Nuclear Physics B272 (1986) 1–76
North-Holland, Amsterdam

HIGGS BOSONS IN SUPERSYMMETRIC MODELS (I)*

John F. GUNION

University of California, Physics Department, Davis, CA 95616, USA

Howard E. HABER

University of California, Physics Department, Santa Cruz, CA 95064 and Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94305, USA

Received 3 June 1985
(Revised 6 August 1985)

We describe the properties of Higgs bosons in a class of supersymmetric theories. We consider models in which the low-energy sector contains two weak complex doublets and perhaps one complex gauge-singlet Higgs field. Supersymmetry is assumed to be either softly or spontaneously broken, thereby imposing a number of restrictions on the Higgs boson parameters. We elucidate the Higgs boson masses and present Feynman rules for their couplings to the gauge bosons, fermions and scalars of the theory. We also present Feynman rules for vertices which are related by supersymmetry to the above couplings. Exact analytic expressions are given in two useful limits – one corresponding to the absence of the gauge-singlet Higgs field and the other corresponding to the absence of a supersymmetric Higgs mass term.

1. Introduction

With the recent discovery of the W and Z gauge bosons [1], the experimental confirmation of the Glashow-Weinberg-Salam [2] (GWS) model of electroweak interactions is nearly complete. The final ingredient which remains to be clarified is the mechanism of electroweak symmetry breaking. In the GWS model, symmetry breaking is triggered by the Higgs mechanism. The main consequence is the appearance of physical elementary scalar fields (the Higgs bosons) in the theory. Unfortunately, the present theory hardly constrains the properties of the Higgs bosons. The fact that $\rho \equiv m_W^2/(m_Z \cos^2\theta_W) \approx 1$ suggests that the low-energy world

* Work supported in part by the Department of Energy, contracts DE-AC03-76SF00515 (SLAC) and DE-AT03-76ER70191 MOD AO26 (UCD) and by National Science Foundation Grant No. PHY8115541 (UCSC).

consists of Higgs bosons which are weak SU(2) doublets and perhaps gauge singlets.★ However the masses of these Higgs bosons and many of their couplings to fermions and scalars are not constrained at all by the theory.

Although the Higgs boson masses are a priori free parameters, it is generally assumed that such masses must be somewhat below 1 TeV. Otherwise, one finds that the Higgs self-couplings become strong and it is no longer appropriate to treat the GWS model as a weak-coupling theory [3]. This observation has led to a number of puzzles (which have been referred to in the literature as the hierarchy [4] and naturalness [5] problems). Basically, it is difficult to understand how an elementary scalar field can be so light ($m_H \lesssim 1$ TeV). The "natural" value for a scalar boson mass is $g\Lambda$, where Λ is the mass scale of some underlying fundamental theory (such as the grand unification mass $M_{GUT} \sim 10^{15}$ GeV or the Planck mass $M_P \sim 10^{19}$ GeV) and g is some coupling strength. In addressing the above problems, various solutions have been proposed. The only solution which keeps the scalar Higgs bosons as elementary fields is supersymmetry [6]. In supersymmetric theories, scalar masses are related by the supersymmetry to fermion masses which can be naturally light due to approximate chiral symmetries. An equivalent but more technical way of saying this is that the unrenormalized theory is free from quadratic divergences.

In supersymmetric models, it is postulated that all known fermions have scalar partners. Unfortunately, it seems impossible to identify some of these states as the Higgs bosons of the GWS model. The reason is that the scalar partners of quarks carry color quantum numbers and the scalar partners of leptons carry lepton number. In order that the theory not spontaneously break color and/or electromagnetism, only the scalar neutrino could acquire a vacuum expectation value. This possibility would lead to lepton number violation in the theory. As shown in ref. [7], one cannot entirely rule out this scenario, although no realistic model exists where a scalar neutrino vacuum expectation value alone is responsible for the electroweak symmetry breaking of the GWS model. One must therefore add Higgs bosons and their fermionic partners in addition to the quark and lepton supersymmetric multiplets.

Supersymmetry imposes a new requirement on the Higgs multiplet structure of the theory. In the standard model, only one Higgs doublet is required to give mass to the quarks and leptons. In the supersymmetric model, *two* Higgs doublets are needed to give mass to both up-type and down-type quarks (and the corresponding leptons) [6, 8]. This requirement arises from a technical property of supersymmetric models. The interaction of Higgs bosons and fermions arises from the superpotential given by:

$$W_F = \varepsilon_{ij} \left[f\hat{H}_1^i \hat{L}^j \hat{R} + f_1 \hat{H}_1^i \hat{Q}^j \hat{D} + f_2 \hat{H}_2^j \hat{Q}^i \hat{U} \right], \tag{1.1}$$

★ It is possible to have $\rho = 1$ either automatically with certain higher Higgs representatives (e.g. $I_W = 3$, $y = 4$, see ref. [44]) or by artificially adjusting the parameters of the model. We shall neglect these alternatives on the basis of simplicity.

where $\hat{H}_1$ and $\hat{H}_2$ are the Higgs superfields, $\hat{Q}$ and $\hat{L}$ are the SU(2) weak-doublet quark and lepton superfields, respectively, $\hat{U}$ and $\hat{D}$ are SU(2)-singlet quark superfields and $\hat{R}$ is an SU(2) weak-singlet charged lepton superfield. (See table 1 for a summary of the quantum numbers of the various fields.) The SU(2) indices i, j are contracted in a gauge invariant way. Supersymmetry forbids the appearance of H_1^* and H_2^* in eq. (1). Because of gauge invariance (in this case, the hypercharge), an H_1QU coupling is prohibited; hence, no up-quark mass can be generated if H_2 is omitted.

Thus, the minimal supersymmetric extension of the GWS model is a two-Higgs doublet model. Furthermore, supersymmetry imposes non-trivial constraints on the Higgs boson sector of the model. Even if we assume that the supersymmetry is spontaneously or softly broken, it must be true that the dimension-four terms of the Higgs potential respect the supersymmetry. The consequences of this observation will be a major focus of this paper.

We propose to study the Higgs sector of the minimal supersymmetric extension of the standard electroweak model. For the sake of generality, we shall admit all possible soft-supersymmetric-breaking terms [9] with arbitrary coefficients, i.e. terms of dimension two or three which do not reintroduce quadratic divergences to the unrenormalized theory. This is in fact a feature of low-energy supergravity models;

TABLE 1

Superfield	Boson fields	Fermionic partners	$SU(2)_W$	y
Gauge multiplets				
$\hat{V}$	V^a	λ^a	triplet	0
$\hat{V}'$	V'	λ'	singlet	0
Matter multiplets				
$\hat{L}$	scalar leptons $\tilde{L} = (\tilde{\nu}, \tilde{e}_L^-)$	$(\nu, e^-)_L$	doublet	-1
$\hat{R}$	scalar leptons $\tilde{R} = \tilde{e}_R^+$	e_L^c	singlet	2
$\hat{Q}$	scalar quarks $\tilde{Q}^j = (\tilde{u}_L, \tilde{d}_L)$	$(u, d)_L$	doublet	$\frac{1}{3}$
$\hat{U}$	scalar quarks $\tilde{U} = \tilde{u}_R^*$	u_L^c	singlet	$-\frac{4}{3}$
$\hat{D}$	scalar quarks $\tilde{D} = \tilde{d}_R^*$	d_L^c	singlet	$\frac{2}{3}$
$\hat{H}_1$	Higgs bosons H_1^i	$(\psi_{H_1}^0, \psi_{H_1}^-)$	doublet	-1
$\hat{H}_2$	Higgs bosons H_2^j	$(\psi_{H_2}^+, \psi_{H_2}^0)$	doublet	1
$\hat{N}$	Higgs bosons N	ψ_N	singlet	0

We list the gauge and matter multiplets of the supersymmetric SU(2) × U(1) model. The charge Q is obtained via $Q = T_3 + \frac{1}{2}y$. The labels are as follows: $a = 1,2,3$ labels the SU(2) triplet of gauge bosons and $i, j = 1,2$ are SU(2) indices. Labels referring to multiple generations of quarks, leptons and their scalar partners are suppressed.

in addition, these models suggest particular values for some of the coefficients of the soft terms introduced.* We shall comment on some of the possible values of these coefficients at the end of this paper.

The use of the term "minimal" above is somewhat ambiguous. In the literature there have been two basic choices. First, one may take a minimal $SU(2) \times U(1)$ model of electroweak interactions with two Higgs doublets and add supersymmetric partners. Unfortunately, the supersymmetric version of this model fails to break the $SU(2) \times U(1)$ gauge symmetry. This is not a problem since by adding appropriate soft-supersymmetry breaking terms, one can arrange for the $SU(2) \times U(1)$ gauge invariance to be spontaneously broken. In the low-energy supergravity models, this scenario occurs as follows. The resulting lagrangian of the model appropriate at the Planck scale M_P has the supersymmetry softly broken and the $SU(2) \times U(1)$ gauge invariance unbroken. When the renormalization group equations are used to evolve down from M_P to energies of order m_W, at least one of the SU(2) weak-doublet Higgs fields acquires a negative mass-squared, indicating that $SU(2) \times U(1)$ has spontaneously broken [8,12–13]. All lagrangians we write down in this paper are appropriate to the energy scale of order m_W.

A second approach is to add a complex scalar field which is an $SU(2) \times U(1)$ gauge singlet to two-Higgs doublet model [14–17]. One can now write down a supersymmetric version of this model where the $SU(2) \times U(1)$ gauge symmetry is spontaneously broken. Although this model has an extra field, it is in some ways simpler than the model described previously. In low-energy supergravity models based on this picture, the $SU(2) \times U(1)$ is already broken at tree level [15–16]. Of course, one must check that the evolution down to scales of order m_W does not upset this picture.

The plan of this paper is as follows. In sect. 2, we discuss the GWS model with two-Higgs doublets in generality (with no particular reference to supersymmetry). In sect. 3 we construct the most general Higgs sector in a softly-broken supersymmetric $SU(2) \times U(1)$ model with two Higgs doublets and one Higgs singlet. Our parameters are chosen so that the $SU(2) \times U(1)$ spontaneously breaks to $U(1)_{EM}$. We then make a few assumptions regarding the parameters of the model. This will allow us to obtain analytic expressions for the masses of all the physical Higgs bosons and their interactions. In sect. 4 we derive the Feynman rules for the interaction of the Higgs bosons with all particles of the supersymmetric spectrum. For completeness, we derive the couplings of the higgsinos to quarks and scalar-quarks in sect. 5. Although these interactions do not explicitly involve the Higgs bosons, they are supersymmetric analogs to some of the Higgs boson couplings discussed in this paper. This will require some careful discussion regarding the mixing of gauginos and higgsinos which we include for completeness in appendix A. The Feynman rules

* For a review of the low-energy supergravity approach and a complete set of references, see refs. [10] and [11].

presented in this paper provide a useful supplement to the rules given in the appendix of ref. [18]. These rules have been obtained assuming one generation of quarks and leptons. Extensions to the case of more than one generation are discussed in appendix B. In sect. 6 we discuss the parameters of the Higgs potential in the context of currently fashionable models of "low energy" supergravity. Some final comments appear in sect. 7. We shall apply the results of this paper to interesting physical processes in a follow-up paper [19].

2. Two-Higgs doublet models – generalities

First, we shall discuss some general properties of the Higgs doublet models [20–22]. We shall then apply the results to the supersymmetric case in the next section as well as allowing for the possible addition of an $SU(2)_W \times U(1)$ gauge singlet scalar field.

Consider two complex $y=1$, $SU(2)_W$ doublet scalar fields, ϕ_1 and ϕ_2. The Higgs potential which spontaneously breaks $SU(2)\times U(1)$ down to $U(1)_{EM}$ can be written in the following form* [20]:

$$
\begin{aligned}
V(\phi_1,\phi_2) = {} & \lambda_1\left(\phi_1^\dagger\phi_1 - v_1^2\right)^2 + \lambda_2\left(\phi_2^\dagger\phi_2 - v_2^2\right)^2 \\
& +\lambda_3\left[\left(\phi_1^\dagger\phi_1 - v_1^2\right) + \left(\phi_2^\dagger\phi_2 - v_2^2\right)\right]^2 \\
& +\lambda_4\left[\left(\phi_1^\dagger\phi_1\right)\left(\phi_2^\dagger\phi_2\right) - \left(\phi_1^\dagger\phi_2\right)\left(\phi_2^\dagger\phi_1\right)\right] \\
& +\lambda_5\left[\mathrm{Re}\left(\phi_1^\dagger\phi_2\right) - v_1v_2\cos\xi\right]^2 \\
& +\lambda_6\left[\mathrm{Im}\left(\phi_1^\dagger\phi_2\right) - v_1v_2\sin\xi\right]^2 + \lambda_7 .
\end{aligned}
\tag{2.1}
$$

A few comments should be useful here. First, by hermiticity the λ_i are all real parameters. Second, λ_7 appears for convenience only; in practice, all constant terms in eq. (2.1) can be dropped. However, when we discuss the supersymmetric case, it is convenient to choose λ_7 such that the minimum of the potential is $V=0$ in the supersymmetric limit. Third, if the $\lambda_i \geqslant 0$, then the minimum of the potential is manifestly

$$
\langle\phi_1\rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \qquad \langle\phi_2\rangle = \begin{pmatrix} v_2 e^{i\xi} \\ 0 \end{pmatrix}, \tag{2.2}
$$

thus breaking $SU(2)_W \times U(1)$ down to $U(1)_{EM}$ as desired. In fact, the allowed range of the λ_i corresponding to this desired minimum is somewhat larger. It can be easily determined by working out the mass spectrum of the physical Higgs bosons and demanding that all the squared masses be non-negative.

* This potential is the most general one subject to two constraints: (a) gauge invariance, and (b) the discrete symmetry $\phi_i \to -\phi_i$ is violated only softly (here, it is violated by dimension-two terms). The latter constraint is a technical one, which is related to insuring that flavor changing neutral currents are not too large [20]. It is automatically satisfied in the supersymmetric models we study here.

In the next section, we will see that supersymmetry imposes the condition $\lambda_5 = \lambda_6$ on eq. (2.1). In this case, we may redefine ϕ_2 via $\phi_2 \to e^{i\xi}\phi_2$ and remove the phase ξ from the potential. As a result, the vacuum expectation values of ϕ_1 and ϕ_2 can be chosen to be *real and positive*.

Therefore, in this section we will not consider the most general potential as given in eq. (2.1). Instead we will derive all our results assuming that $\xi = 0$ (although we will take $\lambda_5 \neq \lambda_6$). This, in fact, corresponds to the most general *CP*-invariant two-Higgs doublet model.

Our major task is to compute the Higgs boson mass matrix. This is most easily done in a real basis where:

$$\phi_1 \equiv \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \to \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix},$$

$$\phi_2 \equiv \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix} = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix} \to \begin{pmatrix} \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \end{pmatrix}. \tag{2.3}$$

The method is described in the appendix of ref. [21]. Here we provide the results and correct a few minor errors in ref. [21]. First, one rewrites eq. (2.1) (with $\xi = 0$) in terms of the ϕ_i $(i = 1, \dots, 8)$. The Higgs-boson squared mass matrix is obtained from:

$$M_{ij}^2 = \frac{1}{2} \left. \frac{\partial^2 V}{\partial\phi_i\, \partial\phi_j} \right|_{\text{minimum}}, \tag{2.4}$$

where "minimum" means setting $\langle\phi_3\rangle = v_1$, $\langle\phi_7\rangle = v_2$ and $\langle\phi_k\rangle = 0$ for all other k. Note that the factor of $\frac{1}{2}$ is needed in eq. (2.4) because of the normalization of the scalar fields as defined in eq. (2.3). When $\xi = 0$ in eq. (2.1), the scalar boson squared mass matrix separates into a series of 2×2 mass matrices. Diagonalization is straightforward and we summarize the results below.

2.1. INDICES 1, 2, 5, AND 6

These are the charged Higgs bosons. The positive and negative states decouple and have equal mass-squared matrices:

$$\lambda_4 \begin{pmatrix} v_1^2 & -v_1 v_2 \\ -v_1 v_2 & v_2^2 \end{pmatrix}. \tag{2.5}$$

Diagonalizing the charged Higgs-boson mass-squared matrices results in two zero-mass Goldstone boson states:

$$G^{\pm} = \phi_1^{\pm}\cos\beta + \phi_2^{\pm}\sin\beta\,, \tag{2.6}$$

where $\phi^- \equiv (\phi^+)^*$, and two massive charged Higgs boson states

$$H^{\pm} = -\phi_1^{\pm}\sin\beta + \phi_2^{\pm}\cos\beta\,, \tag{2.7a}$$

$$m_{H^{\pm}}^2 = \lambda_4\left(v_1^2 + v_2^2\right), \tag{2.7b}$$

where

$$\tan\beta \equiv \frac{v_2}{v_1}\,. \tag{2.8}$$

2.2. INDICES 4 AND 8

The resulting mass-squared matrix is identical to eq. (2.5) with λ_4 replaced by λ_6. Hence we obtain one zero-mass neutral Goldstone boson and one massive neutral

$$G^0 = \sqrt{2}\left(\mathrm{Im}\,\phi_1^0\cos\beta + \mathrm{Im}\,\phi_2^0\sin\beta\right), \tag{2.9a}$$

$$H_3^0 = \sqrt{2}\left(-\mathrm{Im}\,\phi_1^0\sin\beta + \mathrm{Im}\,\phi_2^0\cos\beta\right), \tag{2.9b}$$

$$m_{H_3^0}^2 = \lambda_6\left(v_1^2 + v_2^2\right). \tag{2.9c}$$

The factors of $\sqrt{2}$ are needed in order that these fields have conventional kinetic energy terms.

2.3. INDICES 3 AND 7

The mass-squared matrix is:

$$\begin{pmatrix} 4v_1^2(\lambda_1 + \lambda_3) + v_2^2\lambda_5 & (4\lambda_3 + \lambda_5)v_1v_2 \\ (4\lambda_3 + \lambda_5)v_1v_2 & 4v_2^2(\lambda_2 + \lambda_3) + v_1^2\lambda_5 \end{pmatrix}. \tag{2.10}$$

The physical states are:

$$H_1^0 = \sqrt{2}\left[\left(\mathrm{Re}\,\phi_1^0 - v_1\right)\cos\alpha + \left(\mathrm{Re}\,\phi_2^0 - v_2\right)\sin\alpha\right],$$

$$H_2^0 = \sqrt{2}\left[-\left(\mathrm{Re}\,\phi_1^0 - v_1\right)\sin\alpha + \left(\mathrm{Re}\,\phi_2^0 - v_2\right)\cos\alpha\right]. \tag{2.11}$$

If we define:

$$A = 4v_1^2(\lambda_1 + \lambda_3) + v_2^2\lambda_5\,,$$

$$B = (4\lambda_3 + \lambda_5)v_1v_2\,,$$

$$C = 4v_2^2(\lambda_2 + \lambda_3) + v_1^2\lambda_5\,, \tag{2.12}$$

then the masses and mixing angles are defined as:

$$m^2_{H_1^0,\,H_2^0} = \tfrac{1}{2}\left[A + C \pm \sqrt{(A-C)^2 + 4B^2}\,\right], \tag{2.13a}$$

$$\sin 2\alpha = \frac{2B}{\sqrt{(A-C)^2 + 4B^2}}\,, \tag{2.13b}$$

$$\cos 2\alpha = \frac{A-C}{\sqrt{(A-C)^2 + 4B^2}}\,. \tag{2.13c}$$

In eq. (2.13a) the mass of H_1^0 (H_2^0) corresponds to the plus (minus) sign, respectively.

To get the Feynman rules for the interactions of the Higgs bosons, we employ the unitary gauge. This consists of setting the Goldstone fields $G^{\pm}$ and G^0 to zero. In this gauge,

$$\phi_1^+ = -H^+ \sin\beta\,, \tag{2.14a}$$

$$\phi_2^+ = H^+ \cos\beta\,, \tag{2.14b}$$

$$\phi_1^0 = v_1 + \sqrt{\tfrac{1}{2}}\left(H_1^0\cos\alpha - H_2^0\sin\alpha - iH_3^0\sin\beta\right), \tag{2.14c}$$

$$\phi_2^0 = v_2 + \sqrt{\tfrac{1}{2}}\left(H_1^0\sin\alpha + H_2^0\cos\alpha + iH_3^0\cos\beta\right). \tag{2.14d}$$

By inserting the expressions given by eq. (2.14) into the interaction lagrangian, one obtains the desired interactions of the physical Higgs bosons. Since CP is conserved (for $\xi = 0$), one finds (by analyzing the $H_i^0 q\bar{q}$ couplings) that H_1^0 and H_2^0 are scalars and H_3^0 is a pseudoscalar.

3. The Higgs sector in a minimal supersymmetric model

We now turn to the implications of supersymmetry for the properties of the Higgs bosons [8, 23]. We shall analyze a "minimal" supersymmetric extension of the Standard Model consisting of two Higgs doublets and perhaps one SU(2) × U(1) singlet Higgs field. A list of the fields in our model, which also defines our notation, is provided in table 1. Details of this model can be found in the appendices of ref. [18].*

In order to use the results of sect. 2, we must be careful in our notation. In supersymmetric models, one employs two Higgs-doublet fields of opposite hypercharge: H_1 with $y=-1$ and H_2 with $y=1$. The relations between these fields and the ϕ_i of sect. 2 are:

$$(\phi_1)^j = \varepsilon_{ij} H_1^{i*},$$

$$(\phi_2)^j = H_2^j, \tag{3.1}$$

where i, j are SU(2) indices and $\varepsilon_{12} = -\varepsilon_{21} = 1$, $\varepsilon_{11} = \varepsilon_{22} = 0$. That is,

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} \phi_1^{0*} \\ -\phi_1^- \end{pmatrix}, \tag{3.2}$$

where $\phi_1^- \equiv (\phi_1^+)^*$ and the asterisk indicates complex conjugation.

As described in the introduction, we propose to analyze the most general Higgs potential corresponding to a softly broken supersymmetric theory. To obtain this potential, we first consider the superpotential of an unbroken supersymmetric theory made up of the fields listed in table 1. The most general superpotential (which conserves baryon number and lepton number) is:

$$W = h\varepsilon_{ij} H_1^i H_2^j N + \mu\varepsilon_{ij} H_1^i H_2^j - rN + \tfrac{1}{2}MN^2 + \tfrac{1}{3}\Lambda N^3 + W_{\mathrm{F}}, \tag{3.3}$$

where

$$W_F = \varepsilon_{ij}\left[f H_1^i \tilde{L}^j \tilde{R} + f_1 H_1^i \tilde{Q}^j \tilde{D} + f_2 H_2^j \tilde{Q}^i \tilde{U} \right], \tag{3.4}$$

where we have replaced the superfields by their component scalar field, the definitions of the scalar fields are provided in table 1. The scalar potential is computed by [24]

$$V = \tfrac{1}{2}\left[D^a D^a + (D')^2 \right] + F_i^* F_i, \tag{3.5}$$

* Our notation follows that of ref. [18] with the following exceptions: (i) what we call r here [eq. (3.3)] is called $-s$ there; (ii) what we call v_i here [eq. (3.7)] is called $\sqrt{\frac{1}{2}}\, v_i$ there; (iii) what we call $\tan\beta$ here [eq. (2.8)] is called $\cot\theta_v$ there; and (iv) the Higgs-boson-quark-Yukawa couplings are denoted by f_i here.

where

$$F_i = \frac{\partial W}{\partial A_i}, \tag{3.6a}$$

$$D^a = \tfrac{1}{2} g A_i^* \sigma_{ij}^a A_j, \tag{3.6b}$$

$$D' = \tfrac{1}{2} g' y_i A_i^* A_i + \xi. \tag{3.6c}$$

In the above expressions, A_i collectively denotes all scalar fields appearing in the theory. We shall henceforth assume that the Fayet-Iliopoulos term [25] ξ in eq. (3.6c) is negligible.

We have described above how to calculate the scalar potential in the supersymmetric model. We now add all possible explicit soft-supersymmetry breaking terms to the model. The allowable terms have been derived in ref. [9]; the relevant terms for the scalar potential fall into two classes. The first class consists of all possible dimension-two terms consistent with gauge invariance. The second class consists of those gauge invariant dimension-three terms which do not mix the scalar fields with their complex conjugates. These terms correspond in form precisely to the cubic terms of the superpotential W [eqs. (3.3), (3.4)] plus their hermitian conjugates.

The resulting scalar potential is the one we shall analyze. We make the following assumptions about this potential. First, the Higgs doublet fields H_1 and H_2 acquire vacuum expectation values:

$$\langle H_1 \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \qquad \langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}. \tag{3.7}$$

By appropriate choice of phases for the Higgs fields, v_1 and v_2 are real and non-negative. Second, we assume that the scalar-quark and scalar-lepton fields do not acquire vacuum expectation values. We then can ignore W_F in eqs. (3.3) and (3.4) when studying the Higgs-boson mass matrix. Third, note that we can make a shift in the N field such that the parameter M in eq. (3.3) disappears. We will simply set $M = 0$ with no loss of generality. The scalar potential as a function of H_1, H_2 and N can then be written as:

$$\begin{aligned} V = {} & \tfrac{1}{8} g^2 \left[4 |H_1^{i*} H_2^i|^2 - 2 (H_1^{i*} H_1^i)(H_2^{j*} H_2^j) + (H_1^{i*} H_1^i)^2 + (H_2^{i*} H_2^i)^2 \right] \\ & + \tfrac{1}{8} g'^2 (H_2^{i*} H_2^i - H_1^{i*} H_1^i)^2 + | h H_1^i H_2^j \varepsilon_{ij} - r + \Lambda N^2 |^2 \\ & + |h|^2 (H_1^{i*} H_1^i + H_2^{i*} H_2^i) N^* N + |\mu|^2 (H_1^{i*} H_1^i + H_2^{i*} H_2^i) \\ & + (H_1^{i*} H_1^i + H_2^{i*} H_2^i) (\mu^* h N + \text{h.c.}) + V_{\text{soft}}, \end{aligned} \tag{3.8}$$

$$\begin{aligned} V_{\text{soft}} = {} & m_1^2 (H_1^{i*} H_1^i) + m_2^2 (H_2^{i*} H_2^i) - (m_{12}^2 \varepsilon_{ij} H_1^i H_2^j + \text{h.c.}) + m_4^2 N^* N \\ & + (m_5^2 N^2 + \text{h.c.}) + m_3 (\varepsilon_{ij} h A_1 H_1^i H_2^j N + \tfrac{1}{3} \Lambda A_2 N^3 + \text{h.c.}). \end{aligned} \tag{3.9}$$

The parameters m_i and m_{12} have dimensions of mass, r has dimensions of mass-squared and A_1, A_2 are dimensionless. We will study the terms involving scalar-quark and scalar-lepton fields in sect. 4.

We proceed to compute the spectrum of physical Higgs bosons and their masses. In the most general case [eqs. (3.8), (3.9)], numerical methods are required to obtain some of the physical Higgs masses and eigenstates. We are interested in certain special cases where the Higgs masses and eigenstates can be computed analytically.

