way they are?
- Similarly, why are the electric charges what they are?
- SM incompatible with massive neutrinos

Isospin

- Analogous to spin-$\frac{1}{2}$, described by SU(2)
- Strong isospin states up/down equivalent to spin-$\frac{1}{2}$ projections: $\pm\frac{1}{2}$
- Quite descriptive due to the proximity of the masses of the u,d quarks
- Strong interaction acts on multiplets of the total isospin, e.g. $2 \otimes 2 = 3 \oplus 1$: $\pi^{0,\pm} \oplus \eta$

Hypercharge

- Utility in assigning these particle families a particular number: Y
- Gell-Mann-Nishijima formula: $Q = I_3 + \frac{Y}{2}$

Weak isospin

- Extend idea to leptons, create second doublet: $\begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}$
- Weak isospin mediated by $W^{\pm}, Z^0$
- Observed asymmetry in chirality, only left-handed particles transform non-trivially under SU(2)
- Extend hypercharge as well, Gell-Mann-Nishijima still holds if the weak isospin doublet has $Y = -1$
- Right-handed particles have no weak isospin projections

Gauge bosons, symmetry breaking

- SM gauge group: $SU(3) \otimes SU(2) \otimes U(1)$
- SU(3) gauge bosons: 8 massless gluons
- Unified electroweak gauge bosons (massless): $W^{0,\pm}, B$, with γ, Z^0 being linear combinations of W^0 and B
- $SU(2) \otimes U(1)$ symmetry broken, electromagnetic and weak force behave differently, $W^{\pm}, Z^0$ bosons become massive

SM Summary

- 16 total fermions in $SU(3) \otimes SU(2) \otimes U(1)$
- $\nu_R = (1 \otimes 1)_0$
- $e_R = (1 \otimes 1)_{-2}$
- Left-handed lepton doublet $(1 \otimes 2)_{-1}$
- Right-handed down quark triplet $d_R = (3 \otimes 1)_{-\frac{2}{3}}$
- Right-handed up quark triplet $u_R = (3 \otimes 1)_{\frac{4}{3}}$
- Left-handed quark sextuplet $Q_L = (3 \otimes 2)_{\frac{1}{3}}$
- 16 fermions and 16 anti-fermions: $16 \oplus \bar{16} = 32$

Why SU(5)?

- Sensible to start with a rank 4 group
- SM has 4 commuting generators, $T_{3,8}$ from SU(3), σ_3 from SU(2) and the U(1) generator
- Is it possible to embed the SM into SU(5)?
- By construction, the diagonal generators can be made equivalent to those of the SM

Fitting SU(3)⊗SU(2)⊗U(1) in SU(5)

- Represent matrices in SU(5) as block diagonal, with block elements that are SU(3), SU(2) matrices multiplied by an appropriate U(1) factor
- $$A = \left(\begin{array}{c|c} \alpha^{-2} M & 0 \\ \hline 0 & \alpha^3 N \end{array}\right), \alpha \in U(1), M \in SU(3), N \in SU(2)$$
- kernel of $\phi : (\alpha, M, N) \mapsto A$ is $(\alpha, \alpha^2, \alpha^{-3})$
- Satisfying determinant condition requires that α be a sixth root of unity: $\mathbb{Z}_6$

Fitting SU(3)⊗SU(2)⊗U(1) in SU(5) cont.

- To be able to embed the SM group (G_{SM}) into SU(5), the kernel must act trivially
- With some algebra, this yields constraints on the hypercharge
- e.g. for left-handed quarks, $Y = 2k + \frac{1}{3}$, $k \in \mathbb{Z}$
- Structure introduced to the hypercharges
- Via Gell-Mann-Nishijima, electric charge determined

Where the first generation particles fit

- Examining the structure of that matrix A, its action would be on a vector made up of a doublet and a triplet
- The fundamental $5 = (1 \otimes 2)_1 \oplus (3 \otimes 1)_{-\frac{2}{3}}$
- Utilise $\bar{5} = (1 \otimes 2)_{-1} \oplus (\bar{3} \otimes 1)_{\frac{2}{3}}$, identify with doublet of leptons: $(\nu, e)^T$ and triplet of $\bar{d}$
- The antisymmetric part of the product of two 5s: $5 \otimes_A 5 = 10$ can describe the remainder
- $10 = (1 \otimes 1)_2 \oplus (3 \otimes 2)_{\frac{1}{3}} \oplus (\bar{3} \otimes 1)_{-\frac{4}{3}}$
- Identify $\bar{e}$, quark sextuplet, $\bar{u}$ triplet

Gauge bosons

- $5^2 - 1 = 24$ generators
- $$T^a = \begin{pmatrix} \frac{\lambda^a}{2} & 0 \\ 0 & 0 \end{pmatrix}, T^i = \begin{pmatrix} 0 & 0 \\ 0 & \frac{\sigma^i}{2} \end{pmatrix}$$
- λ^a and σ^i are the Gell-Mann and Pauli matrices respectively.
- for $a = 3, 8$ and $i = 3$, there are 3 diagonal matrices. We expect one more
- $\tilde{Y} = \frac{1}{\sqrt{60}} \text{diag}(-2, -2, -2, 3, 3)$
- $24 = (8 \otimes 1)_0 \oplus (1 \otimes 3)_0 \oplus (1 \otimes 1)_0 \oplus (\bar{3} \otimes 2)_{\frac{5}{6}} \oplus (3 \otimes 2)_{-\frac{5}{6}}$
- Twelve off-diagonal matrices, linear combinations of which give 12 additional bosons
- These X,Y bosons can mediate processes that don't conserve baryon or lepton number

