<u>INSTRUCTIONS</u>: This is a closed book exam. You may consult one (twosided) 8 $1/2" \times 11"$ sheet of paper of personal notes. However, you may not collaborate and/or plagiarize anyone else's work. Failure to comply with these rules will result in a zero on this exam.

The use of calculators for the numerical work is strongly encouraged. You have one hour and ten minutes to complete this exam (unless special prior arrangements have been made). The point count of each problem is specified below; use this information to manage your time wisely.

Please remember to write your name on this exam above. Answers to the multiple choice questions should be indicated by circling your chosen answer. This exam also provides blank sheets for all your work on the short problems.

PART I: Multiple choice questions

Only one of the choices given in each question is the correct answer. For each multiple choice question, please circle your chosen answer. No explanation for your choice is required. Each multiple choice problem is worth 5 points.

1. The density of salt water is larger than that of fresh water. A ship initially floats in a fresh water lake. Subsequently, it floats in the salt-water ocean. In which case is the buoyant force greater on the boat?

- (a) in salt water
- (b) in fresh water
- (c) the buoyant force is the same in both cases
- (d) cannot be determined from the information given

2. A water supply maintains a constant volume rate of flow for water in a hose. You want to change the opening of the nozzle so that water leaving the nozzle reaches a height that is four times the current maximum height the water reaches with the nozzle vertical. To do so, you must

- (a) decrease the area of the opening by a factor of 16.
- (b) decrease the area of the opening by by a factor of 4.
- (c) decrease the area of the opening by by a factor of 2.
- (d) increase the area of the opening by by a factor of 2.
- (e) increase the area of the opening by by a factor of 4.

3. A mass on a spring undergoes simple harmonic motion. When the mass is at maximum displacement from its equilibrium position in the positive direction, then which one of the following statements about the mass is true?

- (a) The velocity is nonzero.
- (b) The kinetic energy is at its maximum.
- (c) The acceleration is positive and its magnitude at its maximum.
- (d) The acceleration is negative and its magnitude is at its maximum.
- (e) The acceleration is zero.

4. Consider a damped harmonic oscillator subject to an oscillating external driving force. Suppose we double the magnitude of the driving force. How does this affect the natural resonant frequency of the oscillator?

- (a) The natural resonant frequency is increased by a factor of four.
- (b) The natural resonant frequency is increased by a factor of two.
- (c) The natural resonant frequency is unchanged.
- (d) The natural resonant frequency is reduced by a factor of two.
- (e) The natural resonant frequency is reduced by a factor of four.

5. A pendulum is made by suspending a hoop of radius R and mass m from a pivot point located on the hoop, such that the plane of the hoop lies in the vertical plane, as shown in the figure below. The period of this pendulum is T. If a hoop with the same radius but a mass four times as great is suspended in the same way, what will be its period of oscillation?

- (a) 4T
- (b) 2T
- (c) T
- (d) T/2
- (e) T/4



The hoop is attached at a *fixed* pivot point indicated by the larger solid black dot above, and hangs in the vertical plane. The smaller solid black dot indicates the center of the hoop, and its radius is indicated by the dashed line. The hoop oscillates initially in the direction indicated by the arrow.

6. The speed of transverse waves on a thin wire is 150 m/s. The density of the material that the wire is made of is 5000 kg/m^3 . The wire has a 0.5 mm diameter circular cross-section. What is the tension that the wire is under?

- (a) 76.2 N
- (b) 22.1 N
- (c) $88.4 \,\mathrm{N}$
- (d) $0.147 \,\mathrm{N}$
- (e) $56.7 \,\mathrm{N}$

PART II: Short problems

To earn full credit on the following problems, you must exhibit the steps that lead to your final result (and will depend on the clarity of your method of solution as well as on your final answer). Problems 7 and 8 are worth 20 points each, and problem 9 is worth 30 points.

7. A siphon of uniform cross-sectional area is used to drain water from a tank as shown in the figure below. Assume steady flow without viscosity or friction.

(a) The water emerges at a height h = 1 m below the surface of the water in the tank as shown in the figure. Compute the speed of the outflow (labeled \vec{v} in the figure below) when it emerges from the siphon.

HINT: You may assume that the downward velocity of the surface of the water in the tank is negligible.



(b) In order for the flow to be continuous, the pressure at any point inside the siphon must always be *above* the vapor pressure of water.* Given that the vapor pressure of water at room temperature is 0.023 atm, determine the *maximal* height of the siphon (denoted by y in the figure above) for which the flow of the water is continuous.

HINT: If y is too large, then the pressure inside the siphon at its highest point will be lower than the vapor pressure of the water, in which case the water will start to boil and the flow of water will no longer be continuous!

REMINDER: $1 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2$.

^{*}The vapor pressure of water, which is a function of temperature, is defined to be the value of the pressure at which the liquid water boils. For example, at $T = 100^{\circ}$ C, the vapor pressure of water is 1 atm. At room temperature, $T = 20^{\circ}$ C, the vapor pressure of water is 0.023 atm.

8. A light spring with spring constant k = 90 N/m is attached vertically to a table as shown in figure (a) below.



A 2 g balloon is filled with helium to a volume of 5 m^3 and is then connected to the spring, causing the spring to stretch as shown in figure (b) above. Compute the extension distance L (shown in the figure above) when the balloon is in equilibrium. You may neglect the weight of the spring.

DATA: The density of helium is $\rho_{He} = 0.179 \text{ kg/m}^3$. The density of air is $\rho_{air} = 1.29 \text{ kg/m}^3$. These data are given for standard temperature and pressure (0°C and 1 atm pressure), which you can assume for this problem.

9. A block with a mass of m = 0.5 kg is attached to a spring with a spring constant k = 50 N/m. Let x be the displacement from the equilibrium position. You may neglect the mass of the spring. At equilibrium, the center-of-mass of the block is located at x = 0, as shown in the figure below.



Suppose that at time t = 0, the block has its maximum speed of 20 m/s and is moving to the left.

(a) Write down an equation for the displacement of the center-of-mass of the block as a function of time, x(t). Determine the numerical values of the amplitude and the angular frequency of the oscillation.

(b) Find the minimum time interval required for the center-of-mass of the block to move from its location at t = 0 to a point 1 m to the left of its equilibrium point.

(c) For what values of x is the potential energy three times the kinetic energy?

HINT: Use the conservation of energy to write the square of the velocity v^2 directly in terms of x^2 by comparing the total energy at any point of the oscillation with the total energy at the point of maximal displacement. You can then immediately solve for x (without ever specifying the time t).