Case 1: $\mu = \langle N \rangle = A_l = 0$. In this case, there is no mixing between N and the doublet Higgs fields. Consequently, we may use all the results of sect. 2. The required translation is:

$$\phi_1^\dagger \phi_1 = H_1^{i*} H_1^i, \tag{3.10a}$$

$$\phi_2^\dagger \phi_2 = H_2^{i*} H_2^i, \tag{3.10b}$$

$$\phi_1^\dagger \phi_2 = \varepsilon_{ij} H_1^i H_2^j. \tag{3.10c}$$

Finally, a useful relation is:

$$|H_1^{i*} H_2^i|^2 + |\varepsilon_{ij} H_1^i H_2^j|^2 = \left(H_1^{i*} H_1^i\right)\left(H_2^{j*} H_2^j\right). \tag{3.11}$$

We then find:

$$\begin{aligned} V = {} & m_1^2 H_1^{i*} H_1^i + m_2^2 H_2^{i*} H_2^i - \left[\left(m_{12}^2 + hr^*\right)\varepsilon_{ij} H_1^i H_2^j + \text{h.c.}\right] \\ & + \tfrac{1}{8}\left(g^2 + g'^2\right)\left[\left(H_1^{i*} H_1^i\right)^2 + \left(H_2^{i*} H_2^i\right)^2\right] \\ & + \tfrac{1}{4}\left(g^2 - g'^2\right)\left(H_1^{i*} H_1^i\right)\left(H_2^{j*} H_2^j\right) \\ & + \left(|h|^2 - \tfrac{1}{2} g^2\right)|\varepsilon_{ij} H_1^i H_2^j|^2 + |r|^2, \end{aligned} \tag{3.12}$$

where we have ignored terms involving N. We have retained the constant term $|r|^2$ for later convenience. Note in particular that no term of the form

$$\left(\varepsilon_{ij} H_1^i H_2^j\right)^2 + \text{h.c.} \tag{3.13}$$

appears above. This implies that $\lambda_5 = \lambda_6$ in eq. (2.1). Therefore, within the pure H_1, H_2 sector of the theory, we may absorb the phase of $m_{12}^2 + hr^*$ into the definition of H_2 and set $\xi = 0$ in eq. (2.1). We emphasize that the same logic allows us to choose v_1 and v_2 to be non-negative. Henceforth, we shall take the parameters m_{12}^2, h and r to be real. Note, however, that with the conventions above, *CP*-violating phases may reappear in the interaction of H_1 and H_2 with other fields in the theory.

Comparing eq. (3.12) to eq. (2.1) (with $\xi = 0$) and using eqs. (3.10), we obtain the following results:

$$\lambda_2 = \lambda_1 , \tag{3.14a}$$

$$\lambda_3 = \tfrac{1}{8}\left(g^2 + g'^2\right) - \lambda_1 , \tag{3.14b}$$

$$\lambda_4 = 2\lambda_1 - \tfrac{1}{2}g'^2 , \tag{3.14c}$$

$$\lambda_5 = \lambda_6 = h^2 - \tfrac{1}{2}\left(g^2 + g'^2\right) + 2\lambda_1 , \tag{3.14d}$$

$$\lambda_7 = r^2 - h^2 v_1^2 v_2^2 - \tfrac{1}{8}\left(v_1^2 - v_2^2\right)^2 \left(g^2 + g'^2\right) , \tag{3.14e}$$

$$m_1^2 = 2\lambda_1 v_2^2 - \tfrac{1}{2}m_Z^2 , \tag{3.14f}$$

$$m_2^2 = 2\lambda_1 v_1^2 - \tfrac{1}{2}m_Z^2 , \tag{3.14g}$$

$$m_{12}^2 = h\left(v_1 v_2 h - r\right) - \tfrac{1}{2}v_1 v_2\left(g^2 + g'^2 - 4\lambda_1\right) , \tag{3.14h}$$

where the Z^0 mass is given by $m_Z^2 = \frac{1}{2}(v_1^2 + v_2^2)(g^2 + g'^2)$. These results indicate that supersymmetry imposes strong constraints on the Higgs-doublet model of sect. 2.

As a check, let us consider the supersymmetric limit by setting $V_{\rm soft} = 0$ in eq. (3.8) (i.e. $m_1^2 = m_2^2 = m_{12}^2 = 0$). We then find from eqs. (3.14f), (3.14g) and (3.14h) that

$$v_1 = v_2 , \tag{3.15a}$$

$$\lambda_1 = \tfrac{1}{4}\left(g^2 + g'^2\right) , \tag{3.15b}$$

$$r = v_1 v_2 h . \tag{3.15c}$$

Inserting these values into eq. (3.14e) gives $\lambda_7 = 0$, i.e. the value of the potential at the supersymmetric minimum is zero.

Using eqs. (3.14a)–(3.14h) and the results of sect. 2, we may immediately obtain the spectrum of physical Higgs particles. The results are:

$$m_{H^\pm}^2 = \tfrac{1}{2}\left(4\lambda_1 - g'^2\right)\left(v_1^2 + v_2^2\right) , \tag{3.16}$$

$$m_{H_3^0}^2 = m_{H^\pm}^2 - m_W^2 + h^2\left(v_1^2 + v_2^2\right) , \tag{3.17}$$

$$m_{H_1^0, H_2^0}^2 = \tfrac{1}{2}\left[m_{H_3^0}^2 + m_Z^2 \pm \sqrt{\left(m_{H_3^0}^2 + m_Z^2\right)^2 - 4m_Z^2 m_{H_3^0}^2 \cos^2 2\beta - 32 h^2 v_1^2 v_2^2 \lambda_1}\,\right] , \tag{3.18}$$

$$\tan 2\alpha = \tan 2\beta \left(\frac{m_{H_1^0}^2 + m_{H_2^0}^2 - 2h^2\left(v_1^2 + v_2^2\right)}{m_{H_3^0}^2 - m_Z^2} \right) , \tag{3.19}$$

where $H^{\pm}$ are the charged Higgs fields, H_i^0 $(i=1,2,3)$ are the neutral Higgs fields, $\tan\beta \equiv v_2/v_1$ and α is the mixing angle which leads to the H_1^0, H_2^0 eigenstates. As usual, $m_W^2 = \frac{1}{2}g^2(v_1^2+v_2^2)$ and $m_Z^2 = \frac{1}{2}(g^2+g'^2)(v_1^2+v_2^2)$.

The results of eqs. (3.17)–(3.19) have been obtained in refs. [8] and [23] in the case of $h=0$. In that case, we see that one of the neutral Higgs scalars must have mass less than or equal to m_Z and that the charged Higgs scalar must be heavier than m_W. Neither of these two conditions needs to be true for $h \neq 0$. Note that even when $h \neq 0$, the mass relation:

$$m_{H_1^0}^2 + m_{H_2^0}^2 = m_{H_3^0}^2 + m_Z^2 \tag{3.20}$$

still holds. The supersymmetric limit is also of interest. In this limit, the complex N scalar consists of two degenerate states of mass $m_N^2 = h^2(v_1^2+v_2^2)$. In addition, eqs. (3.15b) and (3.16) imply that $m_{H^{\pm}} = m_W$ from which it follows that $m_{H_3^0} = m_{H_1^0} = m_N$ and $m_{H_2^0} = m_Z$. This result was expected. In the supersymmetric limit, the $H^{\pm}$ become the scalar superpartners of the $W^{\pm}$ (along with some appropriate combination of the gauginos and higgsinos) and one scalar field, H_2^0, becomes the scalar superpartner of the Z^0 [14]. The remaining neutral Higgs fields are degenerate and live in their own chiral superfield along with the appropriate higgsino.

Case 2: $\mu \neq 0$, N field not present. This case corresponds to taking $h=m_3=m_4=m_5=r=\Lambda=0$ in eqs. (3.8) and (3.9). Again, the results of sect. 2 are applicable. In this case, eqs. (3.14d–h) are replaced by

$$\lambda_5 = \lambda_6 = 2\lambda_1 - \tfrac{1}{2}(g^2+g'^2), \tag{3.21a}$$

$$\lambda_7 = -\tfrac{1}{8}(v_1^2-v_2^2)^2(g^2+g'^2), \tag{3.21b}$$

$$m_1^2 = -|\mu|^2 + 2\lambda_1 v_2^2 - \tfrac{1}{2}m_Z^2, \tag{3.21c}$$

$$m_2^2 = -|\mu|^2 + 2\lambda_1 v_1^2 - \tfrac{1}{2}m_Z^2, \tag{3.21d}$$

$$m_{12}^2 = \tfrac{1}{2}v_1 v_2(4\lambda_1 - g^2 - g'^2), \tag{3.21e}$$

whereas eqs. (3.14a–c) remain unchanged. The masses of the physical Higgs bosons and the mixing angle α are given by eqs. (3.16)–(3.19) with $h=0$. We may obtain a useful expression for the mass of H_3^0 as follows. Using eqs. (3.21c–e), we find

$$m_1^2 + m_2^2 + 2|\mu|^2 = m_{12}^2(\tan\beta + \cot\beta), \tag{3.22}$$

$$\lambda_1 = \tfrac{1}{4}(g^2+g'^2) + \frac{m_{12}^2}{2v_1v_2}. \tag{3.23}$$

Using eqs. (2.9c) and (3.21a), we end up with

$$m^2_{H^0_3} = m_1^2 + m_2^2 + 2|\mu|^2 . \tag{3.24}$$

We have already noted that we may choose v_1 and v_2 both non-negative, which implies (by our convention) that $0 \leqslant \beta \leqslant \frac{1}{2}\pi$. Furthermore, if we use eq. (2.13) (which by our definition implies that $m_{H^0_1} \geqslant m_{H^0_2}$), it follows that $\sin 2\alpha \leqslant 0$ for this case; so we may take $-\frac{1}{2}\pi \leqslant \alpha \leqslant 0$. One interesting limit is $v_1 = v_2$; in this case, $\beta = -\alpha = \frac{1}{4}\pi$, and $m_{H^0_2} = 0$ (at tree level). Useful formulas for $\sin(\alpha \pm \beta)$ and $\cos(\alpha \pm \beta)$ in terms of the neutral Higgs boson masses (these factors often appear in the Feynman rules, see sects. 4 and 5) may be found in ref. [19].

The supersymmetric limit consists of setting $m_1 = m_2 = m_{12} = 0$. However, in this limit, eqs. (3.21c–e) are inconsistent (under the assumption that $\mu \neq 0$ and a nonvanishing vacuum expectation value). The reason for the problem here is simply that the potential V [eqs. (3.8) an (3.9)] with $h = m_3 = m_4 = m_5 = r = \Lambda = 0$ does not spontaneously break $SU(2) \times U(1)$ (i.e. $v_1 = v_2 = 0$). Thus, in a supersymmetric model with only two Higgs doublets but with no singlet Higgs fields, soft-supersymmetry breaking terms are required in order to (spontaneously) break the $SU(2) \times U(1)$ gauge symmetry.

Case 3: $\mu \neq 0$ or $A_1 \neq 0$, N field present. This is the general case where the potential is given by eqs. (3.8) and (3.9). We shall simply indicate some of the resulting complexities.

First, let us assume that $\langle N \rangle = 0$. This depends on the values of the parameters r, Λ, m_4^2 and m_5^2 which are relevant in determining the mass matrix of the two states $\mathrm{Re}\, N$ and $\mathrm{Im}\, N$. The term

$$\left(H_1^{i*}H_1^i + H_2^{i*}H_2^i\right)\left(\mu^* hN + \text{h.c.}\right) + hA_1 m_3 \varepsilon_{ij} H_1^i H_2^j N + \text{h.c.} \tag{3.25}$$

leads to mixing of the complex N scalar with all three physical Higgs scalars $H_i^0 (i = 1, 2, 3)$. This would require a 5×5 neutral Higgs boson mass matrix. Note that this implies CP-violation in the Higgs sector which has entered due to the complex couplings of N with the other scalar fields. If we impose CP-conservation on the Higgs parameters, then some simplification occurs: namely, $\mathrm{Re}\, N$ mixes with H_1^0 and H_2^0 and $\mathrm{Im}\, N$ mixes with H_3^0 as can be seen from eq. 3.25. If we now allow for $\langle N \rangle \neq 0$, no new complexities arise.

For the remainder of this paper, we shall concentrate on cases 1 and 2, described above. There are a number of reasons for this choice. First, we believe that it is useful to have analytic expressions for the Higgs-boson masses and eigenstates. Second, we think that the approximations used in obtaining those expressions are sensible. In models without a singlet Higgs field (case 2), our results are completely general. In models (e.g. case 1) with the singlet field N, we have the convenience of a minimal supersymmetric extension of the standard model in which $SU(2) \times U(1)$ is spontaneously broken at the tree level.

4. Feynman rules for Higgs-boson interactions

In this section we compute the Higgs-boson interactions under the assumptions stated in cases 1 and 2, described in sect. 3. The upshot of those assumptions is that if a singlet field is present, it does not mix with the neutral weak-doublet Higgs fields. This allows us to use eqs. (2.14a–d); in terms of the notation of sect. 3, we obtain:

$$H_2^1 = H^+\cos\beta\,, \tag{4.1a}$$

$$H_1^2 = H^-\sin\beta\,, \tag{4.1b}$$

$$H_1^1 = v_1 + \sqrt{\tfrac{1}{2}}\left(H_1^0\cos\alpha - H_2^0\sin\alpha + iH_3^0\sin\beta\right), \tag{4.1c}$$

$$H_2^2 = v_2 + \sqrt{\tfrac{1}{2}}\left(H_1^0\sin\alpha + H_2^0\cos\alpha + iH_3^0\cos\beta\right), \tag{4.1d}$$

where $\tan\beta = v_2/v_1$ and α is given by eq. (3.19). As discussed previously, we may choose our phases such that v_1 and v_2 are real and non-negative; hence $0 \leqslant \beta \leqslant \frac{1}{2}\pi$.

In supersymmetric models, the Higgs bosons interact with gauge bosons, quarks, leptons, other Higgs bosons and their supersymmetric partners. We shall describe

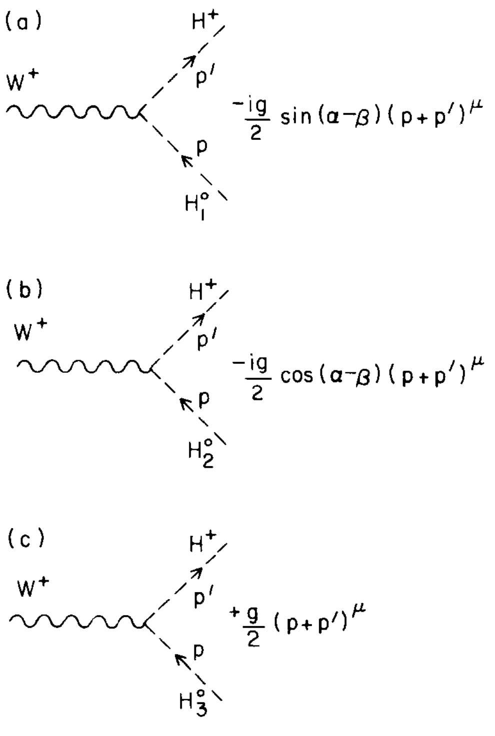


Fig. 1. Feynman rules for $W^+H^+H_i^0$ vertices. The direction of momentum is indicated above.

each of these interactions in turn. We rely heavily here on the appendices of ref. [18] where much of the interaction lagrangian for a supersymmetric extension of the standard model has been discussed in great detail.

4.1. INTERACTION WITH GAUGE BOSONS

One starts with an interaction lagrangian consisting of HHV and HHVV terms (H = Higgs boson, V = vector gauge boson). For example, the interaction with the

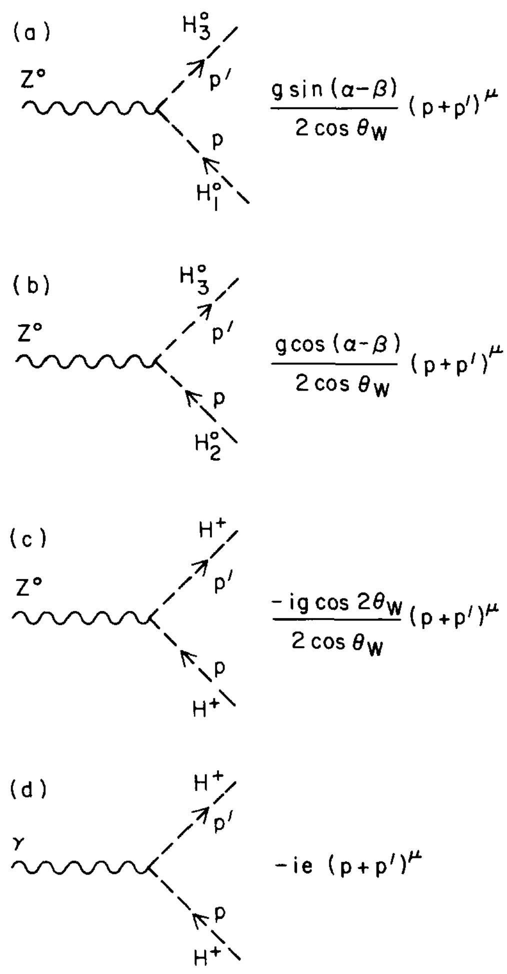


Fig. 2. Feynman rules for $(Z^0,\gamma)H^+H^-$ and $Z^0H_i^0H_j^0$. Note that Bose symmetry forbids $i=j$. In addition, CP-invariance forbids a $Z^0H_1^0H_2^0$ vertex.

photon field A_μ is

$$\mathscr{L}_{\text{int}} = ieA_\mu\left(H_1^{2*} \overleftrightarrow{\partial}^\mu H_1^2 - H_2^{1*} \overleftrightarrow{\partial}^\mu H_2^1 \right) + e^2 A_\mu A^\mu \left(|H_1^2|^2 + |H_2^1|^2 \right). \qquad (4.2)$$

We also need the interaction with the Z^0 and $W^\pm$ gauge bosons. The required expression is given by eq. (C.98) and (C.99) of ref. [18]. One merely has to substitute for H_1^i and H_2^i as given by eq. (4.1). We simply quote the result

$$\mathscr{L}_{\text{int}} = \mathscr{L}_{\text{HHV}} + \mathscr{L}_{\text{HVV}} + \mathscr{L}_{\text{HHVV}}\,, \qquad (4.3)$$

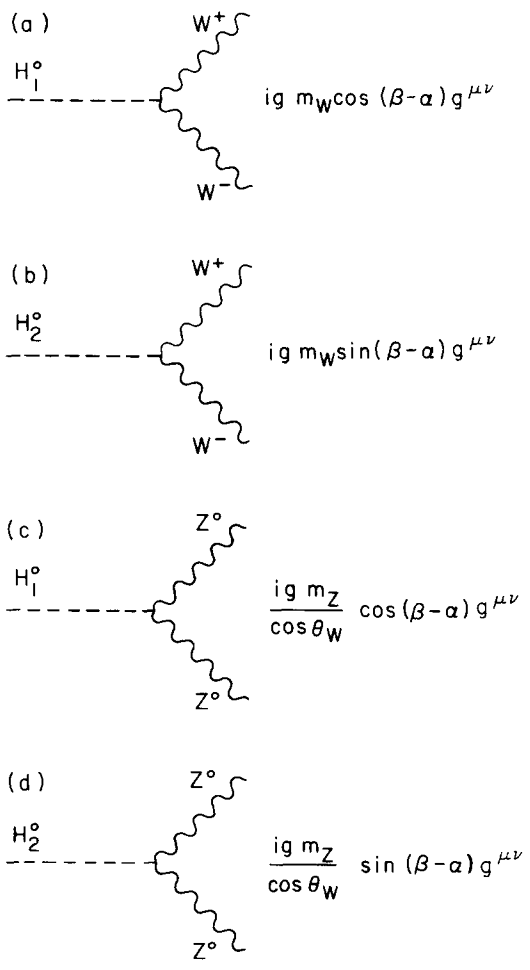


Fig. 3. Feynman rules for $H_i^0 W^+ W^-$ and $H_i^0 Z^0 Z^0$ vertices ($i = 1, 2$). All other possible trilinear HVV vertices vanish at tree level.

where

$$\mathscr{L}_{\mathrm{HHV}} = -\tfrac{1}{2} i g W_\mu^+ H^- \overleftrightarrow{\partial}^\mu \left[H_1^0 \sin(\alpha-\beta) + H_2^0 \cos(\alpha-\beta) + i H_3^0 \right] + \text{h.c.}$$

$$- \frac{ig}{2\cos\theta_{\mathrm{W}}} Z_\mu \left\{ i H_3^0 \overleftrightarrow{\partial}^\mu \left[H_1^0 \sin(\alpha-\beta) + H_2^0 \cos(\alpha-\beta) \right] \right.$$

$$\left. - \left(2\sin^2\theta_{\mathrm{W}} - 1\right) H^- \overleftrightarrow{\partial}^\mu H^+ \right\} - ie A_\mu H^- \overleftrightarrow{\partial}^\mu H^+ , \quad (4.4)$$

$$\mathscr{L}_{\mathrm{HVV}} = g m_{\mathrm{W}} W_\mu W^\mu \left[H_1^0 \cos(\beta-\alpha) + H_2^0 \sin(\beta-\alpha) \right]$$

$$+ \frac{g m_Z}{2\cos\theta_{\mathrm{W}}} Z_\mu Z^\mu \left[H_1^0 \cos(\beta-\alpha) + H_2^0 \sin(\beta-\alpha) \right], \quad (4.5)$$

$$\mathscr{L}_{\mathrm{HHVV}} = \tfrac{1}{4} g^2 W_\mu W^\mu \left[\left(H_1^0\right)^2 + \left(H_2^0\right)^2 + \left(H_3^0\right)^2 + 2 H^+ H^- \right]$$

$$+ \frac{g^2}{8\cos^2\theta_{\mathrm{W}}} Z_\mu Z^\mu \left[\left(H_1^0\right)^2 + \left(H_2^0\right)^2 + \left(H_3^0\right)^2 + 2\cos^2 2\theta_{\mathrm{W}} H^+ H^- \right]$$

$$+ e^2 A_\mu A^\mu H^+ H^- + \frac{eg\cos 2\theta_{\mathrm{W}}}{\cos\theta_{\mathrm{W}}} A_\mu Z^\mu H^+ H^-$$

$$- \tfrac{1}{2} g \left(e A^\mu - \frac{g\sin^2\theta_{\mathrm{W}}}{\cos\theta_{\mathrm{W}}} Z^\mu \right)$$

$$\times \left\{ W_\mu^+ H^- \left[H_1^0 \sin(\beta-\alpha) - H_2^0 \cos(\beta-\alpha) - i H_3^0 \right] + \text{h.c.} \right\} . \quad (4.6)$$

Note that $W_\mu W^\mu \equiv W_\mu^+ W^{-\mu}$. These results have been previously obtained in a nonsupersymmetric two-Higgs doublet model in ref. [26]. Except for a difference in sign convention for the coupling constant g, our results are in agreement. [We choose $\partial_\mu + igW_\mu^a T^a$ for our covariant derivative.] The relevant Feynman rules are given in figs. 1–6. We emphasize a few features. First, note the presence of $ZH_3^0H_1^0$ and $ZH_3^0H_2^0$ couplings; whereas, CP-invariance forbids a $ZH_1^0H_2^0$ vertex. The Higgs sector is effectively CP conserving (more on this later); as we shall see in the next subsection, H_3^0 is a CP-odd scalar and H_1^0 and H_2^0 are CP-even. (Bose statistics forbid a ZH^0H^0 vertex.) Second, there is no tree-level W^+ZH^- vertex; this is a general feature of two-Higgs doublet models [27].

Finally, note that there are no couplings of the field N to vector bosons for the obvious reason that N is an $\mathrm{SU(2)} \times \mathrm{U(1)}$ gauge singlet.

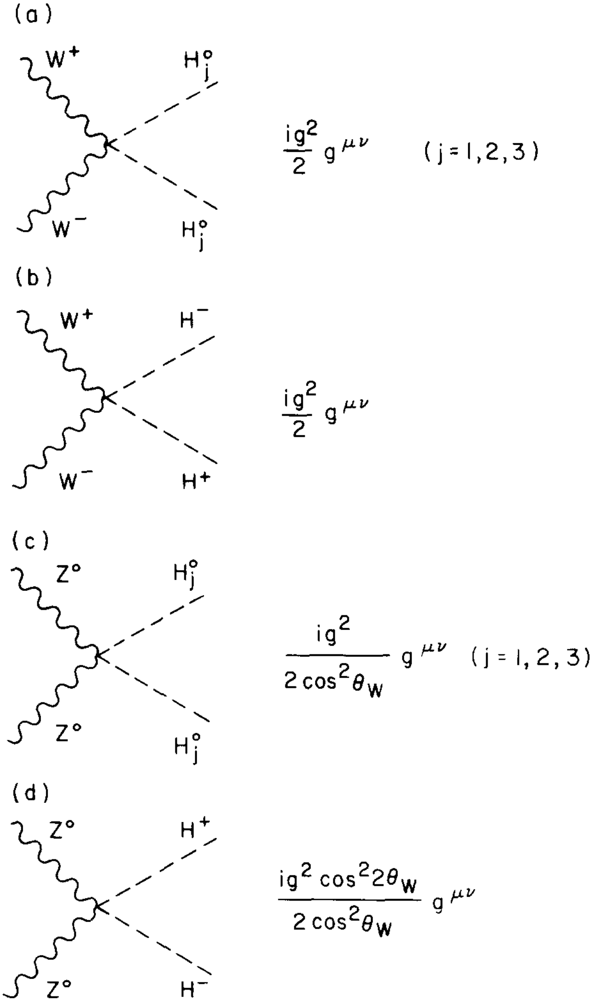


Fig. 4. Feynman rules for four-point Higgs boson-gauge boson couplings (I).

4.2. INTERACTION WITH QUARKS AND LEPTONS

The Higgs-quark-quark coupling is conveniently written down, using two-component spinors for the quarks, as follows:

$$\mathscr{L}_{\text{int}} = -f_1\left[\psi_{Q_2}\psi_D H_1^1 - \psi_{Q_1}\psi_D H_1^2\right] - f_2\left[\psi_{Q_1}\psi_U H_2^2 - \psi_{Q_2}\psi_U H_2^1\right] + \text{h.c.} \tag{4.7}$$

The four-component quark spinors are defined by:

$$u = \begin{pmatrix} \psi_{Q_1} \\ \bar{\psi}_U \end{pmatrix}, \qquad d = \begin{pmatrix} \psi_{Q_2} \\ \bar{\psi}_D \end{pmatrix}. \tag{4.8}$$

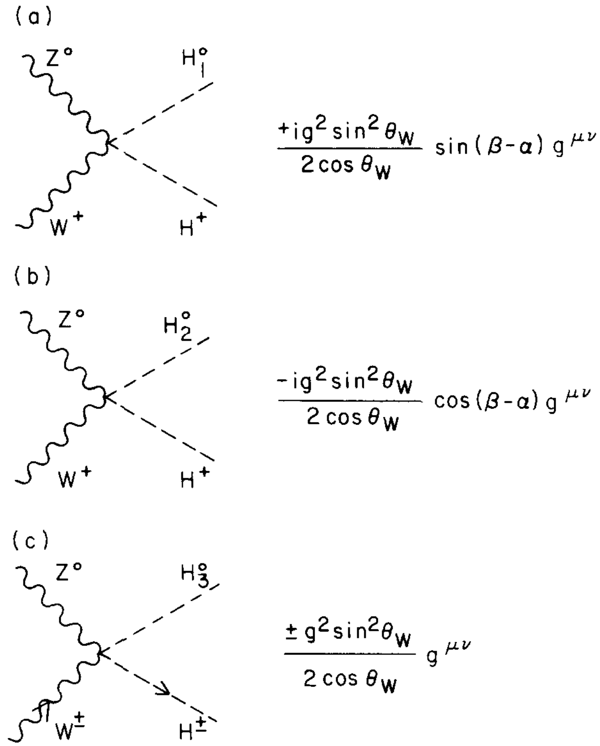


Fig. 5. Feynman rules for four-point Higgs boson-gauge boson couplings (II).