Proton Decay

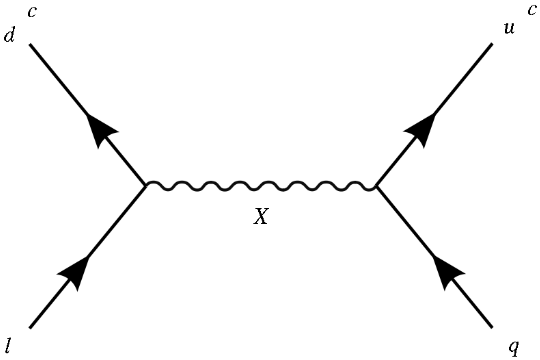
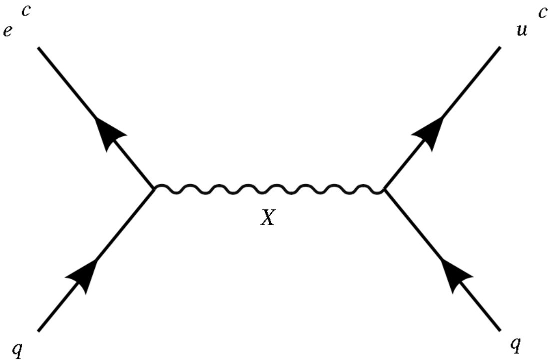


Figure: Retrieved from wikipedia. public domain.

Proton Decay



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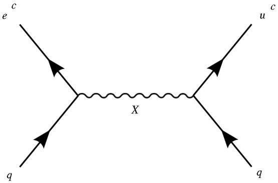
Proton Decay



- $\Gamma(qqq \to q\bar{q}\bar{l} = \pi^0 e^+) \approx \frac{m_p^5}{M_{GUT}^4}$
- Experimental searches for proton decay constrain $M_{GUT} \gtrsim 10^{16} GeV$

Higgs, symmetry breaking, doublet-triplet problem

- For the Higgs, again use fundamental 5 containing a triplet and a doublet.
- Symmetry breaking $SU(5) \to SU(3) \otimes SU(2) \otimes U(1)$
- Doublet energy goes to the weak energy scale (100 GeV)
- Energy of triplet would remain at GUT energy scale, 10^{14} GeV difference: fine tuning problem
- SO(10) encounters same problem

Weinberg angle

- $\sin^2(\theta_W) = \frac{g'^2}{g^2+g'^2}$ where g', g are the couplings for hypercharge and weak isospin respectively
- Denotes the rotation of the basis vectors γ, Z^0 in the W^0, B plane after electroweak symmetry breaking
- Can express it in terms of single SU(5) coupling g, replacing the g in the above equation.
- Inspect the action of $\tilde{Y}$ on the states in the $\bar{5}$ and we can read off $\tilde{Y} = \frac{\sqrt{60}}{6} Y$
- Comparing Y to $\tilde{Y}$ yields $(g/g')^2 = \frac{5}{3}$
- Predicts $\sin^2(\theta_W^{GUT}) = \frac{3}{8}$

SU(5) summary

- A single gauge coupling
- Structure of hypercharge, electric charge
- Energy scale large due to proton stability
- Weinberg angle prediction is motivating but simple scheme untenable
- Not all SM particles embedded into single irrep
- Fine tuning problem present

Pati-Salam model

- Idea: promote right handed singlets to doublets
- Weak isospin now acts like $SU(2) \otimes SU(2)$, with each SU(2) acting on a doublet of different handedness
- Additionally, can think of lepton number as a fourth colour. $SU(3) \rightarrow SU(4)$
- Full model: $SU(4) \otimes SU(2) \otimes SU(2)$

SO(10)

- What can be gained by adding another commuting generator?
- B-L conservation is a naturally motivated choice: gives rise to Pati-Salam
- Can represent SM fermions inside a single irrep: $16 = 10 \oplus \bar{5} \oplus 1$
- All particles inside a single irrep

Breaking

- Can break symmetry in various ways, some examples:
- $SO(10) \to SU(3) \otimes SU(2) \otimes U(1)$
- $SO(10) \to SU(4) \otimes SU(2) \otimes SU(2)$
- $SO(10) \to SU(3) \otimes SU(2) \otimes SU(2) \otimes U(1)$
- $SO(10) \to SU(5) \otimes U(1)$

2^5

- We saw that the 5 of SU(5) was an SU(3) colour triplet and an SU(2) isospin doublet
- With $16 \oplus \bar{16} = 32$ SM fermions (and anti-fermions), can represent every particle in a 5 element vector, whose elements are binary flags for some kind of charge: $2^5 = 32$
- Can think of a binary flag as some SU(2) projection
- $\psi = (r, g, b, up, down)^T$

2^5 representation

ψ	Y	r	g	b	u	d
ν^c	0	0	0	0	0	0
e^c	2	0	0	0	1	1
u_r	1/3	1	0	0	1	0
d_r	1/3	1	0	0	0	1
u_g	1/3	0	1	0	1	0
d_g	1/3	0	1	0	0	1
u_b	1/3	0	0	1	1	0
d_b	1/3	0	0	1	0	1
u_r^c	-4/3	0	1	1	0	0
u_g^c	-4/3	1	0	1	0	0
u_b^c	-4/3	1	1	0	0	0
d_r^c	2/3	0	1	1	1	1
d_g^c	2/3	1	0	1	1	1
d_b^c	2/3	1	1	0	1	1
ν	-1	1	1	1	0	1
e	-1	1	1	1	1	1

Summary

- Various ways to unite leptons and quarks
- Hypercharge seen to be less arbitrary, electric charge follows
- Still a large number of parameters to be inserted
- SU(5): $M_{GUT} \gtrsim 10^{16} GeV$
- SU(5) and SO(10) encounter fine tuning issues
- Extending these theories with SUSY possible, though thoroughly outside the scope here

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