Converting to four-component notation and using eqs. (4.1a–d), we first identify the quark masses which arise due to vacuum expectation values of the Higgs fields:

$$f_1 = \frac{g m_{\mathrm{d}}}{\sqrt{2}\, m_{\mathrm{W}} \cos\beta}\,, \qquad f_2 = \frac{g m_{\mathrm{u}}}{\sqrt{2}\, m_{\mathrm{W}} \sin\beta}\,. \tag{4.9}$$

Using eq. (4.9), we may compute the trilinear interaction terms:

$$\begin{aligned}
\mathscr{L}_{\mathrm{H}\psi\bar{\psi}} = &-\frac{g m_{\mathrm{u}}}{2 m_{\mathrm{W}} \sin\beta}\left[\bar{u}u\left(H_1^0 \sin\alpha + H_2^0 \cos\alpha\right) - i\bar{u}\gamma_5 u H_3^0 \cos\beta\right] \\
&-\frac{g m_{\mathrm{d}}}{2 m_{\mathrm{W}} \cos\beta}\left[\bar{d}d\left(H_1^0 \cos\alpha - H_2^0 \sin\alpha\right) - i\bar{d}\gamma_5 d H_3^0 \sin\beta\right] \\
&+\frac{g}{2\sqrt{2}\, m_{\mathrm{W}}}\left\{H^+\bar{u}\left[\left(m_{\mathrm{d}}\tan\beta + m_{\mathrm{u}}\cot\beta\right)\right.\right. \\
&\qquad\left.\left. + \left(m_{\mathrm{d}}\tan\beta - m_{\mathrm{u}}\cot\beta\right)\gamma_5\right] + \mathrm{h.c.}\right\}.
\end{aligned} \tag{4.10}$$

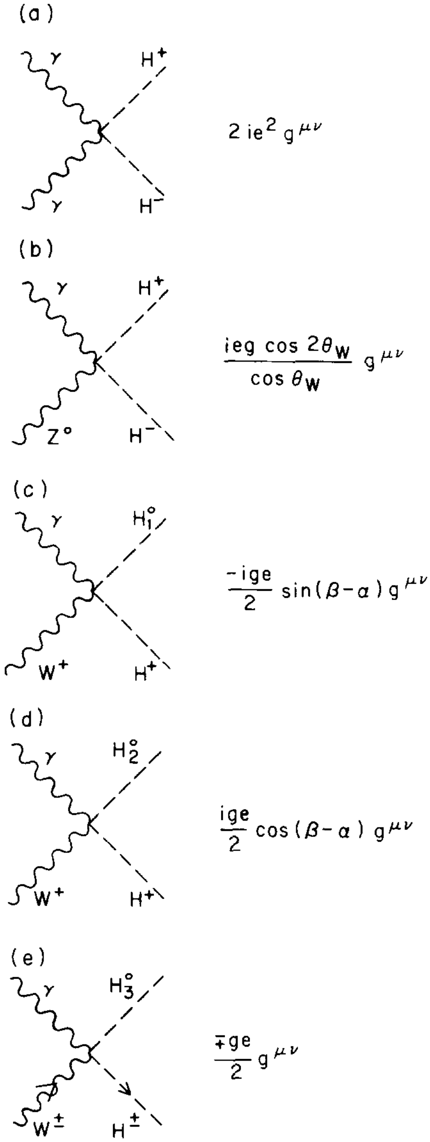


Fig. 6. Feynman rules for four-point Higgs boson-gauge boson couplings (III). Note that in (e) the sign of the rule depends on the direction of the flow of electric charge (as indicated).

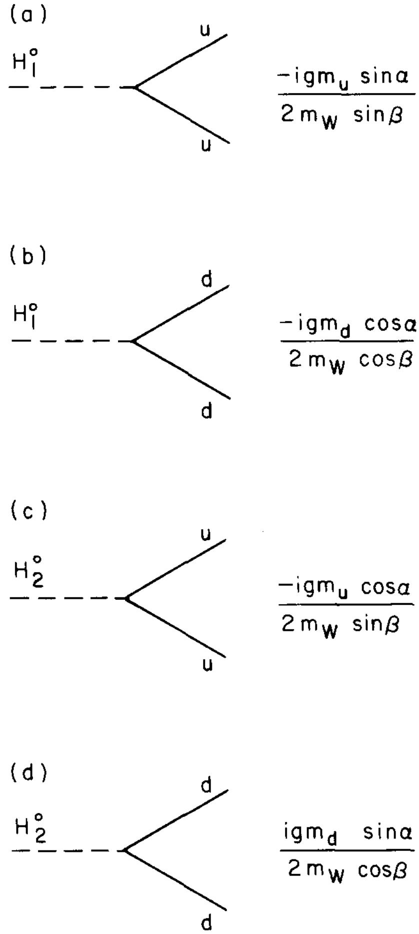


Fig. 7. Feynman rules for $H_i^0\bar{u}u$ and $H_i^0\bar{d}d$, ($i = 1, 2$).

The Feynman rules are displayed in figs. 7 and 8. As we have mentioned before, eq. (4.10) allows us to identify H_1^0 and H_2^0 as CP-even ($J^{PC} = 0^{++}$) and H_3^0 as CP-odd ($J^{PC} = 0^{-+}$). Because the Higgs sector is effectively CP-conserving, the neutral states must separately conserve C and P in their interactions.

Note that there are no couplings of the SU(2) × U(1) gauge singlet scalar field N to quarks. This follows simply from gauge invariance. Otherwise, one would be able to construct gauge-invariant mass terms for the quarks, which is not possible.

The interactions with leptons are easily obtained by replacing (u, d) with (ν, e^-). Note that although we have discussed only one generation of quarks, the extension to the multi-generation case is straightforward (see appendix B). The particular form of eq. (4.7) is a consequence of eq. (1.1) which implies that H_1 alone is responsible

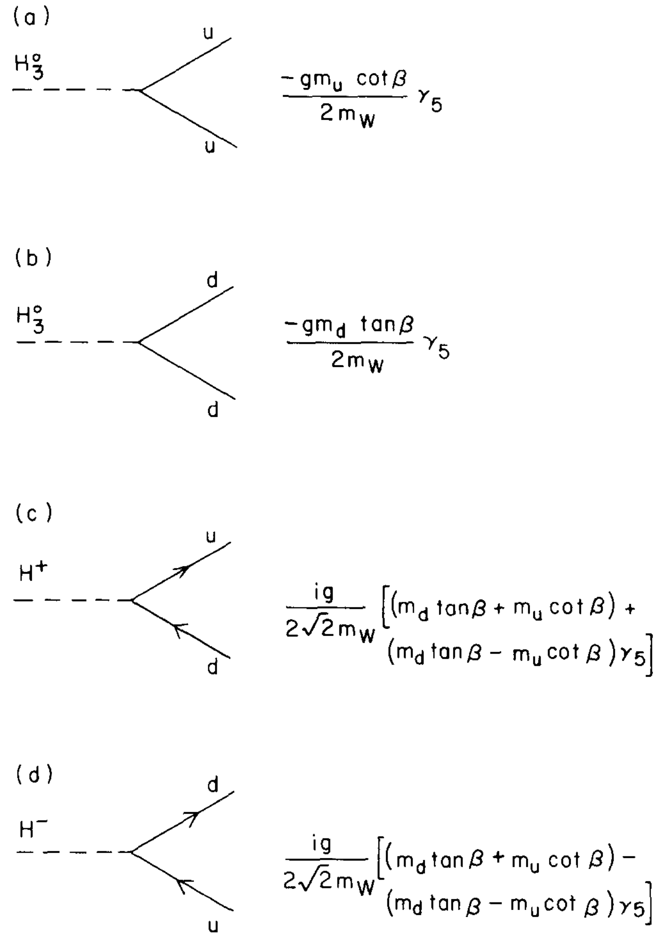


Fig. 8. Feynman rules for $H_3^0 u\bar{u}$, $H_3^0 d\bar{d}$ and H^+ud. In the charged Higgs-boson interactions, all quark mixing angles have been neglected. (See appendix B.)

for the mass of down-type quarks and H_2 alone is responsible for the mass of up-type quarks. General theorems [28] tell us that such models have no flavor changing neutral currents at tree level. In addition, the charged Higgs-quark couplings involve the Kobayashi-Maskawa matrix in the same way as the $W^{\pm}qq'$ couplings. If the neutrinos are massless, no such matrix is required in the lepton sector. Henceforth, we will ignore the presence of other quark and lepton generations for the sake of simplicity.

4.3. SELF-COUPLING OF THE HIGGS BOSONS

It is a straightforward, although tedious task to insert eqs. (4.1a–d) into eq. (3.12) to obtain the desired interaction terms. The trilinear pieces are of the most interest since if the masses are appropriate, then the decay of one Higgs boson into two other Higgs bosons is allowed. In a model with no Higgs-singlet field, the end result is

$$\begin{aligned}\mathscr{L}_{\mathrm{HHH}} = &-g m_{\mathrm{W}} H^+ H^- \left[H_1^0 \cos(\beta-\alpha) + H_2^0 \sin(\beta-\alpha) \right] \\ &- \frac{g m_{\mathrm{Z}}}{4\cos\theta_{\mathrm{W}}} \left[H_1^0 \cos(\beta+\alpha) + H_2^0 \sin(\beta+\alpha) \right] \\ &\times \Big\{ \cos 2\alpha \left[(H_1^0)^2 - (H_2^0)^2 \right] - 2 H_1^0 H_2^0 \sin 2\alpha \\ &\qquad - \left[(H_3^0)^2 + 2 H^+ H^- \right] \cos 2\beta \Big\} . \qquad (4.11)\end{aligned}$$

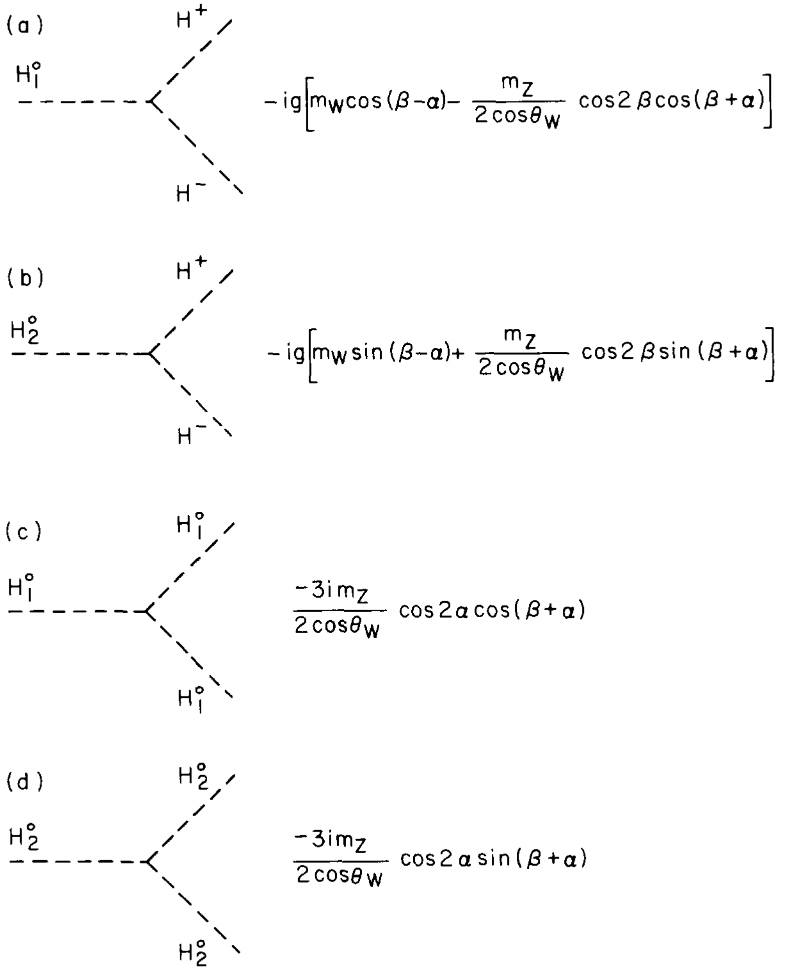


Fig. 9. Feynman rules for $H^+H^-H_i^0$ and $[H_i^0]^3$ vertices ($i = 1,2$). *CP*-invariance forbids $i = 3$.

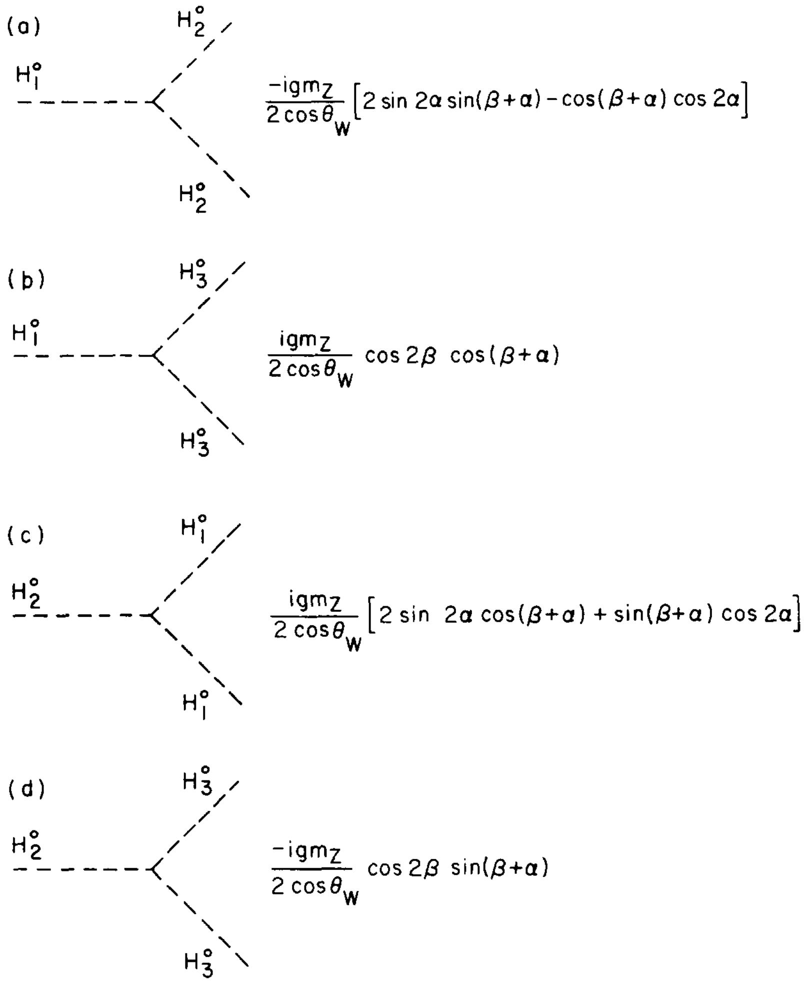


Fig. 10. Feynman rules for $H_i^0 H_j^0 H_j^0$ vertices ($i \neq j$). *CP*-invariance forbids vertices where H_3^0 occurs singly.

The Feynman rules are displayed in figs. 9 and 10. Note that the restrictions of supersymmetry have led to a very simple form for $\mathscr{L}_{\mathrm{HHH}}$. Expressions for three-Higgs couplings in a general (nonsupersymmetric) two-Higgs doublet model are notoriously complicated as illustrated in the last two papers of ref. [26].

There are also three-Higgs vertices involving the N field. If we consider case 1 of sect. 3, the only vertices involved are of the form $N_i N_j H_k^0$ or $N_i N_j N_k$, where N_1 and N_2 are the mass-eigenstates obtained by diagonalizing the $(\mathrm{Re}\,N, \mathrm{Im}\,N)$ mass matrix. These interactions are easily obtained from eqs. (3.8) and (3.9) by inserting the expressions given by eq. (4.1) and picking out the trilinear terms. The exact terms obtained depend on the unknown N mass matrix, so we will not dwell on them. The quartic Higgs couplings are of lesser interest and will be omitted here.

4.4. INTERACTION WITH SCALAR-QUARKS AND SCALAR-LEPTONS

We begin with a discussion of the scalar-quark and scalar-lepton sector of the theory. In eqs. (3.8) and (3.9), we omitted the scalar-quark and scalar-lepton fields. These terms arise from three sources. First, there are the F-terms [see eq. (3.5) and eq. (3.6a)] due to the presence of W_F [eq. (3.4)] in the superpotential. Second, there are the D-terms [see eq. (3.5) and eq. (3.6b, c)]. Finally, we must add the most general set of soft supersymmetry breaking terms to the scalar potential. We write

$$V = V_F + V_D + V_{\rm soft}, \tag{4.12}$$

where

$$\begin{aligned} V_F = {} & \left(h^* H_1^{i*} N^* + \mu^* H_1^{i*} + f_2 \tilde{Q}^{i*} \tilde{U}^*\right)\left(h H_1^i N + \mu H_1^i + f_2 \tilde{Q}^i \tilde{U}\right) \\ & + \left(h^* H_2^{i*} N^* + \mu^* H_2^{i*} + f_1 \tilde{Q}^{i*} \tilde{D}^*\right)\left(h H_2^i N + \mu H_2^i + f_1 \tilde{Q}^i \tilde{D}\right) \\ & + f_1^2 |\varepsilon_{ij} H_1^i Q^j|^2 + f_2^2 |\varepsilon_{ij} H_2^i Q^j|^2 \\ & + \left(f_1 H_1^{i*} \tilde{D}^* - f_2 H_2^{i*} \tilde{U}^*\right)\left(f_1 H_1^i \tilde{D} - f_2 H_2^i \tilde{U}\right), \end{aligned} \tag{4.13}$$

$$\begin{aligned} V_D = {} & \tfrac{1}{8} g^2 \left\{4|H_1^{i*} \tilde{Q}^i|^2 + 4|H_2^{i*} \tilde{Q}^i|^2 - 2(\tilde{Q}^{i*} \tilde{Q}^i)\left[H_1^{i*} H_1^i + H_2^{i*} H_2^i\right] + \left(\tilde{Q}^{i*} \tilde{Q}^i\right)^2\right\} \\ & + \tfrac{1}{8} g'^2 \left[H_2^{i*} H_2^i - H_1^{i*} H_1^i + y_q \tilde{Q}^{i*} \tilde{Q}^i + y_u \tilde{U}^* \tilde{U} + y_d \tilde{D}^* \tilde{D}\right]^2, \end{aligned} \tag{4.14}$$

$$\begin{aligned} V_{\rm soft} = {} & \tilde{M}_Q^2 \tilde{Q}^{i*} \tilde{Q}^i + \tilde{M}_u^2 \tilde{U}^* \tilde{U} + \tilde{M}_d^2 \tilde{D}^* \tilde{D} \\ & + m_6\left(\varepsilon^{ij} f_1 A_d H_1^i \tilde{Q}^j \tilde{D} - \varepsilon^{ij} f_2 A_u H_2^i \tilde{Q}^j \tilde{U} + \text{h.c.}\right), \end{aligned} \tag{4.15}$$

where $y_{\rm q} = \frac{1}{3}$, $y_{\rm u} = -\frac{4}{3}$, $y_{\rm d} = \frac{2}{3}$. We have omitted the terms involving scalar-leptons; they are easy to obtain from the above expressions with appropriate choice of the hypercharges. Presumably, the mass terms in $V_{\rm soft}$ are responsible for making the scalar quarks sufficiently heavy such that they would not have been observed to date.

However, contributions to the masses of the scalar-quarks also arise from other terms. First the supersymmetric piece of the scalar-quark masses arises from V_F when the Higgs bosons acquire vacuum expectation values. Mass terms may also arise in a similar way from V_D and $V_{\rm soft}$. To compute them, insert eqs. (4.1a–d) in eqs. (4.13)–(4.15) [and use eq. (4.9)]. We shall henceforth use more conventional notation for the scalar-quarks.

$$\tilde{Q}^i = \begin{pmatrix} \tilde{u}_{\rm L} \\ \tilde{d}_{\rm L} \end{pmatrix}, \qquad \tilde{U}^* = \tilde{u}_{\rm R}, \qquad \tilde{D}^* = \tilde{d}_{\rm R}. \tag{4.16}$$

Notice the complex conjugation in eq. (4.16). This has been inserted so that the electric charges of $\tilde{u}_{\rm L}$ and $\tilde{u}_{\rm R}$ are equal to $e_{\rm u} \equiv +\frac{2}{3}$; similarly the electric charge of $\tilde{d}_{\rm L}$ and $\tilde{d}_{\rm R}$ is given by $e_{\rm d} \equiv -\frac{1}{3}$. We find for the scalar-quark mass terms:

$$\begin{aligned} -\mathscr{L}_{\rm m} = {} & \tilde{u}_{\rm L}^* \tilde{u}_{\rm L} \left[\tilde{M}_Q^2 + m_{\rm Z}^2 \cos(2\beta) \left(\tfrac{1}{2} - e_{\rm u} \sin^2\theta_{\rm W} \right) + m_{\rm u}^2 \right] \\ & + \tilde{u}_{\rm R}^* \tilde{u}_{\rm R} \left[\tilde{M}_U^2 + m_{\rm Z}^2 \cos(2\beta) e_{\rm u} \sin^2\theta_{\rm W} + m_{\rm u}^2 \right] \\ & + \tilde{d}_{\rm L}^* \tilde{d}_{\rm L} \left[\tilde{M}_Q^2 - m_{\rm Z}^2 \cos(2\beta) \left(\tfrac{1}{2} + e_{\rm d} \sin^2\theta_{\rm W} \right) + m_{\rm d}^2 \right] \\ & + \tilde{d}_{\rm R}^* \tilde{d}_{\rm R} \left[\tilde{M}_D^2 + m_{\rm Z}^2 \cos(2\beta) e_{\rm d} \sin^2\theta_{\rm W} + m_{\rm d}^2 \right] \\ & + \left(\tilde{d}_{\rm R}^* \tilde{d}_{\rm L} + \tilde{d}_{\rm L}^* \tilde{d}_{\rm R} \right) m_{\rm d} \left(A_{\rm d} m_6 + \mu \tan\beta \right) \\ & + \left(\tilde{u}_{\rm R}^* \tilde{u}_{\rm L} + \tilde{u}_{\rm L}^* \tilde{u}_{\rm R} \right) m_{\rm u} \left(A_{\rm u} m_6 + \mu \cot\beta \right). \end{aligned} \tag{4.17}$$

Thus, in general, the scalar-quark eigenstates are

$$\tilde{q}_1 = \tilde{q}_{\rm L} \cos\theta_{\rm q} + \tilde{q}_{\rm R} \sin\theta_{\rm q}, \tag{4.18a}$$

$$\tilde{q}_2 = -\tilde{q}_{\rm L} \sin\theta_{\rm q} + \tilde{q}_{\rm R} \cos\theta_{\rm q}. \tag{4.18b}$$

One needs to diagonalize a 2×2 mass matrix. General formulas can be found in ref. [29]; see also eqs. (C.2)–(C.4) of ref. [18].

The interaction terms $\mathscr{L}_{\mathrm{H\tilde{q}\tilde{q}}}$ can be found by using the familiar procedure. It is convenient to express the results in the $\tilde{q}_{\mathrm{L}} - \tilde{q}_{\mathrm{R}}$ basis

$$\begin{aligned}
\mathscr{L}_{\mathrm{H\tilde{q}\tilde{q}}} = & \frac{g}{\sqrt{2}\, m_{\mathrm{W}}}\left(m_{\mathrm{d}}^2 \tan\beta + m_{\mathrm{u}}^2 \cot\beta - m_{\mathrm{W}}^2 \sin 2\beta\right)\left(H^+ \tilde{u}_{\mathrm{L}}^* \tilde{d}_{\mathrm{L}} + \mathrm{h.c.}\right) \\
& + \frac{g m_{\mathrm{u}} m_{\mathrm{d}} (\cot\beta + \tan\beta)}{\sqrt{2}\, m_{\mathrm{W}}}\left(H^+ \tilde{u}_{\mathrm{R}}^* \tilde{d}_{\mathrm{R}} + \mathrm{h.c.}\right) \\
& + \frac{g m_{\mathrm{d}}}{\sqrt{2}\, m_{\mathrm{W}}}\left(m_6 A_{\mathrm{d}} \tan\beta - \mu\right)\left(H^+ \tilde{u}_{\mathrm{L}}^* \tilde{d}_{\mathrm{R}} + \mathrm{h.c.}\right) \\
& + \frac{g m_{\mathrm{u}}}{\sqrt{2}\, m_{\mathrm{W}}}\left(m_6 A_{\mathrm{u}} \cot\beta - \mu\right)\left(H^+ \tilde{u}_{\mathrm{R}}^* \tilde{d}_{\mathrm{L}} + \mathrm{h.c.}\right) \\
& - \frac{g m_{\mathrm{Z}}}{\cos\theta_{\mathrm{W}}} \sum_i \left[\left(T_{3i} - e_i \sin^2\theta_{\mathrm{W}}\right) \tilde{q}_{i\mathrm{L}}^* \tilde{q}_{i\mathrm{L}} + e_i \sin^2\theta_{\mathrm{W}} \tilde{q}_{i\mathrm{R}}^* \tilde{q}_{i\mathrm{R}}\right] \\
& \quad \times \left[H_1^0 \cos(\alpha + \beta) - H_2^0 \sin(\alpha + \beta)\right] \\
& - \frac{g m_{\mathrm{d}}^2}{m_{\mathrm{W}} \cos\beta}\left(\tilde{d}_{\mathrm{L}}^* \tilde{d}_{\mathrm{L}} + \tilde{d}_{\mathrm{R}}^* \tilde{d}_{\mathrm{R}}\right)\left(H_1^0 \cos\alpha - H_2^0 \sin\alpha\right) \\
& - \frac{g m_{\mathrm{u}}^2}{m_{\mathrm{W}} \sin\beta}\left(\tilde{u}_{\mathrm{L}}^* \tilde{u}_{\mathrm{L}} + \tilde{u}_{\mathrm{R}}^* \tilde{u}_{\mathrm{R}}\right)\left(H_1^0 \sin\alpha + H_2^0 \cos\alpha\right) \\
& - \frac{g m_{\mathrm{d}}}{2 m_{\mathrm{W}} \cos\beta}\left(\tilde{d}_{\mathrm{R}}^* \tilde{d}_{\mathrm{L}} + \tilde{d}_{\mathrm{L}}^* \tilde{d}_{\mathrm{R}}\right) \\
& \quad \times \left[(\mu \sin\alpha + m_6 A_{\mathrm{d}} \cos\alpha) H_1^0 + (\mu \cos\alpha - m_6 A_{\mathrm{d}} \sin\alpha) H_2^0\right] \\
& - \frac{g m_{\mathrm{u}}}{2 m_{\mathrm{W}} \sin\beta}\left(\tilde{u}_{\mathrm{R}}^* \tilde{u}_{\mathrm{L}} + \tilde{u}_{\mathrm{L}}^* \tilde{u}_{\mathrm{R}}\right) \\
& \quad \times \left[(\mu \cos\alpha + m_6 A_{\mathrm{u}} \sin\alpha) H_1^0 + (-\mu \sin\alpha + m_6 A_{\mathrm{u}} \cos\alpha) H_2^0\right] \\
& - \frac{i g m_{\mathrm{d}}}{2 m_{\mathrm{W}}}\left(m_6 A_{\mathrm{d}} \tan\beta - \mu\right)\left(\tilde{d}_{\mathrm{R}}^* \tilde{d}_{\mathrm{L}} - \tilde{d}_{\mathrm{L}}^* \tilde{d}_{\mathrm{R}}\right) H_3^0 \\
& - \frac{i g m_{\mathrm{u}}}{2 m_{\mathrm{W}}}\left(m_6 A_{\mathrm{u}} \cot\beta - \mu\right)\left(\tilde{u}_{\mathrm{R}}^* \tilde{u}_{\mathrm{L}} - \tilde{u}_{\mathrm{L}}^* \tilde{u}_{\mathrm{R}}\right) H_3^0 \\
& - h\left[m_{\mathrm{u}} \cot\beta N \tilde{u}_{\mathrm{L}}^* \tilde{u}_{\mathrm{R}} + m_{\mathrm{d}} \tan\beta N \tilde{d}_{\mathrm{L}}^* \tilde{d}_{\mathrm{R}} + \mathrm{h.c.}\right]. \qquad (4.19)
\end{aligned}$$

TABLE 2

	$\tilde{u}_L^*\tilde{d}_L$	$\tilde{u}_R^*\tilde{d}_R$	$\tilde{u}_L^*\tilde{d}_R$	$\tilde{u}_R^*\tilde{d}_L$
$\tilde{u}_1^*\tilde{d}_1$	$\cos\theta_u\cos\theta_d$	$\sin\theta_u\sin\theta_d$	$\cos\theta_u\sin\theta_d$	$\sin\theta_u\cos\theta_d$
$\tilde{u}_1^*\tilde{d}_2$	$-\cos\theta_u\sin\theta_d$	$\sin\theta_u\cos\theta_d$	$\cos\theta_u\cos\theta_d$	$-\sin\theta_u\sin\theta_d$
$\tilde{u}_2^*\tilde{d}_1$	$-\sin\theta_u\cos\theta_d$	$\cos\theta_u\sin\theta_d$	$-\sin\theta_u\sin\theta_d$	$\cos\theta_u\cos\theta_d$
$\tilde{u}_2^*\tilde{d}_2$	$\sin\theta_u\sin\theta_d$	$\cos\theta_u\cos\theta_d$	$-\sin\theta_u\cos\theta_d$	$-\cos\theta_u\sin\theta_d$

Using this table, we can convert Feynman rules for $X\tilde{q}\tilde{q}$ vertices (where X is a one- or two-particle state) from the $\tilde{q}_L-\tilde{q}_R$ basis to the $\tilde{q}_1-\tilde{q}_2$ basis. If $V(X\tilde{u}_i^*\tilde{d}_j)$ is the desired Feynman rule ($i, j = 1$ or 2), then

$$V(X\tilde{u}_i^*\tilde{d}_j) = \sum_{k,l=L,R} T_{ijkl} V(X\tilde{u}_k^*\tilde{d}_l),$$

where T_{ijkl} is the appropriate entry in the table above. For the case of identical scalar-quarks, simply replace the symbol u (or d) with d (or u) in all expressions.

One clarification is required. The term in eq. (4.19) proportional to m_Z contains a sum over $q_i = u, d$. In the sum we must remember that* $e_{\tilde{u}_R} = e_{\tilde{u}_L} \equiv +\frac{2}{3}$ and $e_{\tilde{d}_R} = e_{\tilde{d}_L} \equiv -\frac{1}{3}$. As usual $T_3 = +\frac{1}{2}, -\frac{1}{2}$ for $\tilde{u}_L, \tilde{d}_L$ respectively. The Feynman rules for the $H\tilde{q}\tilde{q}$ vertices in the $\tilde{q}_L-\tilde{q}_R$ basis are given in figs. 11–15. Note that we do not display separately the rules for $H_2^0\tilde{q}\tilde{q}$ vertices. These may be obtained from the rules for $H_1^0\tilde{q}\tilde{q}$ vertices (figs. 12–13) by making the replacement: $\alpha \to \alpha + \frac{1}{2}\pi$ and β unchanged. In reality, the appropriate Feynman rules to use are those involving the scalar-quark mass eigenstates [given by eq. (4.18)]. These rules can be easily obtained from figs. 11–15 by making use of table 2. Schematically, if $V(H\tilde{u}_i\tilde{d}_j)$ and $V(H\tilde{u}_k\tilde{d}_l)$ are the Feynman rules in the $\tilde{q}_1-\tilde{q}_2$ and $\tilde{q}_L-\tilde{q}_R$ bases respectively, then

$$V(X\tilde{u}_i^*\tilde{d}_j) = \sum_{k,l=L,R} T_{ijkl} V(X\tilde{u}_k^*\tilde{d}_l), \tag{4.20}$$

where the T_{ijkl} are the appropriate entries in table 2. We give two examples. For H_3^0 interactions,

$$V(H_3^0\tilde{q}_2^*\tilde{q}_1) = V(H_3^0\tilde{q}_R^*\tilde{q}_L). \tag{4.21}$$

* A comment at this point is appropriate. Consider the following expression which appears in eqs. (4.19) and (4.23):

$$(T_{3i} - e_i\sin^2\theta_W)\,\tilde{q}_{iL}^*\tilde{q}_{iL} + e_i\sin^2\theta_W\,\tilde{q}_{iR}^*\tilde{q}_{iR}.$$

The term proportional to e_i changes sign when we go from $\tilde{q}_L$ to $\tilde{q}_R$. The origin of this sign change is related to the fact that we have defined $\tilde{U} = \tilde{u}_R^*$ and $\tilde{D} = \tilde{d}_R^*$ in table 1. Thus, the scalar-quarks which appear in the $\hat{Q}$ *chiral* supermultiplet have the *opposite* electric charge from the scalar-quarks which appear in the $\hat{U}$ and $\hat{D}$ *chiral* supermultiplet.

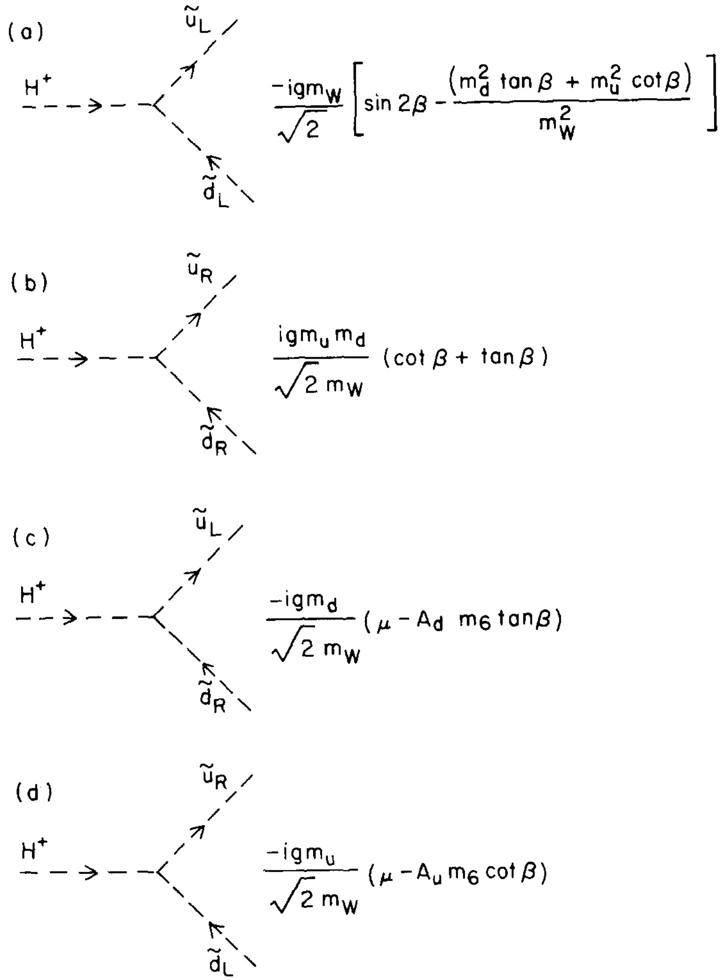


Fig. 11. Feynman rules for the $\tilde{H}^+\tilde{u}\tilde{d}$ vertices in the $\tilde{q}_L - \tilde{q}_R$ basis. To get appropriate rules in the $\tilde{q}_1 - \tilde{q}_2$ basis, see table 2 and discussion in text following eq. (4.19).

A more complicated example would be:

$$V(H_1^0\tilde{u}_1\tilde{u}_1) = \frac{-igm_Z}{\cos\theta_W}\cos(\beta+\alpha)\left[\cos^2(\theta_u)\left(\tfrac{1}{2} - e_u\sin^2\theta_W\right) + \sin^2(\theta_u)e_u\sin^2\theta_W\right]$$
$$-\frac{igm_u^2\sin\alpha}{m_W\sin\beta} - \frac{igm_u\sin 2\theta_u}{2m_W\sin\beta}\left[A_u m_6\sin\alpha + \mu\cos\alpha\right]. \tag{4.22}$$

It is important to note that even in the limit of zero quark masses ($m_u = m_d = 0$), some terms survive in eq. (4.19). These terms originated from V_D [eq. (4.14)].

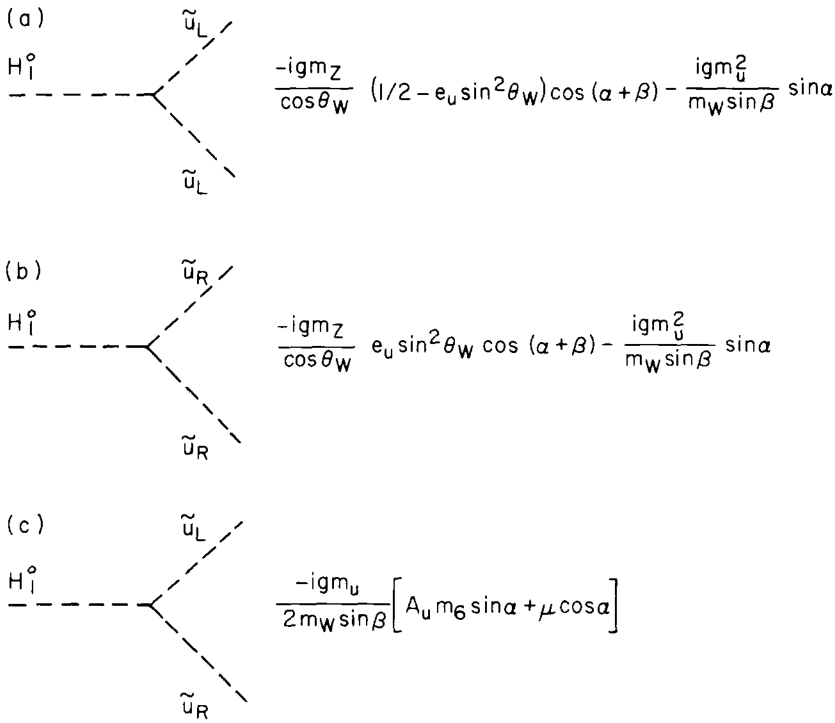


Fig. 12. Feynman rules for the $H_1^0\tilde{u}\tilde{u}$ vertices in the $\tilde{q}_L - \tilde{q}_R$ basis. Rules for the $H_2^0\tilde{u}\tilde{u}$ vertices are obtained by the following replacement: $\alpha \to \alpha + \frac{1}{2}\pi$, and β unchanged [i.e. $\sin\alpha \to \cos\alpha$, $\cos\alpha \to -\sin\alpha$ and $\cos(\beta+\alpha) \to -\sin(\beta+\alpha)$. To get appropriate rules in the $\tilde{q}_1 - \tilde{q}_2$ basis, see table 2 and discussion in text following eq. (4.19)].

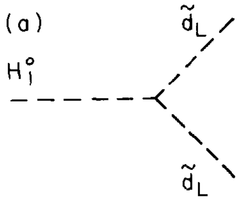
Explicitly, we have:

$$\begin{aligned}\mathscr{L}_{H\tilde{q}\tilde{q}}(m_u = m_d = 0) = & -\sqrt{\tfrac{1}{2}}\, g m_W \sin 2\beta\left(H^+ \tilde{u}_L^* \tilde{d}_L + \text{h.c.}\right) \\ & - \frac{g m_Z}{\cos\theta_W}\left[H_1^0 \cos(\alpha+\beta) - H_2^0 \sin(\alpha+\beta)\right] \\ & \times\left[\left(\tfrac{1}{2} - e_u \sin^2\theta_W\right)\tilde{u}_L^* \tilde{u}_L + \tilde{u}_R^* \tilde{u}_R e_u \sin^2\theta_W \right. \\ & \left. - \left(\tfrac{1}{2} + e_d \sin^2\theta_W\right)\tilde{d}_L^* \tilde{d}_L + \tilde{d}_R^* \tilde{d}_R e_d \sin^2\theta_W\right]. \quad (4.23)\end{aligned}$$

The interpretation of this term in the supersymmetric limit is as follows. As mentioned in sect. 3, the $H^\pm$ become the scalar superpartners of the $W^\pm$. Similarly, one combination of the neutral Higgs scalars becomes the scalar superpartner of the Z^0. Hence, eq. (4.23) is related by supersymmetry to the $Wq\bar{q}'$ and $Zq\bar{q}$ interactions.

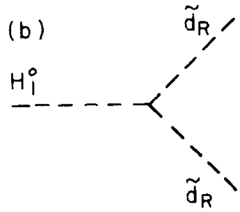
The structure of eq. (4.23) is quite interesting. Suppose we attempt to produce H_1^0 or H_2^0 via gluon-gluon fusion. A class of contributing diagrams is shown in fig. 16. If $\tilde{u}_L$, $\tilde{u}_R$, $\tilde{d}_L$ and $\tilde{d}_R$ are all degenerate in mass, then the sum total of the contribu-

(a)



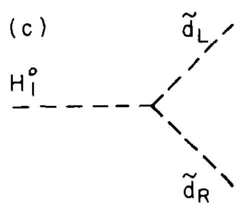
$$\frac{igm_Z}{\cos\theta_W}\left(1/2+e_d\sin^2\theta_W\right)\cos(\alpha+\beta)-\frac{igm_d^2}{m_W\cos\beta}\cos\alpha$$

(b)



$$\frac{-igm_Z}{\cos\theta_W}\, e_d\sin^2\theta_W\,\cos(\alpha+\beta)-\frac{igm_d^2}{m_W\cos\beta}\cos\alpha$$

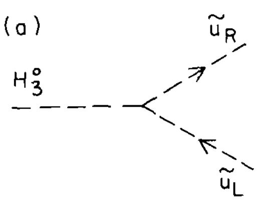
(c)



$$\frac{-igm_d}{2m_W\cos\beta}\left[A_d m_6\cos\alpha+\mu\sin\alpha\right]$$

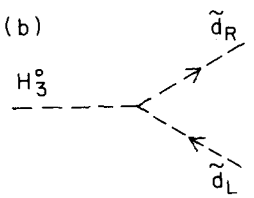
Fig. 13. Feynman rules for the $H_1^0\tilde{d}\tilde{d}$ vertices in the $\tilde{q}_L-\tilde{q}_R$ basis. See caption to fig. 12 for the recipe for obtaining rules for the $H_2^0\tilde{d}\tilde{d}$ vertices and the appropriate rules in the $\tilde{q}_1-\tilde{q}_2$ basis.

(a)



$$\frac{gm_u}{2m_W}\left(m_6A_u\cot\beta-\mu\right)$$

(b)



$$\frac{gm_d}{2m_W}\left(m_6A_d\tan\beta-\mu\right)$$

Fig. 14. Feynman rules for the $H_3^0\tilde{u}_R\tilde{u}_L$ and $H_3^0\tilde{d}_R\tilde{d}_L$ vertices. To obtain the appropriate rules in the $\tilde{q}_1-\tilde{q}_2$ basis, simply replace L with 1 and R with 2. The directions of the scalar-quark momenta are indicated by the arrows. Reversing the arrows leads to an extra factor of -1 as depicted in fig. 15.

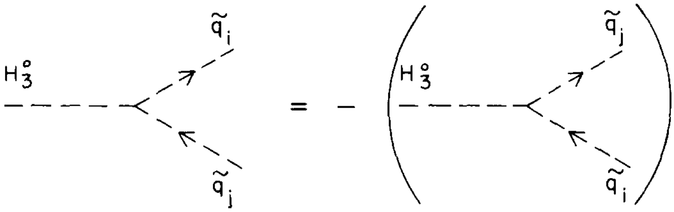


Fig. 15. Behaviour of the Feynman rules for $H_3^0\tilde{q}\tilde{q}$ vertices under a change of sign of the scalar-quark momentum. The indices i, j refer to either the $\tilde{q}_L - \tilde{q}_R$ or $\tilde{q}_1 - \tilde{q}_2$ bases. Note that this rule implies that for $i = j$, the vertex vanishes.

tions of scalar-quark loops due to eq. (4.23) vanishes! The remaining contributions which enter according to eq. (4.19) are all proportional to quark masses. However, in the supersymmetric limit, the scalar-quarks are not all degenerate but are equal in mass to the corresponding quarks. Thus, amusingly, we find that in this limit, the total contribution of the terms of eq. (4.23) to fig. 16 is also proportional to the quark mass.

For completeness, we mention the interaction of scalar-quarks with the gauge singlet N-field (case 1 of sect. 3). Using eq. (4.13) we can immediately write down the Feynman rules for the $\mathrm{N}\tilde{q}\tilde{q}$ vertices. The interaction terms are as follows:

$$\mathscr{L}_{\mathrm{N}\tilde{q}\tilde{q}} = -m_u \cot\beta\left[h^* N^* \tilde{u}_L \tilde{u}_R^* + \text{h.c.}\right] - m_d \tan\beta\left[h^* N^* \tilde{d}_L \tilde{d}_R^* + \text{h.c.}\right]. \quad (4.24)$$

The precise Feynman rules require knowledge of the mass eigenstates N_1 and N_2 obtained by diagonalizing the $(\mathrm{Re}\, N, \mathrm{Im}\, N)$ mass matrix.

We now turn to the quartic interactions of the form $\mathrm{HH}\tilde{q}\tilde{q}$. These terms are required, for example, in the calculation of multi-Higgs production via gluon fusion.

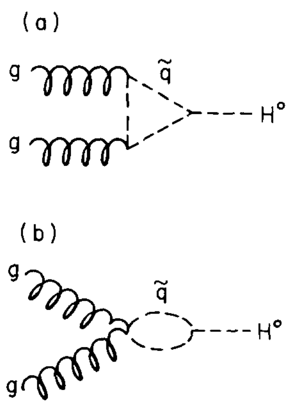
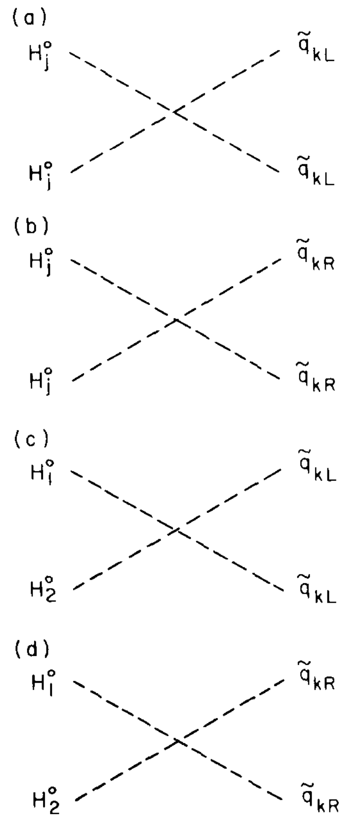


Fig. 16. A class of diagrams which contribute to the production of neutral Higgs bosons via gluon fusion. The internal loop consists of all possible flavors of scalar-quarks, $\tilde{q}_L$ and $\tilde{q}_R$.

Note that because these terms are dimension four, they arise only from the supersymmetric part of the theory. However, these interaction terms are sensitive to the soft-supersymmetry breaking sector of the theory to the extent that it is this sector which determines the precise scalar-quark and Higgs boson mass eigenstates.

There are two sources for the $\mathrm{HH}\tilde{q}\tilde{q}$ interaction terms: the F-terms given by eq. (4.13) and the D-terms given by eq. (4.14). The computation involves inserting eqs. (4.1a–d) into these terms and extracting the quartic pieces. The results are fairly involved, and we summarize them in Feynman rules given in figs. 17 and 18. (See eq. (B.22) in appendix B for the extension to the case of more than one generation of scalar-quarks.) We may also consider case 1 of sect. 3, i.e., a neutral gauge singlet complex field N which does not mix with the doublet Higgs fields. In this case, we get additional four-point interactions which result from eq. (4.13). The relevant



(a) $$\frac{ig^2}{2}\left[C_j\left(\frac{T_{3k}-e_k\sin^2\theta_W}{\cos^2\theta_W}\right)-\frac{m_q^2}{m_W^2}D_{jk}\right]$$

(b) $$\frac{ig^2}{2}\left[\frac{C_j e_k\sin^2\theta_W}{\cos^2\theta_W}-\frac{m_q^2}{m_W^2}D_{jk}\right]$$

(c) $$\frac{ig^2\sin 2\alpha}{4}\left[T_{3k}+\left(\frac{T_{3k}-e_k\sin^2\theta_W}{\cos^2\theta_W}\right)-\frac{m_q^2}{m_W^2}D_k\right]$$

(d) $$\frac{ig^2\sin 2\alpha}{4}\left[\frac{e_k\sin^2\theta_W}{\cos^2\theta_W}-\frac{m_q^2}{m_W^2}D_k\right]$$

Fig. 17. Feynman rules for four-point interactions among scalar-quarks and neutral Higgs bosons. The index j labels the neutral Higgs bosons, while $k=1,2$ corresponds to up-type and down-type flavors, respectively. For definitions of the quantum numbers T_3 and e, see table 1. The coefficients C_j, D_{jk} and D_k are given in table 3. (Note that there is no $\mathrm{H}_3^0\mathrm{H}_j^0\tilde{q}\tilde{q}$ vertex, $j=1,2$.)

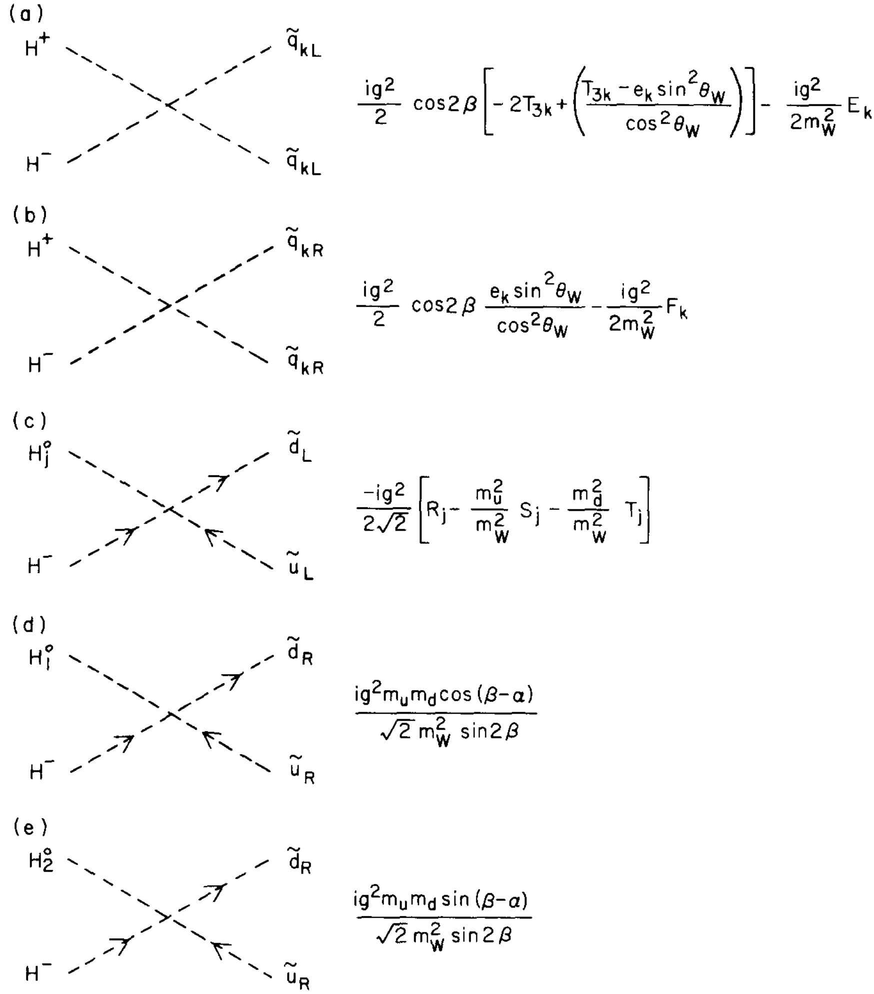


Fig. 18. Feynman rules for the four-point interaction among scalar-quarks and Higgs bosons. See caption to fig. 17. The coefficients E_k, F_k, R_j, S_j and T_j are defined in table 3. (Note that there is no $H_3^0H^-\tilde{d}_R\tilde{u}_R$ vertex.)

interaction term is given by:

$$\begin{aligned}\mathscr{L}_{NH\tilde{q}\tilde{q}} = &-\frac{ghm_u}{2m_W}N\tilde{u}_R\tilde{u}_L^*\left[\frac{\cos\alpha}{\sin\beta}H_1^0-\frac{\sin\alpha}{\sin\beta}H_2^0+iH_3^0\right]\\ &-\frac{ghm_d}{2m_W}N\tilde{d}_R\tilde{d}_L^*\left[\frac{\sin\alpha}{\cos\beta}H_1^0+\frac{\cos\alpha}{\cos\beta}H_2^0+iH_3^0\right]\\ &+\frac{gh}{\sqrt{2}\,m_W}\left[m_u\tilde{u}_R\tilde{d}_L^*NH^-+m_d\tilde{d}_R\tilde{u}_L^*NH^+\right]+\text{h.c.}\end{aligned} \tag{4.25}$$

TABLE 3

(j)	C_j	D_{j1}	D_{j2}
1	$-\cos 2\alpha$	$(\sin^2\alpha)/\sin^2\beta$	$(\cos^2\alpha)/\cos^2\beta$
2	$\cos 2\alpha$	$(\cos^2\alpha)/\sin^2\beta$	$(\sin^2\alpha)/\cos^2\beta$
3	$\cos 2\beta$	$\cot^2\beta$	$\tan^2\beta$
(j)	R_j	S_j	T_j
1	$\sin(\alpha+\beta)$	$(\sin\alpha\cos\beta)/\sin^2\beta$	$(\cos\alpha\sin\beta)/\cos^2\beta$
2	$\cos(\alpha+\beta)$	$(\cos\alpha\cos\beta)/\sin^2\beta$	$-(\sin\alpha\sin\beta)/\cos^2\beta$
3	$i\cos 2\beta$	$i\cot^2\beta$	$-i\tan^2\beta$
(k)	D_k	E_k	F_k
1	$1/\sin^2\beta$	$m_d^2\tan^2\beta$	$m_u^2\cot^2\beta$
2	$-1/\cos^2\beta$	$m_u^2\cot^2\beta$	$m_d^2\tan^2\beta$

We list coefficients which appear in the rules given in figs. 17 and 18 for $HH\tilde{q}\tilde{q}$ four-point vertices. The index j labels the neutral Higgs boson, while $k=1,2$ corresponds to up-type and down-type flavors, respectively.

To derive Feynman rules from eq. (4.25), one would have to determine the proper N eigenstates.

It may also turn out that the proper scalar-quark mass eigenstates are mixtures of $\tilde{q}_L$ and $\tilde{q}_R$ as discussed below eq. (4.19). As before, we may use the results of table 2 to convert rules in the $\tilde{q}_L-\tilde{q}_R$ basis to the $\tilde{q}_1-\tilde{q}_2$ basis. All one has to do is to make use of eq. (4.20) where X here stands for the appropriate two-Higgs-boson combination.

Finally, we note that the Feynman rules for $HH\tilde{\ell}\tilde{\ell}$ vertices involving scalar-leptons may be obtained from figs. 17 and 18 by using the appropriate values for the T_3 and e quantum numbers, as well as the appropriate masses.

4.5. INTERACTION WITH CHARGINOS AND NEUTRALINOS

In this section we compute the interaction of the Higgs bosons with the supersymmetric partners of the gauge and Higgs bosons (the gauginos and higgsinos). After the spontaneous breaking of SU(2) × U(1), the gauginos and higgsinos with the same electric charge can mix. This mixing is model dependent [30–32] and is discussed in Appendix A. (For further details, see appendix C of ref. [18].) The resulting mass eigenstates are called charginos, $\tilde{\chi}^{\pm}$, and neutralinos, $\tilde{\chi}^0$. We proceed now to compute the $H\tilde{\chi}\tilde{\chi}$ interaction terms.

The source of the (dimension-four) interaction terms (in two-component notation) is [24, 18],

$$\mathscr{L}_{\text{int}} = ig\sqrt{2}\, T^a_{ij}\lambda^a\psi_j A^*_i - \tfrac{1}{2}\left(\frac{\partial^2 W}{\partial A_i\, \partial A_j}\right)\psi_i\psi_j + \text{h.c.}, \tag{4.26}$$

where W is given by eq. (3.3) (including terms involving the N field, if desired) and ψ and A stand for generic two-component fermion and scalar fields. Writing out the results explicitly,

$$\begin{aligned}\mathscr{L}_{\text{int}} = {} & ig\left(H_1^{1*}\lambda^+\psi^-_{H_1} + H_1^{2*}\lambda^-\psi^0_{H_1} + H_2^{1*}\lambda^+\psi^0_{H_2} + H_2^{2*}\lambda^-\psi^+_{H_2}\right)\\ & + \sqrt{\tfrac{1}{2}}\, i\left(g\lambda^3 - g'\lambda'\right)\left(H_1^{1*}\psi^0_{H_1} - H_2^{2*}\psi^0_{H_2}\right)\\ & + \sqrt{\tfrac{1}{2}}\, i\left(g\lambda^3 + g'\lambda'\right)\left(H_2^{1*}\psi^+_{H_2} - H_1^{2*}\psi^-_{H_1}\right)\\ & + h\psi_N\left(H_1^2\psi^+_{H_2} + H_2^1\psi^-_{H_1} - H_1^1\psi^0_{H_2} - H_2^2\psi^0_{H_1}\right)\\ & + hN\left(\psi^-_{H_1}\psi^+_{H_2} - \psi^0_{H_1}\psi^0_{H_2}\right) - 2\Lambda N\psi_N\psi_N + \text{h.c.}\end{aligned} \tag{4.27}$$

In addition, there are mass terms which are responsible for the chargino and neutralino mass matrices. They arise from three sources. First, quadratic terms in W when inserted into eq. (4.26) lead to $\psi\psi$ mass terms:

$$\mathscr{L}_{\text{m}}^{\text{susy}} = \mu\left(\psi^0_{H_1}\psi^0_{H_2} - \psi^-_{H_1}\psi^+_{H_2}\right). \tag{4.28}$$

Second, there is a soft-supersymmetry-breaking mass term for the gauginos:

$$\mathscr{L}_{\text{m}}^{\text{soft}} = -M\lambda^a\lambda^a - M'\lambda'\lambda' + \text{h.c.} \tag{4.29}$$

(Note that explicit supersymmetry-breaking mass terms for the higgsinos are *not* soft according to the definition of ref. [9]). Finally, because H_1^1 and H_2^2 acquire vacuum expectation values when we insert eq. (4.1) into eq. (4.27), one finds the following mass terms due to $\text{SU}(2)\times\text{U}(1)$ symmetry breaking:

$$\begin{aligned}\mathscr{L}_{\text{m}}^{\text{breaking}} = {} & ig\left(v_1\lambda^+\psi^-_{H_1} + v_2\lambda^-\psi^+_{H_2}\right)\\ & + \sqrt{\tfrac{1}{2}}\, i\left(g\lambda^3 - g'\lambda'\right)\left(v_1\psi^0_{H_1} + v_2\psi^0_{H_2}\right)\\ & - h\psi_N\left(v_1\psi^0_{H_2} + v_2\psi^0_{H_1}\right) + \text{h.c.},\end{aligned} \tag{4.30}$$

where $\lambda^{\pm} = \sqrt{\tfrac{1}{2}}\,(\lambda^1 \mp i\lambda^2)$.

We shall now sketch the derivation of the $\text{H}\tilde{\chi}^+\tilde{\chi}^-$ rules. For all other cases we simply summarize the final results. Starting with the first term in eq. (4.27), we convert to four-component notation. Then, using the spinor fields defined by eq. (A.11), we find

$$\mathscr{L}_{\rm int} = -g\left\{ H_1^{1*}\overline{\tilde{H}}P_{\rm L}\tilde{W} + H_2^{2*}\overline{\tilde{W}}P_{\rm L}\tilde{H} + {\rm h.c.}\right\}, \tag{4.31}$$

where $P_{\rm R,L} = \frac{1}{2}(1 \pm \gamma_5)$. The $\tilde{W}$ and $\tilde{H}$ fields are not mass eigenstates. To obtain the desired Feynman rules, we express $\tilde{W}$ and $\tilde{H}$ in terms of $\tilde{\chi}_1^+$ and $\tilde{\chi}_2^+$ using eqs. (A.13a–d). Finally, we insert the proper Higgs boson mass eigenstates using eq. (4.1). The end result is

$$\begin{aligned}\mathscr{L}_{\rm H\tilde{\chi}^+\tilde{\chi}^-} = &-g\left(H_1^0\cos\alpha - H_2^0\sin\alpha\right)\overline{\tilde{\chi}}{}_i^+\left[Q_{ij}^*P_{\rm L} + Q_{ji}P_{\rm R}\right]\tilde{\chi}_j^+ \\ &-g\left(H_1^0\sin\alpha + H_2^0\cos\alpha\right)\overline{\tilde{\chi}}{}_i^+\left[S_{ij}^*P_{\rm L} + S_{ji}P_{\rm R}\right]\tilde{\chi}_j^+ \\ &+igH_3^0\sin\beta\,\overline{\tilde{\chi}}{}_i^+\left[Q_{ij}^*P_{\rm L} - Q_{ji}P_{\rm R}\right]\tilde{\chi}_j^+ \\ &+igH_3^0\cos\beta\,\overline{\tilde{\chi}}{}_i^+\left[S_{ij}^*P_{\rm L} - S_{ji}P_{\rm R}\right]\tilde{\chi}_j^+\ ,\end{aligned} \tag{4.32}$$

where summation over i, j is implied and Q and S are defined in terms of the matrices U and V which diagonalize the chargino mass matrix [see eqs. (A.4), (A.5)]:

$$Q_{ij} = \sqrt{\tfrac{1}{2}}\,U_{i2}V_{j1}, \tag{4.33}$$

$$S_{ij} = \sqrt{\tfrac{1}{2}}\,U_{i1}V_{j2}. \tag{4.34}$$

We can rewrite eq. (4.32) in another form by relating S to Q and the chargino mass matrix. From eqs. (4.28)–(4.30) and eq. (A.4), the chargino mass matrix can be written as follows:

$$\begin{aligned}-\mathscr{L}_{\rm m}^{(+)} = \sqrt{2}\,\overline{\tilde{\chi}}{}_i^+\Big\{&\left[g\left(v_1Q_{ij}^* + v_2S_{ij}^*\right) + \sqrt{2}\,m_{\rm W}R_{ij}^*\right]P_{\rm L} \\ &+\left[g\left(v_1Q_{ji} + v_2S_{ji}\right) + \sqrt{2}\,m_{\rm W}R_{ji}\right]P_{\rm R}\Big\}\tilde{\chi}_i^+\ ,\end{aligned} \tag{4.35}$$

where Q and S are defined in eqs. (4.33) and (4.34) and R is defined by

$$R_{ij} = \frac{1}{2m_{\rm W}}\left[M^*U_{i1}V_{j1} + \mu^*U_{i2}V_{j2}\right]. \tag{4.36}$$

However, U and V are chosen specifically such that:

$$-\mathscr{L}_{\rm m}^{(+)} = \tilde{M}_1^{(+)}\overline{\tilde{\chi}}{}_1^+\tilde{\chi}_1^+ + \tilde{M}_2^{(+)}\overline{\tilde{\chi}}{}_2^+\tilde{\chi}_2^+\ . \tag{4.37}$$

Equating eqs. (4.35) and (4.37) leads to:

$$S_{ij} = \frac{1}{\sin\beta}\left[\frac{\tilde{M}_i^{(+)}}{2m_{\rm W}}\delta_{ij} - Q_{ij}\cos\beta - R_{ij}\right]. \tag{4.38}$$

Inserting this expression into eq. (4.32) gives us our desired form:

$$\mathscr{L}_{H\tilde{\chi}^+\tilde{\chi}^-} = -\frac{g\tilde{M}_i^{(+)}}{2m_W\sin\beta}\left[\left(H_1^0\sin\alpha + H_2^0\cos\alpha\right)\bar{\tilde{\chi}}_i^+\tilde{\chi}_i^+ + iH_3^0\bar{\tilde{\chi}}_i^+\gamma_5\tilde{\chi}_i^+\cos\beta\right]$$

$$-\frac{g}{\sin\beta}\bar{\tilde{\chi}}_i^+\left[\left(Q_{ij}^*\sin(\beta-\alpha) - R_{ij}^*\sin\alpha\right)P_L\right.$$

$$\left. + \left(Q_{ji}\sin(\beta-\alpha) - R_{ji}\sin\alpha\right)P_R\right]\tilde{\chi}_j^+H_1^0$$

$$+\frac{g}{\sin\beta}\bar{\tilde{\chi}}_i^+\left[\left(Q_{ij}^*\cos(\beta-\alpha) + R_{ij}^*\cos\alpha\right)P_L\right.$$

$$\left. + \left(Q_{ji}\cos(\beta-\alpha) + R_{ji}\cos\alpha\right)P_R\right]\tilde{\chi}_j^+H_2^0$$

$$-\frac{ig}{\sin\beta}\bar{\tilde{\chi}}_i^+\left[\left(Q_{ij}^*\cos 2\beta + R_{ij}^*\cos\beta\right)P_L\right.$$

$$\left. - \left(Q_{ji}\cos 2\beta + R_{ji}\cos\beta\right)P_R\right]\tilde{\chi}_j^+H_3^0. \qquad (4.39)$$

The corresponding Feynman rules are shown in fig. 19. Note that if Q and R are real matrices then CP is conserved, and indeed the diagonal couplings $H_i\tilde{\chi}_j^+\tilde{\chi}_j^-$ are purely scalar for H_1^0, H_2^0 and pseudoscalar for H_3^0.

Next, we consider the $H^+\tilde{\chi}^-\tilde{\chi}^0$ interactions. Here the analysis is straightforward and we quote the final result:

$$\mathscr{L}_{H^-\tilde{\chi}^+\tilde{\chi}^0} = -H^-\bar{\tilde{\chi}}_i^0\left[Q_{ij}^{\prime L}P_L + Q_{ij}^{\prime R}P_R\right]\tilde{\chi}_j^+ + \text{h.c.}, \qquad (4.40)$$

where we have defined:

$$Q_{ij}^{\prime L} = g\cos\beta\left[N_{i4}^*V_{j1}^* + \sqrt{\tfrac{1}{2}}\left(N_{i2}^* + N_{i1}^*\tan\theta_W\right)V_{j2}^*\right] - hN_{i5}^*V_{j2}^*\sin\beta, \quad (4.41)$$

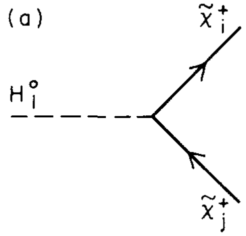
$$Q_{ij}^{\prime R} = g\sin\beta\left[N_{i3}U_{j1} - \sqrt{\tfrac{1}{2}}\left(N_{i2} + N_{i1}\tan\theta_W\right)U_{j2}\right] - h^*N_{i5}U_{j2}\cos\beta. \quad (4.42)$$

The matrix N diagonalizes the neutralino mass matrix as shown in eqs. (A.20)–(A.21). The corresponding Feynman rule is shown in fig. 20.

As an interesting exercise, suppose that the $\tilde{\gamma}$ is a neutralino mass eigenstate, to be identified with $\tilde{\chi}_1^0$. Then it follows from eqs. (A.17) and (A.23) that $N_{11}' = 1$ and $N_{k1}' = N_{1k}' = 0$ for $k \neq 1$. Using eq. (A.23), this implies that $N_{11} = \cos\theta_W$, $N_{12} = \sin\theta_W$ and $N_{1k} = 0$ for $k = 3, 4, (5)$. Inserting these results into eqs. (4.40)–(4.42), we find (using $e = g\sin\theta_W$):

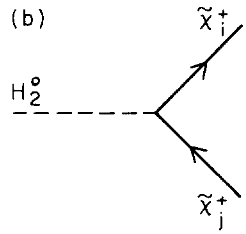
$$\mathscr{L}_{H^-\tilde{\chi}^+\tilde{\gamma}} = -\sqrt{2}\,eH^-\bar{\tilde{\gamma}}\left[V_{j2}^*P_L\cos\beta - U_{j2}P_R\sin\beta\right]\tilde{\chi}_j^+ + \text{h.c.}, \qquad (4.43)$$

(a)



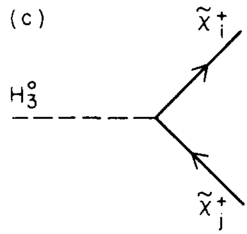
$$\frac{-ig}{2\sin\beta}\left[\frac{\tilde{M}_i^{(+)}\delta_{ij}\sin\alpha}{m_W} + \left(Q_{ij}^*\sin(\beta-\alpha) - R_{ij}^*\sin\alpha\right)(1-\gamma_5) + \left(Q_{ji}\sin(\beta-\alpha) - R_{ji}\sin\alpha\right)(1+\gamma_5)\right]$$

(b)



$$\frac{-ig}{2\sin\beta}\left[\frac{\tilde{M}_i^{(+)}\delta_{ij}\cos\alpha}{m_W} - \left(Q_{ij}^*\cos(\beta-\alpha) + R_{ij}^*\cos\alpha\right)(1-\gamma_5) - \left(Q_{ji}\cos(\beta-\alpha) + R_{ji}\cos\alpha\right)(1+\gamma_5)\right]$$

(c)



$$\frac{g}{2\sin\beta}\left[\frac{\tilde{M}_i^{(+)}\delta_{ij}\cos\beta}{m_W}\gamma_5 + \left(Q_{ij}^*\cos 2\beta + R_{ij}^*\cos\beta\right)(1-\gamma_5) - \left(Q_{ji}\cos 2\beta + R_{ji}\cos\beta\right)(1+\gamma_5)\right]$$

Fig. 19. Feynman rules for the $H^0\tilde{\chi}^+\tilde{\chi}^-$ vertices where $\tilde{\chi}_i^\pm$ are the charginos, with masses $\tilde{M}_i^{(+)}$. The matrices Q_{ij} and R_{ij} are defined in eqs. (4.33) and (4.36), respectively.

which is displayed in fig. 20. In order to make the physical origin of this result clear, it is useful to make use of the "interaction" eigenstate $\tilde{H}$ [see eq. (A.11)]. Using eqs. (A.13c–d), eq. (4.43) may be written as:

$$\mathscr{L}_{H^-\tilde{\chi}^+\tilde{\gamma}} = -\sqrt{2}\,eH^-\bar{\tilde{\gamma}}[P_L\cos\beta - P_R\sin\beta]\tilde{H} + \text{h.c.} \tag{4.44}$$

Thus we see that eq. (4.43) is the supersymmetric version of the $H^+H^-\gamma$ vertex. One final limiting case of interest is the supersymmetric limit [see eqs. (A.7)–(A.8)]. In this limit, the charginos are degenerate in mass with the $W^\pm$ and $H^\pm$ (these particles belong to a common massive supermultiplet). It is convenient to make use of the wiggsinos $\tilde{\omega}_i$ as the chargino mass eigenstates [see eq. (A.9)–(A.10)]. Then in the supersymmetric limit (where $\sin\beta = \cos\beta = \sqrt{\frac{1}{2}}$), we find:

$$\mathscr{L}^{\text{susy}}_{H^-\tilde{\chi}^+\tilde{\gamma}} = -\sqrt{2}\,eH^-\bar{\tilde{\gamma}}(P_L\tilde{\omega}_2^+ - P_R\tilde{\omega}_1^+) + \text{h.c.} \tag{4.45}$$

Finally, we turn to the $H_i^0\tilde{\chi}_j^0\tilde{\chi}_k^0$ interaction. The procedure is similar to the one described above. However, there is one subtlety which must be considered. Because

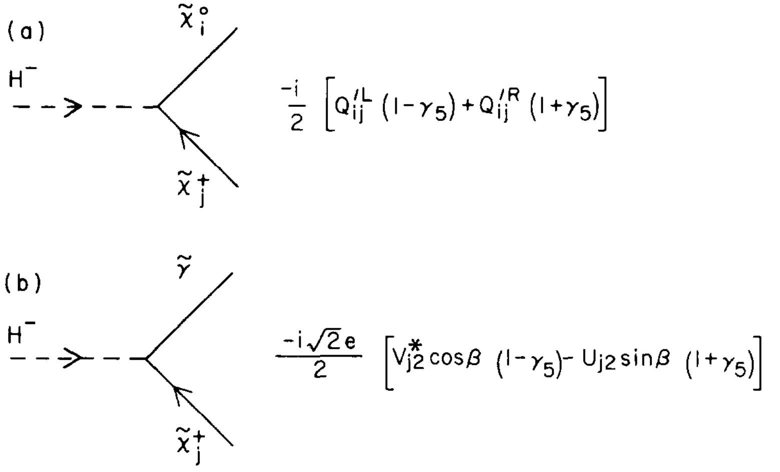


Fig. 20. (a) Feynman rules for the $H^+\tilde{\chi}^-\tilde{\chi}^0$ vertex. The matrices $Q_{ij}^{\prime L}$ and $Q_{ij}^{\prime R}$ are defined in eqs. (4.41) and (4.42), respectively; (b) Feynman rules for the $H^+\tilde{\chi}^-\tilde{\gamma}$ vertex. Here we assume that the photino corresponds to one of the neutralino mass eigenstates ($\tilde{\chi}_1^0$).

$\tilde{\chi}_j^0$ is a Majorana fermion, one must note the following identity which holds for anticommuting four-component Majorana spinors:

$$\bar{\tilde{\chi}}_j^0(1 \pm \gamma_5)\tilde{\chi}_k^0 = \bar{\tilde{\chi}}_k^0(1 \pm \gamma_5)\tilde{\chi}_j^0 . \tag{4.46}$$

This implies that the $H_i^0\tilde{\chi}_j^0\tilde{\chi}_k^0$ interaction must be symmetric under interchange of j and k. Starting from eq. (4.27), we arrive at:

$$\begin{aligned}\mathscr{L}_{H\tilde{\chi}^0\tilde{\chi}^0} = &-\tfrac{1}{2}g\left(H_1^0\cos\alpha - H_2^0\sin\alpha\right)\bar{\tilde{\chi}}_i^0\left(Q_{ij}^{\prime\prime *}P_L + Q_{ij}^{\prime\prime}P_R\right)\tilde{\chi}_j^0 \\ &+\tfrac{1}{2}g\left(H_1^0\sin\alpha + H_2^0\cos\alpha\right)\bar{\tilde{\chi}}_i^0\left(S_{ij}^{\prime\prime *}P_L + S_{ij}^{\prime\prime}P_R\right)\tilde{\chi}_j^0 \\ &+\tfrac{1}{2}igH_3^0\bar{\tilde{\chi}}_i^0\left[\left(Q_{ij}^{\prime\prime *}\sin\beta - S_{ij}^{\prime\prime *}\cos\beta\right)P_L - \left(Q_{ij}^{\prime\prime}\sin\beta - S_{ij}^{\prime\prime}\cos\beta\right)P_R\right]\tilde{\chi}_j^0 ,\end{aligned} \tag{4.47}$$

where

$$gQ_{ij}^{\prime\prime} = \tfrac{1}{2}\left[N_{i3}\left(gN_{j2} - g'N_{j1}\right) + \sqrt{2}\,h^*N_{i4}N_{j5} + (i \leftrightarrow j)\right], \tag{4.48}$$

$$gS_{ij}^{\prime\prime} = \tfrac{1}{2}\left[N_{i4}\left(gN_{j2} - g'N_{j1}\right) - \sqrt{2}\,h^*N_{i3}N_{j5} + (i \leftrightarrow j)\right]. \tag{4.49}$$

We can rewrite eq. (4.47) in another form by using the neutralino mass matrix. Using eqs. (4.28)–(4.30) and eq. (A.20), the neutralino mass matrix can be written as follows:

$$-\mathscr{L}_{\rm m}^{(0)} = \sqrt{\tfrac{1}{2}}\,\bar{\tilde{\chi}}_i^0\Big\{\Big[g\big(v_1 Q_{ij}''^{*} - v_2 S_{ij}''^{*}\big) + \sqrt{2}\,m_{\rm W} R_{ij}''^{*}\Big] P_{\rm L} + \Big[g\big(v_1 Q_{ij}'' - v_2 S_{ij}''\big) + \sqrt{2}\,m_{\rm W} R_{ij}''\Big] P_{\rm R}\Big\}\tilde{\chi}_j^0 , \qquad (4.50)$$

where Q'' and S'' are defined in eqs. (4.48) and (4.49) and R'' is defined by:

$$R_{ij}'' = \frac{1}{2m_{\rm W}}\big[M^* N_{i2}N_{j2} + M'^* N_{i1}N_{j1} - \mu^*\big(N_{i3}N_{j4} + N_{i4}N_{j3}\big)\big] . \qquad (4.51)$$

However, the matrix N is chosen specifically such that:

$$-\mathscr{L}_{\rm m}^{(0)} = \tfrac{1}{2}\tilde{M}_i^{(0)}\bar{\tilde{\chi}}_i^0\tilde{\chi}_i^0 , \qquad (4.52)$$

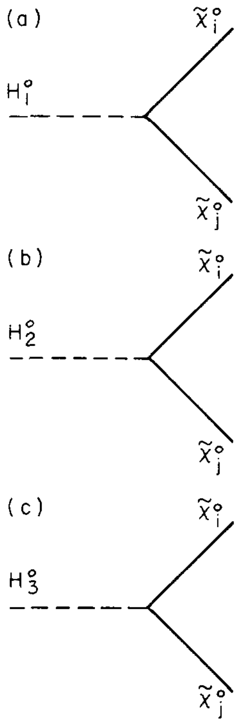
where summation over $i = 1,\ldots,5$ is implied. Equating eqs. (4.50) and (4.52) leads to:

$$S_{ij}'' = -\frac{1}{\sin\beta}\left[\frac{\tilde{M}_i^{(0)}}{2m_{\rm W}}\delta_{ij} - Q_{ij}''\cos\beta - R_{ij}''\right]. \qquad (4.53)$$

We now insert this into eq. (4.47) in order to get the desired form:

$$\begin{aligned}
\mathscr{L}_{{\rm H}\tilde{\chi}^0\tilde{\chi}^0} = &-\frac{g\tilde{M}_i^{(0)}}{4m_{\rm W}\sin\beta}\big[\big(H_1^0\sin\alpha + H_2^0\cos\alpha\big)\bar{\tilde{\chi}}_i^0\tilde{\chi}_i^0 + iH_3^0\bar{\tilde{\chi}}_i^0\gamma_5\tilde{\chi}_i^0\cos\beta\big]\\
&-\frac{g}{2\sin\beta}\bar{\tilde{\chi}}_i^0\big[\big(Q_{ij}''^{*}\sin(\beta-\alpha) - R_{ij}''^{*}\sin\alpha\big)P_{\rm L}\\
&\qquad + \big(Q_{ij}''\sin(\beta-\alpha) - R_{ij}''\sin\alpha\big)P_{\rm R}\big]\tilde{\chi}_j^0 H_1^0\\
&+\frac{g}{2\sin\beta}\bar{\tilde{\chi}}_i^0\big[\big(Q_{ij}''^{*}\cos(\beta-\alpha) + R_{ij}''^{*}\cos\alpha\big)P_{\rm L}\\
&\qquad + \big(Q_{ij}''\cos(\beta-\alpha) + R_{ij}''\cos\alpha\big)P_{\rm R}\big]\tilde{\chi}_j^0 H_2^0\\
&-\frac{ig}{2\sin\beta}\bar{\tilde{\chi}}_i^0\big[\big(Q_{ij}''^{*}\cos 2\beta + R_{ij}''^{*}\cos\beta\big)P_{\rm L}\\
&\qquad - \big(Q_{ij}''\cos 2\beta + R_{ij}''\cos\beta\big)P_{\rm R}\big]\tilde{\chi}_j^0 H_3^0 ,
\end{aligned} \qquad (4.54)$$

where summation over $i, j = 1,\ldots,5$ is implied. Note that Q_{ij}'' and R_{ij}'' are symmetric under interchange of $i \leftrightarrow j$ as required. Eq. (4.54) is closely analogous to the $\mathrm{H}\tilde{\chi}^+\tilde{\chi}^-$ interaction given by eq. (4.39) and the remarks we made there also apply here. Note that the extra factor of $\frac{1}{2}$ between the two equations is simply a consequence of the Majorana nature of the neutralinos. This factor of $\frac{1}{2}$ must be removed when writing down the Feynman rules as shown in fig. 21. These rules allow the index i to run from $1,\ldots,5$. If the model contains no $\mathrm{SU(2)}\times\mathrm{U(1)}$ gauge



(a)
$$\frac{-ig}{2\sin\beta}\left[\frac{\tilde M_i^{(0)}\delta_{ij}\sin\alpha}{m_W} + \left(Q''^*_{ij}\sin(\beta-\alpha) - R''^*_{ij}\sin\alpha\right)(1-\gamma_5) + \left(Q''_{ij}\sin(\beta-\alpha) - R''_{ij}\sin\alpha\right)(1+\gamma_5)\right]$$

(b)
$$\frac{-ig}{2\sin\beta}\left[\frac{\tilde M_i^{(0)}\delta_{ij}\cos\alpha}{m_W} - \left(Q''^*_{ij}\cos(\beta-\alpha) + R''^*_{ij}\cos\alpha\right)(1-\gamma_5) - \left(Q''_{ij}\cos(\beta-\alpha) + R''_{ij}\cos\alpha\right)(1+\gamma_5)\right]$$

(c)
$$\frac{g}{2\sin\beta}\left[\frac{\tilde M_i^{(0)}\delta_{ij}\cos\beta}{m_W}\gamma_5 + \left(Q''^*_{ij}\cos 2\beta + R''^*_{ij}\cos\beta\right)(1-\gamma_5) - \left(Q''_{ij}\cos 2\beta + R''_{ij}\cos\beta\right)(1+\gamma_5)\right]$$

Fig. 21. Feynman rules for the $H^0\tilde\chi_i^0\tilde\chi_j^0$ vertices where $\tilde\chi_i^0$ are the neutralinos with masses $\tilde M_i^{(0)}$. The index runs from 1,...,4 or 5 depending on whether one has a gauge singlet N field (and its higgsino partner) in the theory. The *symmetric* matrices Q''_{ij} and R''_{ij} are defined in eqs. (4.48) and (4.51), respectively.

singlet N-field (and hence no $\tilde\chi_5^0$), one must simply set $N_{5j} = N_{i5} = 0$ above (or equivalently set $h = 0$) and not allow $i = 5$.

Let us once again examine the case where one (or both) of the neutralinos is the photino. As before, we set $N_{11} = \cos\theta_W$, $N_{12} = \sin\theta_W$ and $N_{1k} = 0$ for $k = 3, 4, (5)$. Using eqs. (4.48), (4.51) and (4.53), we find that in this limit,

$$Q''_{1k} = S''_{1k} = 0, \tag{4.55}$$

$$R''_{1k} = \frac{\tilde M_{\tilde\gamma}}{2m_W}\delta_{1k}, \tag{4.56}$$

$$\tilde M_{\tilde\gamma} = M\sin^2\theta_W + M'\cos^2\theta_W. \tag{4.57}$$

Inserting these results into eq. (4.54) we find that

$$\mathscr{L}_{H\tilde\gamma\tilde\gamma} = \mathscr{L}_{H\tilde\chi^0\tilde\gamma} = 0. \tag{4.58}$$

This result is not surprising as there is no corresponding supersymmetric version of these vertices. Note that the fact that $\mathscr{L}_{\mathrm{H}\tilde{\gamma}\tilde{\gamma}} = 0$ is algebraically nontrivial and serves as an additional check on the correct form for eq. (4.54).

The last interaction vertices we consider involve the gauge single N field. As before, these interactions will depend on the unknown N mass matrix. The relevant interaction terms can be obtained from eq. (4.27) and the result is:

$$\mathscr{L}_{\mathrm{N}\tilde{\chi}\tilde{\chi}} = N\Big\{ hU_{i2}^{*}V_{j2}^{*}\bar{\tilde{\chi}}_i^{+}P_{\mathrm{L}}\tilde{\chi}_j^{+} - \tfrac{1}{2}\big[h\big(N_{i3}^{*}N_{j4}^{*} + N_{i4}^{*}N_{j3}^{*}\big) + 4\Lambda N_{i5}^{*}N_{j5}^{*}\big]\bar{\tilde{\chi}}_i^{0}P_{\mathrm{L}}\tilde{\chi}_j^{0}\Big\} + \mathrm{h.c.} \quad (4.59)$$

This completes our study of the interaction of charginos and neutralinos with the Higgs bosons.

5. Feynman rules for related interactions

In this section we discuss Feyman rules for the interaction of quarks and scalar-quarks with charginos and neutralinos, i.e. the $\mathrm{q}\tilde{\mathrm{q}}\tilde{\chi}^{+}$ and $\mathrm{q}\tilde{\mathrm{q}}\tilde{\chi}^{0}$ vertices. There are two contributions to the above vertices. The first contribution is the supersymmetric analog of the $\mathrm{q}\bar{\mathrm{q}}\mathrm{W}^{\pm}$ and $\mathrm{q}\bar{\mathrm{q}}\mathrm{Z}^{0}$ interactions. These have been discussed in detail in appendix C of ref. [18]. The second contribution is the supersymmetric analog of the $\mathrm{q}\bar{\mathrm{q}}\mathrm{H}$ interaction. This contribution is proportional to the quark mass and depends on the properties of the Higgs bosons in the supersymmetric model. The source of these two contributions corresponds to the two terms given in eq. (4.26). In this case the relevant part of W used in eq. (4.26) is given by W_F [see eq. (3.4)].

Consider first the $\mathrm{q}\tilde{\mathrm{q}}\tilde{\chi}^{+}$ interaction. We convert from two-component notation to four-component notation as discussed in sect. 4.5. We then find:

$$\begin{aligned}\mathscr{L}_{\mathrm{q}\tilde{\mathrm{q}}\tilde{\chi}^{+}} &= -g\big[\bar{\tilde{W}}P_{\mathrm{L}}u\tilde{d}_{\mathrm{L}}^{*} + \bar{\tilde{W}}^{\mathrm{c}}P_{\mathrm{L}}\,d\tilde{u}_{\mathrm{L}}^{*}\big] \\ &\quad + \frac{gm_{\mathrm{d}}}{\sqrt{2}\,m_{\mathrm{W}}\cos\beta}\big[\bar{\tilde{H}}P_{\mathrm{L}}u\tilde{d}_{\mathrm{R}}^{*} + \bar{d}P_{\mathrm{L}}\,\tilde{H}^{\mathrm{c}}\tilde{u}_{\mathrm{L}}\big] \\ &\quad + \frac{gm_{\mathrm{u}}}{\sqrt{2}\,m_{\mathrm{W}}\sin\beta}\big[\bar{u}P_{\mathrm{L}}\tilde{H}\tilde{d}_{\mathrm{L}} + \bar{\tilde{H}}^{\mathrm{c}}P_{\mathrm{L}}\,d\tilde{u}_{\mathrm{R}}^{*}\big] + \mathrm{h.c.},\end{aligned} \quad (5.1)$$

where u and d are four-component quark spinors, and the "interaction" eigenstates $\tilde{W}$ and $\tilde{H}$ are defined in eq. (A.11). An unusual feature of eq. (5.1) is the appearance of charge-conjugated states.* (See appendix A of ref. [18] for a summary of our notation.) This arises due to the existence of a nonconserved fermion-number which is a standard feature of supersymmetric models. We shall discuss this further after we have written down the final Feynman rules. The next step is to convert eq. (5.1) to an expression involving the chargino mass eigenstates $\tilde{\chi}_i^+$, $i = 1,2$. This is done by using eqs. (A.13a–d). In addition, we need four additional equations involving the charge conjugated fields. It is easy to derive an appropriate recipe. For example,

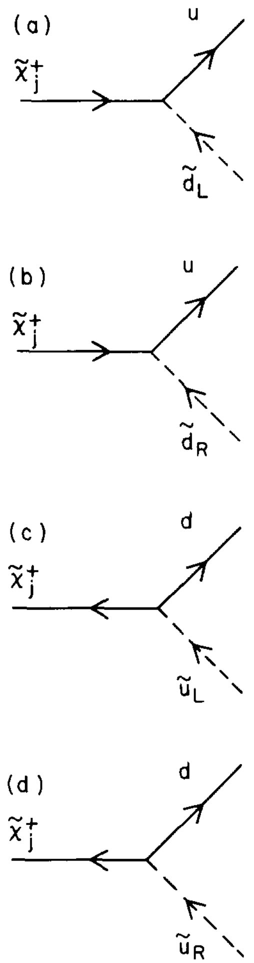
$$P_R \tilde{W}^c = P_R (V_{11}\tilde{\chi}_1^c + V_{21}\tilde{\chi}_2^c) . \tag{5.2}$$

(We employ the notation: $\chi_i^c \equiv (\chi_i^+)^c$ which is a negatively charged fermion.) Thus the recipe is simply to charge conjugate all fields in eqs. (A.13a–d) and interchange the matrices U and V. The final result is:

$$\begin{aligned} \mathscr{L}_{q\tilde{q}\tilde{\chi}^+} = & -g\left[\bar{u}P_R(U_{11}\tilde{\chi}_1^+ + U_{21}\tilde{\chi}_2^+)\tilde{d}_L + \bar{d}P_R(V_{11}\tilde{\chi}_1^c + V_{21}\tilde{\chi}_2^c)\tilde{u}_L\right] \\ & + \frac{gm_d}{\sqrt{2}\, m_W \cos\beta}\left[\bar{u}P_R(U_{12}\tilde{\chi}_1^+ + U_{22}\tilde{\chi}_2^+)\tilde{d}_R + \bar{d}P_L(U_{12}^*\tilde{\chi}_1^c + U_{22}^*\tilde{\chi}_2^c)\tilde{u}_L\right] \\ & + \frac{gm_u}{\sqrt{2}\, m_W \sin\beta}\left[\bar{u}P_L(V_{12}^*\tilde{\chi}_1^+ + V_{22}^*\tilde{\chi}_2^+)\tilde{d}_L + \bar{d}P_R(V_{12}\tilde{\chi}_1^c + V_{22}\tilde{\chi}_2^c)\tilde{u}_R\right] + \text{h.c.} \end{aligned} \tag{5.3}$$

The Feynman rules are given in fig. 22. As mentioned above, the appearance of both chargino fields and their charge-conjugates in eq. (5.3) is a consequence of fermion-number violation which naturally occurs in supersymmetric models. This violation is well understood in the case of neutral Majorana fields. In the present context, $\tilde{\chi}_1^+$ and $\tilde{\chi}_2^+$ are charged Dirac fields. Nevertheless, fermion-number violation may still occur when a given interaction involves both $\tilde{\chi}_i^+$ and $\tilde{\chi}_i^c$ fields. This is apparent in the Feynman rules exhibited in fig. 22. In figs. 22c and 22d the flow of fermion-number as indicated by the direction of the arrows on the (solid) fermion lines is not continuous. This leads to the explicit appearance of the charge-conjugation matrix C in the rules themselves [33, 18]. (The C arises from eq. (5.3) simply because $\chi^c = C\bar{\chi}^T$). It is not difficult to deal with fermion-number violating propagators and vertices. A complete discussion of the appropriate rules can be found in appendix D of ref. [18].

* This feature did not occur in the $H^-\tilde{\chi}^+\tilde{\chi}^0$ vertices [eq. (4.40)]. The reason is that $\tilde{\chi}^0$ is a Majorana field, i.e., $(\tilde{\chi}^0)^c = \tilde{\chi}^0$, so we were above to avoid the appearance of $(\tilde{\chi}^+)^c$ fields. In the present case, if one were to make use of a similar technique, one would end up with the appearance of charge-conjugated quark fields. We prefer not to do that.



(a)
$$\frac{-ig}{2}\left[U_{j1}(1+\gamma_5)-\frac{m_u V_{j2}^*}{\sqrt{2}\, m_W \sin\beta}(1-\gamma_5)\right]$$

(b)
$$\frac{ig m_d U_{j2}}{2\sqrt{2}\, m_W \cos\beta}(1-\gamma_5)$$

(c)
$$\frac{-ig}{2}\left[V_{j1}(1+\gamma_5)-\frac{m_d U_{j2}^*}{\sqrt{2}\, m_W \cos\beta}(1-\gamma_5)\right]C$$

(d)
$$\frac{ig m_u V_{j2}}{2\sqrt{2}\, m_W \sin\beta}(1-\gamma_5)C$$

Fig. 22. Feynman rules for the $q\tilde{q}\tilde{\chi}^+$ vertices. The matrices U and V are defined in eqs. (A.4) and (A.5). The arrows denote direction of flow of electric charge: +1 in the case of $\tilde{\chi}^+$ and e_q in the case of q and $\tilde{q}$ ($e_u = \frac{2}{3}$, $e_d = -\frac{1}{3}$). The charge conjugation matrix, C, appears when there is a discontinuous flow of fermion number as indicated by the arrows. Diagrams should always be read in such a way that the *quark* lines are traversed in the usual direction, i.e. opposite to its arrow. This rule indicates the proper placement of suppressed spinor indices. See appendix D of ref. [18] for a discussion on Feynman rules involving the charge conjugation matrix.

One must also consider four more diagrams which are obtained by reversing all arrows in fig. 22. (Note that the arrows indicate the direction of flow of a particular electric charge: +1 for the $\tilde{\chi}_j^+$, and e_q for q and $\tilde{q}$, where $e_u = \frac{2}{3}$ and $e_d = -\frac{1}{3}$.) The Feynman rules for the four new diagrams are easily stated. First, in all four cases, make the following interchanges: $U \leftrightarrow U^*$, $V \leftrightarrow V^*$, $(1+\gamma_5) \leftrightarrow (1-\gamma_5)$. Second, for the diagrams corresponding to fig. 22c and 22d, remove the factor of C

which appears on the right and insert a factor of $-C^{-1}$ which should be placed on the left. (This rule arises because $\bar{\chi}^c = -C^{-1}\chi^T$.) This is illustrated in fig. 23.

We next consider the $q\tilde{q}\tilde{\chi}^0$ interaction. After converting to four component notation, we find:

$$\begin{aligned}
\mathscr{L}_{q\tilde{q}\tilde{\chi}^0} = & -\sqrt{\tfrac{1}{2}}\,\tilde{u}_L^*\left[g\overline{\tilde{W}}_3 P_L u + y_q g'\overline{\tilde{B}}P_L u\right] \\
& -\sqrt{\tfrac{1}{2}}\,\tilde{d}_L^*\left[-g\overline{\tilde{W}}_3 P_L d + y_q g'\overline{\tilde{B}}P_L d\right] \\
& -\sqrt{\tfrac{1}{2}}\,g'\left[y_u \bar{u} P_L \tilde{B}\tilde{u}_R + y_d \bar{d} P_L \tilde{B}\tilde{d}_R\right] \\
& -\frac{g m_d}{\sqrt{2}\,m_W \cos\beta}\left[\overline{\tilde{H}}_1 P_L d\tilde{d}_R^* + \bar{d}P_L \tilde{H}_1 \tilde{d}_L\right] \\
& -\frac{g m_u}{\sqrt{2}\,m_W \sin\beta}\left[\overline{\tilde{H}}_2 P_L u\tilde{u}_R^* + \bar{u}P_L \tilde{H}_2 \tilde{u}_L\right] + \text{h.c.},
\end{aligned} \tag{5.4}$$

where the "interaction" eigenstates $\tilde{W}_3$, $\tilde{B}$, $\tilde{H}_1$ and $\tilde{H}_2$ are defined in eq. (A.24). Note that even in models with a gauge singlet Higgs field, N, the higgsino field $\tilde{N}$ does not appear in eq. (5.4).

It is straightforward to convert eq. (5.4) into an equation involving the chargino mass eigenstates $\tilde{\chi}_i^0$, by using eqs. (A.25a–b) and similar equations involving $\tilde{W}_3$ and $\tilde{B}$. In addition, we find it convenient to replace the matrix elements N_{j1} and N_{j2} by N'_{j1} and N'_{j2} defined in eq. (A.23).

One last trick is to eliminate the hypercharges y_q, y_u and y_d in favor of the electric charges $e_u = \frac{2}{3}$ and $e_d = -\frac{1}{3}$. This is done most easily by using $y_q = -1 + 2e_u = 1 + 2e_d$, $y_u = -2e_u$, and $y_d = -2e_d$. The final result is:

$$\begin{aligned}
\mathscr{L}_{q\tilde{q}\tilde{\chi}^0} = & -\sqrt{2}\,\bar{q}_i\left\{\frac{g m_{q_i} N_{j,5-i}^*}{2 m_W B_i}P_L + \left[e e_i N'_{j1} + \frac{g}{\cos\theta_W}N'_{j2}\left(T_{3i} - e_i \sin^2\theta_W\right)\right]P_R\right\}\tilde{\chi}_j^0 \tilde{q}_{iL} \\
& +\sqrt{2}\,\bar{q}_i\left[\left(e e_i N'^*_{j1} - \frac{g e_i \sin^2\theta_W}{\cos\theta_W}N'^*_{j2}\right)P_L + \frac{g m_{q_i} N_{j,5-i}}{2 m_W B_i}P_R\right]\tilde{\chi}_j^0 \tilde{q}_{iR} + \text{h.c.},
\end{aligned} \tag{5.5}$$

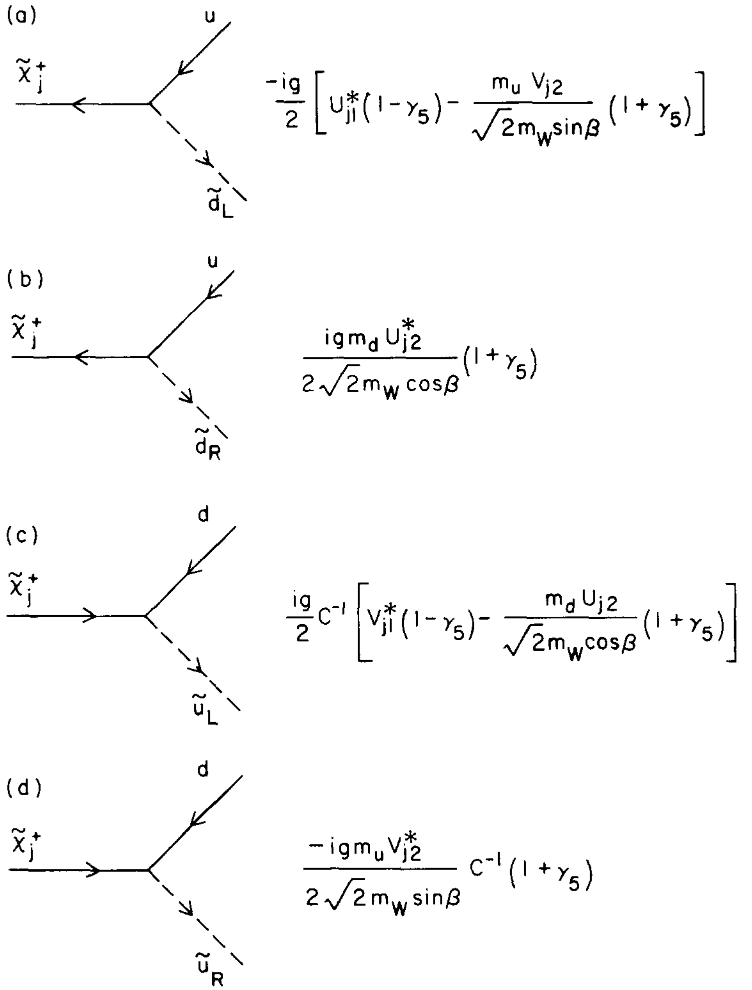


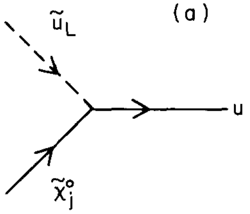
Fig. 23. Feynman rules for the $q\tilde{q}\tilde{\chi}^+$ vertices. See caption to fig. 22. This figure differs from fig. 22 in that all arrows are reversed.

where a summation over $i = 1,2$ and j is implied, and

$$q_i = \begin{pmatrix} u \\ d \end{pmatrix}, \qquad B_i = \begin{pmatrix} \sin\beta \\ \cos\beta \end{pmatrix}. \tag{5.6}$$

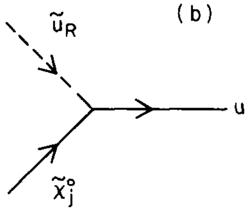
The quantum numbers T_{3i} and e_i are the weak-isospin and electric charge (in units of $e > 0$) of the quarks q_i. We emphasize that $\tilde{q}_{iR}$ and $\tilde{q}_{iL}$ have the same electric charge as the quarks q_i. The Feynman rules are depicted in fig. 24. Note that if the supersymmetric model involves the gauge singlet N field, then one must sum over $j = 1,2,\ldots,5$; otherwise the sum stops at $j = 4$.

(a)



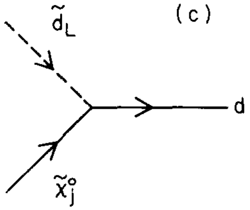
$$\frac{-i}{\sqrt{2}}\left\{\frac{g m_u}{2 m_W \sin\beta} N_{j4}^* (1-\gamma_5) + \left[e e_u N'_{j1} + \frac{g}{\cos\theta_W}\left(1/2 - e_u \sin^2\theta_W\right) N'_{j2}\right](1+\gamma_5)\right\}$$

(b)



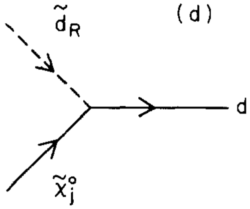
$$\frac{-i}{\sqrt{2}}\left\{\frac{g m_u}{2 m_W \sin\beta} N_{j4} (1+\gamma_5) - \left[e e_u N'^*_{j1} - \frac{g e_u \sin^2\theta_W}{\cos\theta_W} N'^*_{j2}\right](1-\gamma_5)\right\}$$

(c)



$$\frac{-i}{\sqrt{2}}\left\{\frac{g m_d}{2 m_W \cos\beta} N_{j3}^* (1-\gamma_5) + \left[e e_d N'_{j1} - \frac{g}{\cos\theta_W}\left(1/2 + e_d \sin^2\theta_W\right) N'_{j2}\right](1+\gamma_5)\right\}$$

(d)



$$\frac{-i}{\sqrt{2}}\left\{\frac{g m_d}{2 m_W \cos\beta} N_{j3} (1+\gamma_5) - \left[e e_d N'^*_{j1} - \frac{g e_d \sin^2\theta_W}{\cos\theta_W} N'^*_{j2}\right](1-\gamma_5)\right\}$$

Fig. 24. Feynman rules for the $q\tilde{q}\tilde{\chi}^0$ vertices. The quark charges are given by $e_u = \frac{2}{3}$, $e_d = -\frac{1}{3}$. The matrices N and N' are defined in eqs. (A.20), (A.21), and (A.23).

The results obtained in this section can also be used to obtain the couplings of leptons and scalar-leptons. One need only insert the correct quantum numbers (i.e., T_{3i} and e_i) as specified in table 1.

6. Comments on the supersymmetric parameters

In the Feynman rules presented in this paper, many parameters appear that are not fixed by general principles. For example, all possible soft supersymmetry breaking terms (consistent with the gauge symmetry and possibly some discrete symmetries) are allowed a priori; their coefficients must be taken as free parameters in a general approach. This can result in too much freedom when we apply our rules to phenomenological questions. It is often useful to make use of specific models as a guide to suggest (possible) likely values for many of the free parameters. One of the most popular approaches that one finds in the literature is that of low-energy supergravity [10–13]. In this approach, one obtains an effective lagrangian which is relevant at the Planck scale. One then uses the renormalization group equations to obtain the values of the parameters at a scale of order m_W. The resulting parameters are the ones which appear in the Feynman rules given in sects. 4 and 5.

Of course, the results given in this paper are for the most part model-independent. But, given the results of a particular model, one may easily use the techniques and results of this paper to obtain all the Higgs boson vertices which appear. We think it is useful to illustrate some of the aspects of the procedure by which one obtains the appropriate low-energy parameters from a supergravity model. However, it is not our purpose to review supergravity model building techniques here [10, 11]. Fairly detailed models have been studied in the literature (see e.g. ref. [13]) which satisfy the necessary phenomenological requirements. For the purposes of illustration, we exhibit below some of the features of one of the original low-energy supergravity models studied in ref. [12]. Our choice here is motivated by one of simplicity – a minimum of algebra helps to make the procedure quite transparent. Note, however, that this model is certainly not realistic (it requires a very heavy top-quark); the reader is referred to the literature [10, 11, 13] for more realistic examples.

The model of ref. [12] consists of a minimal supersymmetric extension of the standard model, with two Higgs-doublet fields but with no Higgs-singlet field. At the Planck scale (M_P), the parameters of this model satisfy:

$$\tilde{M}_Q \approx \tilde{M}_U \approx \tilde{M}_D \approx m_1 \approx m_2 \approx m_6 \approx \tilde{M}_g , \tag{6.1}$$

$$m_{12}^2 = B\mu \tilde{M}_g , \tag{6.2}$$

$$A_1 \approx A_2 \approx A_u \approx A_d \approx A_0 , \tag{6.3}$$

where A_0 and B are constants of order unity and $\tilde{M}_g$ is the gravitino mass which is expected to be of order m_W. The parameter μ is less certain and one can imagine either $\mu \sim \alpha \tilde{M}_g$ or $\mu \sim \tilde{M}_g$ (where α is some small coupling constant). In the former case, μ is small and to first approximation can be neglected. Then because $m_1^2, m_2^2 > 0$, SU(2) × U(1) is unbroken. However, upon evolution down to scales of order m_W, one finds that $m_2^2 < 0$. In ref. [12], this is triggered by a large Higgs-fermion Yukawa coupling (such as the top-quark). We sketch here some of the details for this particular example. In the evolution of scalar masses, we ignore all couplings except for the top-quark – Higgs-Yukawa coupling. The solution to the renormalization group equations takes the simple form [10, 12]

$$m_2^2(t) = 3C - \tfrac{1}{2}\tilde{M}_g^2 , \tag{6.4}$$

$$\tilde{M}_{U_3}^2(t) = 2C , \tag{6.5}$$

$$\tilde{M}_{Q_3}^2(t) = C + \tfrac{1}{2}\tilde{M}_g^2 , \tag{6.6}$$

where

$$C=\frac{\tilde{M}_g^2}{1-\zeta}\left[1+\tfrac{1}{3}A^2\frac{\zeta}{1-\zeta}\right], \tag{6.7}$$

$$\zeta(t)=\frac{3\alpha_0 t}{\pi}, \tag{6.8}$$

$$t=\ln\frac{m_W}{M_P}, \tag{6.9}$$

$$\alpha=\frac{\alpha_0}{1-\zeta}, \tag{6.10}$$

$$A=\frac{A_0}{1-\zeta}, \tag{6.11}$$

where $\alpha_0 \equiv \lambda^2/4\pi$ is the top-quark Yukawa coupling [analogous to eq. (4.9)], A_0 is given in eq. (6.3), and α and A are the corresponding quantities at the low-energy scale. Note that in eqs. (6.5) and (6.6) we use the subscript 3 to denote the third generation scalar-quark masses. In the approximation we are using, all other scalar-quark and Higgs masses do not run but are fixed at $\tilde{M}_g$ [cf., eq. (6.1)].

We need one boundary condition to fix $t \equiv \ln(m_W/M_P)$. This is obtained by inserting eq. (3.23) into eq. (3.14g), resulting in

$$m_2^2=-\tfrac{1}{2}m_Z^2+\tfrac{1}{2}v_1^2(g^2+g'^2)+m_{12}^2\cot\beta. \tag{6.12}$$

In the approximation where μ is neglected, we may take $m_{12}^2=0$ [see eq. (6.2)]. Also, because m_1^2 does not run (i.e. $m_1^2=\tilde{M}_g^2>0$), it is clear that $v_1=0$. Therefore, eq. (6.12) reduces to $m_2^2(t)=-\frac{1}{2}m_Z^2$. This implies that $C=\frac{1}{6}(\tilde{M}_g^2-m_Z^2)$ and plugging back into eqs. (6.5), (6.6) yields the scalar-quark mass parameters. It then follows that [12]:

$$\tilde{M}_{Q_3}^2=\tfrac{2}{3}\tilde{M}_g^2-\tfrac{1}{6}m_Z^2, \tag{6.13}$$

$$\tilde{M}_{U_3}^2=\tfrac{1}{3}(\tilde{M}_g^2-m_Z^2), \tag{6.14}$$

$$\tilde{M}_{Q_i}^2=\tilde{M}_g^2, \qquad i=1,2, \tag{6.15}$$

$$\tilde{M}_{U_i}^2=\tilde{M}_g^2, \qquad i=1,2, \tag{6.16}$$

$$\tilde{M}_{D_i}^2=\tilde{M}_g^2, \qquad i=1,2,3. \tag{6.17}$$

It is these parameters which are to be inserted into eq. (4.17) to obtain the desired scalar-quark mass matrix. The appropriate value of A [eq. (6.11)] would also have to be used in eq. (4.17). Note, however, that under the assumptions being considered here, the term in the scalar-quark mass matrix which mixes $\tilde{q}_L$ with $\tilde{q}_R$ is non-negligible only for massive quarks (the t-quark or heavier).

Clearly, the above calculation is unrealistic since the approximations we have used implied that $v_1 = 0$. An improvement can be made by taking into account the effects of the parameter μ. According to eq. (6.2), $m_{12}^2 \neq 0$ and this can induce a vacuum expectation value for H_1. Using eqs. (3.21c–e) one finds to leading order in μ

$$v_1 \approx \frac{m_{12}^2 v_2}{m_1^2 + m_2^2}\,. \tag{6.18}$$

Note that $m_1^2 > 0$ and $m_2^2 < 0$. We may now go back and recompute C based on the boundary condition given by eq. (6.12). Now, we may no longer omit the last two terms of eq. (6.12) (they are both of the same order in $\mu \equiv m_{12}^2/B\tilde{M}_g$). We can rewrite eq. (6.12) as:

$$m_2^2 = \tfrac{1}{2} m_Z^2 \cos 2\beta + m_{12}^2 \cot \beta\,. \tag{6.19}$$

The solution for C [from eq. (6.4)] becomes

$$C = \tfrac{1}{6}\left(\tilde{M}_g^2 + m_Z^2 \cos 2\beta + 2 m_{12}^2 \cot \beta\right). \tag{6.20}$$

(This equation reduces to the one we obtained previously, since for $v_1 = 0$, $\beta = 90°$). Plugging into eqs. (6.5) and (6.6), we obtain:

$$\tilde{M}_{Q_3}^2 = \tfrac{2}{3}\tilde{M}_g^2 + \tfrac{1}{6} m_Z^2 \cos 2\beta + \tfrac{1}{3} m_{12}^2 \cot \beta\,, \tag{6.21}$$

$$\tilde{M}_{U_3}^2 = \tfrac{1}{3}\tilde{M}_g^2 + \tfrac{1}{3} m_Z^2 \cos 2\beta + \tfrac{2}{3} m_{12}^2 \cot \beta\,, \tag{6.22}$$

which are to be used in eq. (4.17) to obtain the scalar-quark masses. These mass formulas have been previously obtained in ref. [10].*

Of course, realistic models require numerical solution of a complicated set of renormalization group equations. The resulting scalar-quark masses as well as other parameters of the model must be obtained numerically. Typical results have been presented in refs. [13, 34]. Nevertheless, the analytic formulas displayed above give a rough guide as to possible values for the supersymmetric parameters.

There have also been low-energy supergravity models which make use of the Higgs singlet field N. However, realistic models do not appear to satisfy the requirements of $\mu = 0$, $\langle N \rangle \neq 0$ which we impose in sect. 3 in order to obtain analytic expressions for the neutral Higgs (H_1^0, H_2^0, N) mixing. In particular, we point out that in an interesting model discussed in ref. [16] where all dimensionful parameters in eqs.

* Note that in the notation of ref. [10], their angle α is equal to $\frac{1}{2}\pi - \beta$.

(3.8) (μ, r and Λ) are set to zero, one finds that necessarily $\langle N \rangle \neq 0$ in order to get a realistic particle spectrum. Therefore, in such models with a Higgs singlet field, although the Feynman rules of sect. 4 are still correct, H_1^0, H_2^0 and N will no longer be mass-eigenstates. One will be required to diagonalize numerically a more complicated mass matrix in order to obtain Feynman rules involving physical particles.

The final set of remarks in this section are concerned with the possible appearance of CP-violating phases in the theory [35–37]. We have emphasized in sects. 2 and 3 that in a supersymmetric two-Higgs doublet model, we are free to choose the phases of the weak-doublet fields H_1 and H_2 such that no CP-violating phases appear in the pure H_1, H_2 sector of the theory. This also allows us to choose the vacuum expectation values v_1 and v_2 to be real and non-negative. Having implemented this convention, CP-violating phases can in general appear elsewhere in the theory. These can arise from a number of sources [see eqs. (3.8), (3.9), (4.15) and (4.29)]. First, the parameters μ, M, M', A_1, A_2, $A_{\rm u}$ and $A_{\rm d}$ are in general complex. This can lead to CP-violation in the $\mathrm{H}^0\tilde{\mathrm{q}}\tilde{\mathrm{q}}$ interactions [eq. (4.19)] and the $\mathrm{H}^0\tilde{\chi}\tilde{\chi}$ interactions [eqs. (4.39) and (4.54)]. The easiest way to identify CP-violation is as follows. In a CP-invariant theory, we have shown that H_1^0 and H_2^0 are CP-even states and H_3^0 is a CP-odd state. Violations of these conditions are a signal of CP-violation. For example, in eqs. (4.39) and (4.54), CP-invariance requires the diagonal $\mathrm{H}^0\tilde{\chi}_i\tilde{\chi}_i$ interaction to be of the form

$$\mathscr{L}_{\mathrm{H}^0\tilde{\chi}\tilde{\chi}} = \left(a_i H_1^0 + b_i H_2^0\right)\bar{\tilde{\chi}}_i\tilde{\chi}_i + ic_i H_3^0\bar{\tilde{\chi}}_i\gamma_5\tilde{\chi}_i, \tag{6.23}$$

where a_i, b_i and c_i are *real* constants. In eq. (6.23) $\tilde{\chi}_i$ stands for either a chargino or neutralino field. This implies that the diagonal elements of the coupling matrices Q, R, Q'' and R'' [defined in eqs. (4.33), (4.36), (4.48) and (4.51)] are real. Second, if the singlet N field is present, then h, r, m_5^2 and a possible vacuum expectation value $\langle N \rangle$ (which would depend on some of the previously mentioned parameters) can also be complex. One could choose the phases of N and the scalar-quark fields to eliminate a few of the phases but some non-trivial phases must remain. This could be a serious constraint on supersymmetric models [35–37]. For example, the absence of an observed neutron electric dipole moment [36] requires that such phases be very small (if not absent altogether). A natural explanation of the smallness of such phases would be highly desirable.

One can peruse the Feynman rules for the occurrence of possible sources of CP-violation. Some examples: if there is a singlet Higgs field N, one has in general complex HHN couplings. In general, the $\mathrm{H}\tilde{\mathrm{q}}\tilde{\mathrm{q}}$ couplings [eq. (4.19)] will exhibit CP-violating phases due to the presence of complex μ and A-parameters. It is interesting to note that in the neutral Higgs couplings to quarks, no CP-violating phases occur. Thus, our claim that H_1^0 and H_2^0 are CP-even states and H_3^0 is a CP-odd state remain valid (at least at tree-level) as far as its interactions with the quarks are concerned. Likewise, no CP-violating phases occur in the tree-level interactions of the Higgs bosons with the vector gauge bosons.

In models with three or more generations, the charged Higgs interactions with quarks involve the Cabibbo-Kobayashi-Maskawa (CKM) [38] matrix which possesses the usual *CP*-violating phases. In addition, new generation mixing matrices must be introduced due to the generational mixing of scalar-quarks. These new matrices can also introduce new *CP*-violating phases. The modification of the Feynman rules due to more than one generation of quarks and scalar-quarks is discussed in appendix B.

7. Conclusion

It has been obvious for many years that the Higgs sector of electroweak theories is the most sensitive to the nature of interactions at mass scales higher than those currently probed experimentally. Thus many theoretical uncertainties regarding the Higgs sector have emerged. In particular, there are the problems of hierarchy and naturalness, the number of Higgs doublets, the possibility of higher Higgs representations, composite Higgs and so forth. Of the existing models which propose to solve the hierarchy and naturalness problems, supersymmetric theories are unique in two respects:

(i) They are completely consistent internally and at present suffer no known phenomenological defects.

(ii) They have the potential to solve the hierarchy/naturalness problems while maintaining the elementarity of the Higgs.

In this paper we have chosen to examine in detail minimal supersymmetric theories. At least two Higgs doublets are required in order to give mass to both up and down type quarks. In the absence of other scalar Higgs fields, $SU(2) \times U(1)$ is not broken until soft supersymmetry breaking terms are added. Thus we have also considered the case in which an additional complex scalar field, an $SU(2) \times U(1)$ gauge singlet, is introduced so that $SU(2) \times U(1)$ may be broken at tree level even in the absence of supersymmetry breaking.

While supersymmetric theories provide a direct motivation for a two-Higgs doublet model, they simultaneously impose severe constraints on the otherwise enormously model-dependent self-coupling of the Higgs. Of course, as part of the solution to the hierarchy problem, couplings to new supersymmetric partners of the ordinary particles appear. The purpose of this paper has been to enumerate all the Higgs couplings that are of most immediate phenomenological interest. These include:

(a) couplings to gauge particles, figs. 1–6;
(b) couplings to ordinary fermions, figs. 7–8;
(c) self-couplings, figs. 9–10;
(d) couplings to scalar quarks, figs. 11–15 and 17–18; and
(e) couplings to charginos and neutralinos, figs. 19–21.

For completeness, we have also derived rules for the coupling of quarks and scalar-quarks to charginos and neutralinos shown in figs. 22–24. These are related (in part) by supersymmetry to (b) and (d) above, and are therefore sensitive to the Higgs boson sector of the model.

All of the couplings we have obtained under (a) and (b), above, are the same as those which appear in certain non-supersymmetric two-doublet models in which a fully general choice of vacuum expectation values is allowed for. There are, however, many aspects of these couplings and constraints among them which have not been fully explored in the literature. For instance $\beta \equiv \tan^{-1}(v_2/v_1)$ and the mixing angle α which results from diagonalization of the neutral Higgs boson mass matrix yield potentially enhanced couplings of the charged Higgs couplings to quarks. In some low-energy supergravity models, these angles tend to take on extreme values which could result in unexpected phenomenological consequences. Also, the absence of certain couplings (e.g. no WZH vertex) can have important phenomenological implications for expectations regarding Higgs production.

The Higgs self-couplings become of phenomenological importance when one Higgs is much more massive than others, and its decay into two lighter Higgs is allowed. Trilinear Higgs couplings also yield new sources of single Higgs production through a process analogous to the effective W approximation [41], in which the fusing virtual gauge particles are replaced by virtual Higgs.

The couplings of Higgs bosons and scalar-quarks yield new contributions (through scalar quark loops) to the gluon-gluon fusion mechanism for Higgs production [42]. Due to cancellations, these are not as large as the order g couplings which appear in the Feynman rules of figs. 12–13 might suggest [as discussed below eq. (4.23)], but can result in a significant enchancement to Higgs production cross sections. The order g couplings are certainly important to the phenomenology of Higgs decays if the scalar quarks are sufficiently light that these channels are open.

In a future paper [19] we shall explore some of the above phenomenological consequences of minimal two-Higgs doublet supersymmetric theories. The Higgs sector in such theories, while varied and complex, is tightly constrained. Above threshold for production of Higgs particles, the phenomenology of their production, interaction and decay will provide an important testing ground for the theory and help constrain the nature of supersymmetry breaking. As an example, a Higgs doublet with enhanced couplings to *both* up- and down-quarks would be incompatible with the two-doublet Higgs supersymmetry model [42]. In general, the discovery of multiple Higgs doublets (or convincing evidence for *only one* doublet) would provide important insight into the viability of low-energy supersymmetry.

In the absence of the gauge singlet field, the minimal two-Higgs doublet model requires that one of the neutral Higgs lies below the mass of the Z. It could well appear in toponium decays and other reactions that will soon be available. In such models, the H^+ is always heavier than the W^+. However, in models with a gauge singlet field present, there is a range of parameters for which the H^+ is sufficiently light so that it could appear in W and Z decays. In general the minimal supersymmetric models suggest that some of the Higgs masses are modest in size and perhaps accessible in the near future. Thus, the Higgs sector may play a crucial role in suggesting the nature of new physics beyond the standard model as well as revealing the nature of spontaneous symmetry breaking and the generation of the electroweak scale.

We are grateful to Larry Hall for his comments concerning *CP*-violating phases. In addition, we acknowledge Christian Preitschopf for discussions regarding aspects of supergravity model building.

Appendix A

CHARGINO AND NEUTRALINO MIXING

In this appendix, we will summarize the required formalism needed to obtain the mass-eigenstates in the gaugino-higgsino sector of the theory. For details, we direct the reader to appendix C of ref. [18].* (See also refs. [30–32].)

A.1. Charginos. The charginos, $\tilde{\chi}_i^+$ $(i=1,2)$, are four-component Dirac fermions which arise due to the mixing of the winos, $\tilde{W}^-$, $\tilde{W}^+$, and the charged higgsinos, $\tilde{H}_1^-$ and $\tilde{H}_2^+$. Because there are actually two independent mixings, $(\tilde{W}^-, \tilde{H}_1^-)$ and $(\tilde{W}^+, \tilde{H}_2^+)$, we shall need to define two unitary mixing matrices [31]. We define:

$$\begin{aligned} \psi_j^+ &= \left(-i\lambda^+, \psi_{H_2}^+\right), \\ \psi_j^- &= \left(-i\lambda^-, \psi_{H_1}^-\right), \end{aligned} \qquad j=1,2, \tag{A.1}$$

where we have used the notation of table 1. In eq. (A.1), the fields are two-component fermion fields, with $\lambda^{\pm} = \sqrt{\frac{1}{2}}(\lambda^1 \mp i\lambda^2)$. The mass term in the lagrangian is:

$$\mathscr{L}_{\mathrm{m}} = -\tfrac{1}{2}(\psi^+ \psi^-)\begin{pmatrix} 0 & X^{\mathrm{T}} \\ X & 0 \end{pmatrix}\begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix}, \tag{A.2}$$

where

$$X = \begin{pmatrix} M & m_{\mathrm{W}}\sqrt{2}\,\sin\beta \\ m_{\mathrm{W}}\sqrt{2}\,\cos\beta & \mu \end{pmatrix}, \tag{A.3}$$

where M is a Majorana mass term for the winos, μ is defined in eq. (3.3) and $\tan\beta \equiv v_2/v_1$. Note that $m_{\mathrm{W}}^2 = \frac{1}{2}g^2(v_1^2 + v_2^2)$, where the v_i are defined in eq. (3.7).

* The notation in this appendix is identical to that of ref. [18] with two exceptions. We denote here $\langle H_1^1 \rangle = v_1$, $\langle H_2^2 \rangle = v_2$ and $\tan\beta \equiv v_2/v_1$, whereas in ref. [18] v_i is replaced by $\sqrt{\frac{1}{2}}\,v_i$ and $\tan\beta$ is replaced by $\cot\theta_v$.

We define two-component mass-eigenstates via:

$$\begin{aligned}\chi_i^+ &= V_{ij}\psi_j^+ ,\\ \chi_i^- &= U_{ij}\psi_j^- ,\end{aligned}\qquad i, j = 1,2, \tag{A.4}$$

where U and V are unitary matrices chosen such that:

$$U^*XV^{-1} = M_D , \tag{A.5}$$

where M_D is the diagonal chargino mass matrix. In particular, U and V can be chosen so that the elements of the diagonal matrix M_D are real and *non-negative*. The proper four-component mass-eigenstates are the charginos which are defined in terms of the two-component χ_i^+ fields as:

$$\tilde{\chi}_1^+ = \begin{pmatrix} \chi_1^+ \\ \bar{\chi}_1^- \end{pmatrix}, \qquad \tilde{\chi}_2^+ = \begin{pmatrix} \chi_2^+ \\ \bar{\chi}_2^- \end{pmatrix}. \tag{A.6}$$

The supersymmetric limit can be taken where SU(2) × U(1) remains broken if the model possesses a gauge singlet N-field. In this limit (taking $M = \mu = 0$), we find:

$$\sin\beta = \cos\beta = \sqrt{\tfrac{1}{2}} , \tag{A.7}$$

$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \qquad V = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{A.8}$$

Note that $U \neq V$; the difference in the two matrices has been arranged so that the masses of the chargino eigenvalues are positive. In the above limit, we can write the chargino states as

$$\tilde{\chi}_1^+ = \sqrt{\tfrac{1}{2}}\,(\tilde{\omega}_1^+ + \tilde{\omega}_2^+), \qquad \tilde{\chi}_2^+ = \sqrt{\tfrac{1}{2}}\,(\tilde{\omega}_1^+ - \tilde{\omega}_2^+), \tag{A.9}$$

where ω_1 and ω_2 are the wiggsinos:

$$\tilde{\omega}_1^+ = \begin{pmatrix} -i\lambda^+ \\ \bar{\psi}_{H_1}^- \end{pmatrix}, \qquad \tilde{\omega}_2^+ = \begin{pmatrix} \psi_{H_2}^+ \\ i\bar{\lambda}^- \end{pmatrix}. \tag{A.10}$$

Furthermore, in this limit, the chargino states are the wiggsinos which are degenerate in mass with the $W^\pm$. In fact, from eqs. (3.15) and (3.16) we see that $m_{H^\pm} = m_{W^\pm}$ in this limit. It follows that (H^-; $\tilde{\omega}_1^-$; W^-) and (H^+; $\tilde{\omega}_2^+$; W^+) make up two massive supermultiplets consisting of particles with mass equal to m_W.

It is sometimes convenient to work with four-component fields which are not mass-eigenstates but which lead to simpler expressions for interaction terms. We choose to work with $\tilde{W}$ and $\tilde{H}$ defined by:

$$\tilde{W} = \begin{pmatrix} -i\lambda^+ \\ i\bar{\lambda}^- \end{pmatrix}, \qquad \tilde{H} = \begin{pmatrix} \psi^+_{H_2} \\ \bar{\psi}^-_{H_1} \end{pmatrix}. \tag{A.11}$$

If one has an interaction lagrangian involving $\tilde{W}$ and $\tilde{H}$, it is a simple matter to convert it to the appropriate expression involving the chargino mass-eigenstates. Define:

$$P_L = \tfrac{1}{2}(1 - \gamma_5),$$

$$P_R = \tfrac{1}{2}(1 + \gamma_5), \tag{A.12}$$

which project out respectively the top two components and the bottom two components of a four-component spinor. Then, using eqs. (A.4), (A.6) and (A.11) we find:

$$P_L\tilde{W} = P_L(V^*_{11}\tilde{\chi}_1 + V^*_{21}\tilde{\chi}_2), \tag{A.13a}$$

$$P_R\tilde{W} = P_R(U_{11}\tilde{\chi}_1 + U_{21}\tilde{\chi}_2), \tag{A.13b}$$

$$P_L\tilde{H} = P_L(V^*_{12}\tilde{\chi}_1 + V^*_{22}\tilde{\chi}_2), \tag{A.13c}$$

$$P_R\tilde{H} = P_R(U_{12}\tilde{\chi}_1 + U_{22}\tilde{\chi}_2). \tag{A.13d}$$

Using these equations, one can write out any interaction term involving $\tilde{H}$ and $\tilde{W}$ in terms of the charginos, $\tilde{\chi}$. Note that from eqs. (A.13a)–(A.13d) one can derive additional equations, such as:

$$\bar{\tilde{W}}P_R = (V_{11}\bar{\tilde{\chi}}_1 + V_{21}\bar{\tilde{\chi}}_2)P_R, \tag{A.14}$$

where as usual, $\bar{\psi} \equiv \psi^\dagger\gamma^0$.

A.2. Neutralinos. We turn next to the neutralinos, $\tilde{\chi}^0_j$ which are due to the mixing of the photino, zino and neutral higgsinos. Here, $j = 1,\ldots,4$ in the minimal model with no gauge singlet N-field. If an N-field is included in the model, then the model necessarily contains an extra higgsino resulting in five neutralinos, so we must take $j = 1,\ldots,5$. We shall consider the two possible cases in turn.

In the case with four neutralinos, we define the two-component fermion fields:

$$\psi^0_j = \left(-i\lambda', -i\lambda^3, \psi^0_{H_1}, \psi^0_{H_2}\right). \tag{A.15}$$

Again using the notation of table 1, λ^3 is the neutral wino and λ' is the bino. These fields can also be expressed in terms of the (two-component) photino and zino via:

$$\lambda_z = \lambda^3 \cos\theta_W - \lambda' \sin\theta_W\,, \tag{A.16a}$$

$$\lambda_\gamma = \lambda^3 \sin\theta_W + \lambda' \cos\theta_W\,. \tag{A.16b}$$

Occasionally, it will be useful to define

$$\psi_j'^0 = \left(-i\lambda_\gamma, -i\lambda_z, \psi_{H_1}^0, \psi_{H_2}^0\right) \tag{A.17}$$

in place of eq. (A.15). The mass term in the lagrangian is given by:

$$\mathscr{L}_m = -\tfrac{1}{2}\left(\psi^0\right)^T Y \psi^0 + \text{h.c.}, \tag{A.18}$$

where Y is in general a complex *symmetric* matrix* given by:

$$Y = \begin{pmatrix} M' & 0 & -m_Z \sin\theta_W \cos\beta & m_Z \sin\theta_W \sin\beta \\ 0 & M & m_Z \cos\theta_W \cos\beta & -m_Z \cos\theta_W \sin\beta \\ -m_Z \sin\theta_W \cos\beta & m_Z \cos\theta_W \cos\beta & 0 & -\mu \\ m_Z \sin\theta_W \sin\beta & -m_Z \cos\theta_W \sin\beta & -\mu & 0 \end{pmatrix} \tag{A.19}$$

M' is the Majorana mass for the bino; all other terms above have been previously defined. As usual, $m_Z^2 = \frac{1}{2}(g^2 + g'^2)(v_1^2 + v_2^2)$. We define two-component mass-eigenstates using:

$$\chi_i^0 = N_{ij}\psi_j^0\,, \qquad i, j = 1,\dots,4, \tag{A.20}$$

where N is a unitary matrices satisfying:

$$N^* Y N^{-1} = N_D\,, \tag{A.21}$$

where N_D is the diagonal neutralino mass matrix. One can choose N such that the elements of the diagonal matrix N_D are real and non-negative. The proper four-component mass-eigenstates are the neutralinos which are defined in terms of the two-component $\tilde{\chi}_i^0$ fields as

$$\tilde{\chi}_i^0 = \begin{pmatrix} \chi_i^0 \\ \bar{\chi}_i^0 \end{pmatrix} \qquad (i = 1,\dots 4)\,. \tag{A.22}$$

Note that the $\tilde{\chi}_i^0$ are Majorana fermions.

* The fact that Y is symmetric follows from eq. (4.46) and is due to the Majorana nature of the neutralinos. As a result, only one diagonalizing matrix N [eq. (A.21)] is required in this case.

If we had wished to make use of eq. (A.17) instead of eq. (A.15), then the matrix Y would be replaced by a matrix Y' and the unitary matrices N would be replaced by a new matrix N' given by:

$$\begin{aligned} N'_{j1} &= N_{j1}\cos\theta_W + N_{j2}\sin\theta_W\,, \\ N'_{j2} &= -N_{j1}\sin\theta_W + N_{j2}\cos\theta_W\,, \\ N'_{j3} &= N_{j3}\,, \\ N'_{j4} &= N_{j4}\,. \end{aligned} \tag{A.23}$$

As above, interactions often look simpler in terms of four-component fields which are not mass eigenstates. We define the following four-component (neutral) Majorana spinors:

$$\tilde{B} = \begin{pmatrix} -i\lambda' \\ i\bar{\lambda}' \end{pmatrix}, \qquad \tilde{W}_3 = \begin{pmatrix} -i\lambda^3 \\ i\bar{\lambda}^3 \end{pmatrix}, \qquad \tilde{H}_1 = \begin{pmatrix} \psi^0_{H_1} \\ \bar{\psi}^0_{H_1} \end{pmatrix}, \qquad \tilde{H}_2 = \begin{pmatrix} \psi^0_{H_2} \\ \bar{\psi}^0_{H_2} \end{pmatrix}. \tag{A.24}$$

We may then relate the above spinors [eq. (A.24)] to the mass eigenstates [eq. (A.22)] using relations analogous to those given in eq. (A.13). For example,

$$P_L \tilde{H}_i = P_L \sum_j N^*_{j,i+2}\tilde{\chi}^0_j\,, \tag{A.25a}$$

$$P_R \tilde{H}_i = P_R \sum_j N_{j,i+2}\tilde{\chi}^0_j\,, \tag{A.25b}$$

where we have used the fact that N is unitary, with similar equations for $\tilde{B}$ and $\tilde{W}_3$. It is sometimes convenient to introduce the four-component photino ($\tilde{\gamma}$) and zino ($\tilde{Z}$) Majorana spinors:

$$\tilde{\gamma} = \begin{pmatrix} -i\lambda_\gamma \\ i\bar{\lambda}_\gamma \end{pmatrix}, \qquad \tilde{Z} = \begin{pmatrix} -i\lambda_Z \\ i\bar{\lambda}_Z \end{pmatrix}, \tag{A.26}$$

which are related to $\tilde{W}_3$ and $\tilde{B}$ by the obvious relations [see eq. (A.16)]. Then, to express $\tilde{\gamma}$ and $\tilde{Z}$ in terms of the mass eigenstates $\tilde{\chi}_i$, we need to use the matrix N'. For example,

$$P_L \tilde{\gamma} = P_L \sum_j N'^*_{j1}\tilde{\chi}^0_j\,, \tag{A.27a}$$

$$P_R \tilde{\gamma} = P_R \sum_j N'_{j1}\tilde{\chi}^0_j\,. \tag{A.27b}$$

The analysis above assumed that there were only four neutralino states. If we include the SU(2) × U(1) singlet field N, then the (two-component) higgsino field ψ_N must be included in the discussion. As discussed in case 1 of sect. 3, we can obtain explicit analytic expressions for all results of interest if we assume that $\mu = \langle N \rangle = 0$. In this case we expand our previous definitions. In place of eq. (A.15) we have:

$$\psi_j^0 = \left(-i\lambda', -i\lambda^3, \psi^0_{H_1}, \psi^0_{H_2}, \psi_N \right). \tag{A.28}$$

Eq. (A.18) defines the mass matrix, where Y is now a 5×5 matrix. Setting $\mu = 0$, we obtain:

$$Y = \begin{pmatrix} M' & 0 & \sqrt{\tfrac{1}{2}}(-v_1 g') & \sqrt{\tfrac{1}{2}}(v_2 g') & 0 \\ 0 & M & \sqrt{\tfrac{1}{2}}(v_1 g) & \sqrt{\tfrac{1}{2}}(-v_2 g) & 0 \\ \sqrt{\tfrac{1}{2}}(-v_1 g') & \sqrt{\tfrac{1}{2}}(v_1 g) & 0 & 0 & hv_2 \\ \sqrt{\tfrac{1}{2}}(v_2 g') & \sqrt{\tfrac{1}{2}}(-v_2 g) & 0 & 0 & hv_1 \\ 0 & 0 & hv_2 & hv_1 & 0 \end{pmatrix}. \tag{A.29}$$

To be different, we have replaced m_Z, θ_W and β in eq. (A.19) with v_1, v_2, g and g'. Most of the remaining formulas go through. By using the appropriate generalization of eq. (A.25) (i.e. summing over five possible neutralino states), the physical neutralinos $\tilde{\chi}_i^0$ can then be expressed in terms of the fields of eq. (A.24) and

$$\tilde{N} = \begin{pmatrix} \psi_N \\ \bar{\psi}_N \end{pmatrix}. \tag{A.30}$$

The advantage of including the fifth neutralino state is that it permits a supersymmetric limit which still breaks the SU(2) × U(1) symmetry. In this limit (where $M' = M = \mu = 0$ and $v_1 = v_2$), it is convenient to choose the following basis instead of eq. (A.28):

$$\tilde{\psi}_j'^0 = \left[-i\lambda_\gamma, -i\lambda_z, \sqrt{\tfrac{1}{2}}\left(\psi^0_{H_1} - \psi^0_{H_2}\right), \sqrt{\tfrac{1}{2}}\left(\psi^0_{H_1} + \psi^0_{H_2}\right), \psi_N \right]. \tag{A.31}$$

The mass matrix is then

$$-\mathscr{L}_m^{(0)} = -\sqrt{\tfrac{1}{2}}\, i m_Z \lambda_z \left(\psi^0_{H_1} - \psi^0_{H_2}\right) + hv\psi_N\left(\psi^0_{H_1} + \psi^0_{H_2}\right), \tag{A.32}$$

where $v = v_1 = v_2$. Note that in the supersymmetric limit the neutral Higgs-boson spectrum is $m_{H_3^0} = m_{H_1^0} = m_N$ and $m_{H_2^0} = m_Z$ [see discussion below eq. (3.20)], where

$m_N = \sqrt{2}\,hv$. We therefore define the ziggsino state:

$$\tilde{\zeta} = \begin{pmatrix} \sqrt{\tfrac{1}{2}}\left(\psi^0_{\mathrm{H}_1} - \psi^0_{\mathrm{H}_2}\right) \\ i\bar{\lambda}_{\mathrm{Z}} \end{pmatrix} \tag{A.33}$$

and the higgsino state:

$$\tilde{h} = \begin{pmatrix} \sqrt{\tfrac{1}{2}}\left(\psi^0_{\mathrm{H}_1} + \psi^0_{\mathrm{H}_2}\right) \\ \psi_N \end{pmatrix}, \tag{A.34}$$

which are both four-component *Dirac* spinors. In addition, we have the photino

$$\tilde{\gamma} = \begin{pmatrix} -i\lambda_\gamma \\ i\bar{\lambda}_\gamma \end{pmatrix}, \tag{A.35}$$

which is a four-component Majorana spinor. In the supersymmetric limit, we see that the photino is massless, $\tilde{M}_{\tilde{\zeta}} = m_Z$ and $\tilde{M}_{\tilde{h}} = m_N$. The massive supersymmetric multiplets are then identified as $(H_2^0;\ \tilde{\zeta};\ Z^0)$ and $(H_1^0,\ H_3^0, \mathrm{Re}\sqrt{\tfrac{1}{2}}\,N, \mathrm{Im}\sqrt{\tfrac{1}{2}}\,N;\ \tilde{h})$, and the $(\tilde{\gamma};\ \gamma)$ supermultiplet stays massless.

Finally, we can compute the values of the diagonalizing matrix N [see eqs. (A.21) and (A.29)] which produces the diagonal mass matrix given by eq. (A.32). The result is:

$$N = \begin{pmatrix} \cos\theta_{\mathrm{W}} & \sin\theta_{\mathrm{W}} & 0 & 0 & 0 \\ -\sqrt{\tfrac{1}{2}}\,\sin\theta_{\mathrm{W}} & \sqrt{\tfrac{1}{2}}\,\cos\theta_{\mathrm{W}} & \tfrac{1}{2} & -\tfrac{1}{2} & 0 \\ -\sqrt{\tfrac{1}{2}}\,i\sin\theta_{\mathrm{W}} & \sqrt{\tfrac{1}{2}}\,i\cos\theta_{\mathrm{W}} & -\tfrac{1}{2}i & \tfrac{1}{2}i & 0 \\ 0 & 0 & \tfrac{1}{2} & \tfrac{1}{2} & \sqrt{\tfrac{1}{2}} \\ 0 & 0 & \tfrac{1}{2}i & \tfrac{1}{2}i & -\sqrt{\tfrac{1}{2}}\,i \end{pmatrix}. \tag{A.36}$$

A few subtleties are worth mentioning. The factors of i in the third and fifth rows have been chosen so that the neutralino eigenvalues are all non-negative. This is possible because of the appearance of N^* in eq. (A.21). (An alternative method is to allow for negative mass eigenvalues for some of the neutralinos. Then one must multiply the corresponding neutralino spinors by γ_5.) Using eq. (A.36), one can read off the physical neutralino states by examining the rows of N. For example, the first row corresponds to the photino given in eq. (A.16b). However, using this method, one gets (in terms of four-component fermions) Majorana fermions (i.e. the $\tilde{\chi}_i^0$) rather than the Dirac fermions given by eqs. (A.33) and (A.34). In the supersymmetric limit the resulting Majorana spinors can be defined as follows:

$$\tilde{\zeta}_i = \begin{pmatrix} \psi_i \\ \bar{\psi}_i \end{pmatrix}, \qquad \tilde{h}_i = \begin{pmatrix} \xi_i \\ \bar{\xi}_i \end{pmatrix}, \tag{A.37}$$

where $i=1,2$ and ψ_i and ξ_i are defined below:

$$\psi_1=\sqrt{\tfrac{1}{2}}\left[-i\lambda_z+\sqrt{\tfrac{1}{2}}\left(\psi^0_{H_1}-\psi^0_{H_2}\right)\right], \tag{A.38}$$

$$-i\psi_2=\sqrt{\tfrac{1}{2}}\left[-i\lambda_z-\sqrt{\tfrac{1}{2}}\left(\psi^0_{H_1}-\psi^0_{H_2}\right)\right], \tag{A.39}$$

$$\xi_1=\sqrt{\tfrac{1}{2}}\left[\psi_N+\sqrt{\tfrac{1}{2}}\left(\psi^0_{H_1}+\psi^0_{H_2}\right)\right], \tag{A.40}$$

$$-i\xi_2=\sqrt{\tfrac{1}{2}}\left[\psi_N-\sqrt{\tfrac{1}{2}}\left(\psi^0_{H_1}+\psi^0_{H_2}\right)\right]. \tag{A.41}$$

In terms of our previous notation, $\tilde{\chi}^0_1=\tilde{\gamma}$, $\tilde{\chi}^0_{2,3}=\tilde{\zeta}^0_{1,2}$ and $\tilde{\chi}^0_{4,5}=\tilde{h}^0_{1,2}$. The factors of i in eqs. (A.39) and (A.41) correspond to the factors of i in the matrix N [eq. (A.36)]; these factors insure that the neutralino masses are all non-negative. Of course, the physical content of eq. (A.37) is identical to eqs. (A.33) and (A.34). Namely, a neutral Dirac fermion is equivalent to two Majorana fermions which are degenerate in mass.

The appearance of the factors of i in eqs. (A.39) and (A.41) appear less mysterious if we write out the corresponding four-component equations. In the chiral basis where $\gamma_5=\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$, eq. (A.39) can be written as:

$$i\gamma_5\tilde{\zeta}_2=\sqrt{\tfrac{1}{2}}\left[\tilde{Z}-\sqrt{\tfrac{1}{2}}\left(\tilde{H}_1-\tilde{H}_2\right)\right]. \tag{A.42}$$

In eq. (A.42), the factor of i is an irrelevant phase factor which we shall dispose of in the next section (see eqs. (A.50) and (A.51)). The factor of γ_5 is important and insures that the mass of $\tilde{\zeta}_2$ is non-negative.

The $H\tilde{\chi}^0_i\tilde{\chi}^0_j$ rules are easily obtained in the supersymmetric limit. The matrices Q'' and R'' which appear in eq. (4.54) take on a simple form:

$$Q''=\frac{1}{4\sqrt{2}\cos\theta_W}\begin{pmatrix}0 & 0 & 0 & 0 & 0\\ 0 & 2 & 0 & 1-x & i(1+x)\\ 0 & 0 & 2 & i(1+x) & -1+x\\ 0 & 1-x & i(1+x) & 2x & 0\\ 0 & i(1+x) & -1+x & 0 & 2x\end{pmatrix}, \tag{A.43a}$$

$$R''=0, \tag{A.43b}$$

where

$$x=\frac{\sqrt{2}\,h^*\cos\theta_W}{g}. \tag{A.44}$$

A.3. The problem of negative mass eigenvalues. In the previous two sections of this appendix, we have defined the diagonalizing matrices U,V [eq. (A.5)] and N [eq. (A.21)] such that the diagonal elements of the mass matrices were real and *non-negative*. It is sometimes more convenient to allow the (real) mass eigenvalues to be either positive or negative. If the mass eigenvalue is negative, then one must replace the corresponding four-component eigenspinor $\tilde{\chi}$ by $\gamma_5\tilde{\chi}$ in the interaction lagrangian. Let us see how this works out in practice. Replace eqs. (A.4), (A.5), (A.20) and (A.21) by the following:

$$\chi_i^+ = W_{ij}\psi_j^+ , \qquad (i, j=1,2), \tag{A.45a}$$

$$\chi_i^- = U_{ij}\psi_j^- , \qquad (i, j=1,2), \tag{A.45b}$$

$$\chi_i^0 = Z_{ij}\psi_j^0 , \qquad (i, j=1,\ldots,n), \tag{A.45c}$$

$$U^*XW^{-1} = \mathrm{diag}\left(\eta_1\tilde{M}_1^{(+)}, \eta_2\tilde{M}_2^{(+)}\right), \tag{A.46}$$

$$Z^*YZ^{-1} = \mathrm{diag}\left(\varepsilon_1\tilde{M}_1^{(0)},\ldots, \varepsilon_n\tilde{M}_n^{(0)}\right), \tag{A.47}$$

where n is the number of neutralino states (either four or five in this paper), "diag" means a diagonal matrix (with the diagonal entries listed in parentheses), the $\tilde{M}_i$ are non-negative masses and ε_i and η_i are either ± 1. U, W and Z are unitary matrices. Technically, one determines the matrices by solving the eigenvalue problem for $XX^\dagger$, $X^\dagger X$ and $Y^\dagger Y$. This determines the diagonal elements of eqs. (A.46) and (A.47) up to a sign. We can arrange the phases of these matrices to give non-negative mass eigenvalues as we did in previous sections of the appendix. In this section, we allow for the appearance of negative eigenvalues as shown in eqs. (A.46) and (A.47). The question then arises: how will this change the Feynman rules which we have derived in sects. 4 and 5.

We demand that the lagrangian contain only non-negative masses for the charginos and neutralinos. In two-component notation, what appears in the lagrangian is (summed over i):

$$-\mathscr{L}_m = \eta_i\tilde{M}_i^{(+)}\left(\chi_i^+\chi_i^- + \bar{\chi}_i^+\bar{\chi}_i^-\right) + \varepsilon_i\tilde{M}_i^{(0)}\left(\chi_i^0\chi_i^0 + \bar{\chi}_i^0\bar{\chi}_i^0\right). \tag{A.48}$$

We require that in four-component notation, eq. (A.48) must read:

$$-\mathscr{L}_m = \tilde{M}_i^{(+)}\bar{\tilde{\chi}}_i^+\tilde{\chi}_i^+ + \tilde{M}_i^{(0)}\bar{\tilde{\chi}}_i^0\tilde{\chi}_i^0 . \tag{A.49}$$

This implies that we must define our charginos and neutralino fields as follows:

$$\tilde{\chi}_i^+ = \left(\eta_i P_\mathrm{L} + P_\mathrm{R}\right)\begin{pmatrix} \chi_i^+ \\ \bar{\chi}_i^- \end{pmatrix}, \tag{A.50}$$

$$\tilde{\chi}_i^0 = \left(\varepsilon_i P_\mathrm{L} + P_\mathrm{R}\right)\begin{pmatrix} \chi_i^0 \\ \bar{\chi}_i^0 \end{pmatrix}. \tag{A.51}$$

Note that for $\varepsilon_i = -1$, $P_\mathrm{R} - P_\mathrm{L} = \gamma_5$ which confirms the statement made earlier.

[For $\varepsilon_i = 1$, $P_L + P_R = 1$ and there is no change from eqs. (A.6) and (A.22).] In order to see how this affects interaction terms, all we need to do is determine how eqs. (A.13) and (A.25) change. Clearly, only the equations involving P_L change (since $P_L^2 = P_L$, $P_R^2 = P_R$ and $P_L P_R = 0$). The new results are:

$$P_L \tilde{W} = P_L \left(W_{11}^* \eta_1 \tilde{\chi}_1 + W_{21}^* \eta_2 \tilde{\chi}_2 \right), \tag{A.52}$$

$$P_L \tilde{H} = P_L \left(W_{12}^* \eta_1 \tilde{\chi}_1 + W_{22}^* \eta_2 \tilde{\chi}_2 \right), \tag{A.53}$$

$$P_L \tilde{H}_i = P_L \sum_j Z_{j,i+2}^* \varepsilon_j \tilde{\chi}_j^0, \tag{A.54}$$

$$P_R \tilde{H}_i = P_R \sum_j Z_{j,i+2} \tilde{\chi}_j^0. \tag{A.55}$$

If we compare now with eqs. (A.13a)–(A.13c) and (A.25), we can make the following identification:

$$V_{ij} = \eta_i W_{ij}, \qquad \text{(no sum over } i\text{)}, \tag{A.56}$$

$$N_{ij} \tilde{\chi}_i^0 \to Z_{ij} \tilde{\chi}_i^0, \qquad \text{(no sum over } i\text{)}, \tag{A.57}$$

$$N_{ij}^* \tilde{\chi}_i^0 \to \varepsilon_i Z_{ij}^* \tilde{\chi}_i^0, \qquad \text{(no sum over } i\text{)}. \tag{A.58}$$

In eqs. (A.57) and (A.58), we have used the arrow to mean "make the replacement" since if it were an equality, then eqs. (A.57) and (A.58) would be incompatible. Eqs. (A.56)–(A.58) [or eq. (A.59) below] is the appropriate recipe for using in the Feynman rules stated in sects. 4 and 5 if negative mass eigenvalues are obtained. Note that eq. (A.57) also implies the substitution rule $\bar{\tilde{\chi}}_i^0 N_{ij}^* \to \bar{\tilde{\chi}}_i^0 Z_{ij}^*$. Thus, in a Feynman rule where the $\tilde{\chi}_i^0$ is *annihilated*, N_{ij}^* is replaced by $\varepsilon_i Z_{ij}^*$ [see eq. (A.58)]. But, if the $\tilde{\chi}_i^0$ is *created*, N_{ij}^* is replaced by Z_{ij}^*.

There is a second alternative: eqs. (A.57) and (A.58) can be replaced by:

$$N_{ij} = \varepsilon_i^{1/2} Z_{ij} \qquad \text{(no sum over } i\text{)}. \tag{A.59}$$

This satisfies the requirement that eqs. (A.57) and (A.58) have opposite signs when $\varepsilon_i = -1$ since then $\varepsilon_i^{1/2} = i$ changes sign under complex conjugation.

We give two simple examples of the above procedure by examining the supersymmetric limit. First, the chargino mass matrix, X is off-diagonal and real symmetric. It can therefore be diagonalized by a single real orthogonal matrix $W = U$ and the resulting eigenvalues are $\pm m_W$. By eq. (A.56) we see that $V_{1j} = U_{1j}$ and $V_{2j} = -U_{2j}$ which confirms eq. (A.8). Second, the neutralino mass matrix, Y, is off-diagonal and real symmetric. It can be diagonalized by a real orthogonal matrix Z and has five eigenvalues: 0, $\pm m_Z$, $\pm m_N$. By eq. (A.59), $N_{3j} = iZ_{3j}$ and $N_{5j} = iZ_{5j}$, $N_{ij} = Z_{ij}$ for $i = 1, 2, 4$. This explains the appearance of the factors of i in eq. (A.36). However, when it comes to the Feynman rules involving neutralinos, it is perfectly acceptable to make the replacement (A.57) and (A.58) instead of using eq. (A.59). This procedure has the advantage that it avoids the proliferation of factors of i's in the rules where they are not really needed. In the above example, the two alternatives correspond to defining the eigenstate corresponding to the mass eigenvalue of $-m_Z$ to be $i\gamma_5\tilde{\xi}_2$ or $\gamma_5\tilde{\xi}_2$ (corresponding to eqs. (A.42) and (A.51) respectively). Thus, the respective Feynman rules involving one incoming $\tilde{\xi}_2$ field differ by a factor of i. Of course, in the end, the physical consequences of either set of rules must be identical.

Appendix B

EXTENSION TO MORE THAN ONE GENERATION OF QUARKS AND SCALAR-QUARKS

Although we have confined the discussion in this paper to the case of one generation of quarks (and scalar-quarks), the extension to multigenerations is straightforward. However, one must be careful since, a priori, the Cabibbo-Kobayashi-Maskawa (CKM) [38] angles in the scalar-quark sector can be different from the usual CKM angles which appear in the quark sector [39–40]. The precise details are a model dependent question, although the absence of flavor-changing neutral currents does impose nontrivial (but not impossible) constraints on the model-building [39–40]. In this appendix we shall briefly indicate some of the changes which occur for the mutligeneration case. If we put in the generational indices in eq. (3.4), we obtain for the terms involving the scalar-quarks:

$$W_F = \varepsilon_{ij}\left[f_1^{ab} H_1^i \tilde{Q}_a^j \tilde{D}_b + f_2^{ab} \tilde{Q}_a^i H_2^j \tilde{U}_b \right], \tag{B.1}$$

where f_1 and f_2 are now matrices in generation space.

Eq. (B.1) leads to the following terms in the supersymmetric lagrangian (using two-component notation for the fermions):

$$\mathscr{L} = -\sum_i \left| \frac{\partial W_F}{\partial A_i} \right|^2 - \tfrac{1}{2}\left(\sum_{i,j} \frac{\partial^2 W_F}{\partial A_i\, \partial A_j} \psi_i \psi_j + \text{h.c.} \right), \tag{B.2}$$

where A_i is a generic notation for the scalar fields in eq. (B.1). Our first task is to diagonalize the quark mass matrix thereby identifying f_1 and f_2 in eq. (B.1). Here, we can simply use the same mixing formalism which we employed for the charginos in appendix A. We denote the two-component "interaction" eigenstates as:

$$\psi_{\mathrm{Q}_{ib}} = \left(\psi_{Q_{1b}}, \psi_{Q_{2b}} \right), \tag{B.3a}$$

$$\psi_{\mathrm{R}_{ib}} = \left(\psi_{U_b}, \psi_{D_b} \right), \tag{B.3b}$$

corresponding to the left- and right-handed quarks, respectively, where b is a generation label. The quark eigenstates of definite mass are defined by:

$$\xi_{ia} = V_{iab} \psi_{\mathrm{Q}_{ib}}, \tag{B.4a}$$

$$\eta_{ia} = U_{iab} \psi_{\mathrm{R}_{ib}}, \tag{B.4b}$$

where U_i, V_i $(i=1,2)$ are unitary matrices. The four-component quark spinors are then:

$$u_{0a}=\begin{pmatrix}\psi_{Q_{1a}}\\ \bar{\psi}_{U_a}\end{pmatrix},\qquad d_{0a}=\begin{pmatrix}\psi_{Q_{2a}}\\ \bar{\psi}_{D_a}\end{pmatrix},\tag{B.5}$$

$$u_a=\begin{pmatrix}\xi_{1a}\\ \bar{\eta}_{1a}\end{pmatrix},\qquad d_a=\begin{pmatrix}\xi_{2a}\\ \bar{\eta}_{2a}\end{pmatrix}.\tag{B.6}$$

We can simply transcribe the desired results from eqs. (A.4)–(A.6). The quark mass term is given by

$$-\mathscr{L}_{\mathrm{m}}=\sum_{i=1}^{2}\psi_{\mathrm{R}_{ia}}X_{iab}\psi_{\mathrm{Q}_{ib}}+\text{h.c.}\tag{B.7a}$$

$$=\sum_{i=1}^{2}\eta_{ia}M_{iab}\xi_i+\text{h.c.},\tag{B.7b}$$

where M_i are the diagonal quark mass matrices:

$$M_1\equiv M_{\mathrm{u}}=\operatorname{diag}(m_{\mathrm{u}1},m_{\mathrm{u}2},\ldots,),\tag{B.8a}$$

$$M_2\equiv M_{\mathrm{d}}=\operatorname{diag}(m_{\mathrm{d}1},m_{\mathrm{d}2},\ldots,),\tag{B.8b}$$

and the quark mass matrices X_i are obtained by inserting eq. (B.1) into eq. (B.2) and setting $\langle H_i^j\rangle=v_i\delta_{ij}$.

$$X_{1ab}=v_2 f_2^{ba},\tag{B.9a}$$

$$X_{2ab}=v_1 f_1^{ba}.\tag{B.9b}$$

X_i and M_i are related by:

$$U_i^* X_i V_i^{-1}=M_i.\tag{B.10}$$

From eqs. (A.11)–(A.13), we find, for example:

$$P_{\mathrm{R}}u_{0a}=P_{\mathrm{R}}U_{1ba}u_b,\qquad P_{\mathrm{L}}u_{0a}=P_{\mathrm{L}}V_{1ba}^*u_b,\tag{B.11a}$$

$$P_{\mathrm{R}}d_{0a}=P_{\mathrm{R}}U_{2ba}d_b,\qquad P_{\mathrm{L}}d_{0a}=P_{\mathrm{L}}V_{2ba}^*d_b.\tag{B.11b}$$

These equations immediately yield the CKM matrix (denoted by K):

$$\mathscr{L}_{\mathrm{q_1\bar{q}_2}W^\pm}=-\sqrt{\tfrac{1}{2}}\,g\left(W_\mu^+\bar{u}_b\gamma^\mu P_{\mathrm{L}}K_{bc}d_c+\text{h.c.}\right),\tag{B.12}$$

where

$$K = V_1 V_2^\dagger . \tag{B.13}$$

The GIM mechanism [43] insures that the $q\bar{q}(Z^0, \gamma, H^0)$ vertices are flavor diagonal. However, the CKM matrix appears in the $q_1\bar{q}_2H^{\pm}$ interactions. Using eqs. (B.11a–b), we find

$$\mathscr{L}_{q_1\bar{q}_2H^{\pm}} = \frac{\sin\beta}{v_1}\left(U_2^* X_2 V_1^\dagger\right)_{cd} \bar{d}_c P_L u_d H^- + \text{h.c.}$$

$$+ \frac{\cos\beta}{v_2}\left(U_1^* X_1 V_2^\dagger\right)_{cd} \bar{u}_c P_L d_d H^+ + \text{h.c.}, \tag{B.14}$$

which has been obtained from eqs. (B.1)–(B.2) using eqs. (4.1a–b) and (B.9a–b). This equation may be cast in a familiar form using:

$$\frac{\cos\beta}{v_1} = \frac{\sin\beta}{v_2} = \frac{g}{\sqrt{2}\, m_W} . \tag{B.15}$$

With the help of eqs. (B.10) and (B.13) we obtain:

$$\mathscr{L}_{q_1\bar{q}_2H^{\pm}} = \frac{g}{\sqrt{2}\, m_W} H^+ \bar{u}\left[P_R K M_d \tan\beta + P_L M_u K \cot\beta\right] d + \text{h.c.} \tag{B.16}$$

We now turn to the scalar-quark interactions. First we consider just those terms which appear when there is no supersymmetry breaking. The D-terms [which arise from gauge interactions – see eqs. (3.5–3.6)] are diagonal in the "interaction" basis so we focus on the terms which arise from the first term in eq. (B.2). First consider the scalar-quark mass terms which are obtained by setting $\langle H_i^j \rangle = v_i \delta_{ij}$. The result is:

$$-\mathscr{L}_m^{\text{susy}} = \tilde{d}_{0R}^*\left(X_2 X_2^\dagger\right)\tilde{d}_{0R} + \tilde{u}_{0R}^*\left(X_1 X_1^\dagger\right)\tilde{u}_{0R} + \tilde{d}_{0L}^*\left(X_2^\dagger X_2\right)\tilde{d}_{0L} + \tilde{u}_{0L}^*\left(X_1^\dagger X_1\right)\tilde{u}_{0L} , \tag{B.17}$$

where the scalar-quark fields $\tilde{u}_{0L} = \tilde{Q}_{1a}$, $\tilde{d}_{0L} = \tilde{Q}_{2a}$, $\tilde{u}_{0R} = \tilde{U}_b^*$ and $\tilde{d}_{0R} = \tilde{D}_b^*$, are vectors in generational space and the subscript zero denotes "interaction" eigenstates. In the supersymmetric limit, eq. (B17) is the only source of scalar-quark mass terms and we see that the scalar-quarks and quarks have identical mass matrices. When supersymmetry breaking is introduced, additional contributions to the scalar-quark masses are obtained [see. eq. (4.17)], some of which need not be diagonal in the "interaction" basis. In the scalar-quark sector, one has an additional complication in that mixing is possible between $\tilde{q}_L$ and $\tilde{q}_R$ of different generations. To simplify the remaining discussion, we will neglect $\tilde{q}_L - \tilde{q}_R$ mixing in what follows (see Duncan [40] for further comments). We then introduce the mass eigenstates:

$$\tilde{q}_{iLa} = \tilde{V}_{iab}\tilde{q}_{i0Lb}, \tag{B.18a}$$

$$\tilde{q}_{iRa} = \tilde{U}^*_{iab}\tilde{q}_{i0Rb}, \tag{B.18b}$$

in analogy with eq. (B.4).

Let us now survey the scalar-quark interactions to see how the mixing matrices enter. The $\tilde{q}\tilde{q}W^{\pm}$ interaction involves the super-CKM matrix:

$$\tilde{K} = \tilde{V}_1\tilde{V}_2^{\dagger} \tag{B.19}$$

in analogy with eqs. (B.12), (B.13), whereas the $\tilde{q}\tilde{q}(Z^0, \gamma)$ interactions are flavor diagonal. The $\tilde{q}\tilde{q}H$ and $\tilde{q}\tilde{q}HH$ vertices are more complicated. Before we study these vertices, it is convenient to introduce some additional notation. We define new matrices:

$$\Gamma_i = \tilde{V}_i V_i^{\dagger}, \qquad (i = 1,2), \tag{B.20}$$

$$B_i = \tilde{U}_i^* U_i^{\mathrm{T}}, \qquad (i = 1,2). \tag{B.21}$$

Using this notation, we now exhibit the structure of the $\tilde{q}\tilde{q}H$ and $\tilde{q}\tilde{q}HH$ interaction:

$$\mathscr{L}_{\text{int}} = \mathscr{L}_F + \mathscr{L}_D + \mathscr{L}_{\text{break}}, \tag{B.22a}$$

$$\begin{aligned}\mathscr{L}_F = & -\tilde{d}_R^* B_2 M_d^2 B_2^{\dagger} \tilde{d}_R h_1 - \tilde{u}_R^* B_1 M_u^2 B_1^{\dagger} \tilde{u}_R h_2 \\ & - \tilde{d}_L^* \Gamma_2 \left[M_d^2 h_3 + K^{\dagger} M_u^2 K h_4 \right] \Gamma_2^{\dagger} \tilde{d}_L - \tilde{u}_L^* \Gamma_1 \left[M_u^2 h_5 + K M_d^2 K^{\dagger} h_6 \right] \Gamma_1^{\dagger} \tilde{u}_L \\ & + \left\{ \tilde{d}_R^* B_2 K^{\dagger} M_d M_u B_1^{\dagger} \tilde{u}_R h_7 + \tilde{d}_L^* \Gamma_2 \left[M_d^2 K^{\dagger} h_8 + K^{\dagger} M_u^2 h_9 \right] \Gamma_1^{\dagger} \tilde{u}_L + \text{h.c.} \right\}.\end{aligned} \tag{B.22b}$$

The h_i $(i = 1, \dots, 9)$ are combinations of one or two Higgs fields. Explicit expressions for the h_i are listed in table 4. The terms in eq. (B.22a) which are not proportional to the quark masses have their origin in the D-terms (denoted by $\mathscr{L}_D$) and are generation-diagonal in the "interaction" basis. Finally, $\mathscr{L}_{\text{break}}$ in eq. (B.22a) refers to terms proportional to μ, A_u or A_d. These terms mix $\tilde{q}_L$ with $\tilde{q}_R$ and can make the scalar-quark mixing problem substantially more complicated. We will continue to ignore these terms in this appendix.*

* In some low-energy supergravity models, $\tilde{q}_L - \tilde{q}_R$ mixing tends to be small except for the case of the $\tilde{t}$. Because mixing angles involving the t-quark tend to be small, it should be adequate to deal with the $\tilde{t}_L - \tilde{t}_R$ mixing after the generational mixing has been included. However, the reader should be warned that for some physical applications (such as the electric dipole moment of the neutron), the above approximations are *not* adequate and one must treat the full scalar-quark mixing problem correctly (A.I. Sanda, private communication).

TABLE 4

$$h_1 = h_3 + h_6$$

$$h_2 = h_5 + h_4$$

$$h_3 = \frac{g}{m_W \cos\beta}\left\{\phi_2 + \frac{g}{4m_W\cos\beta}\left[(\phi_2)^2 + \left(H_3^0\right)^2\sin^2\beta\right]\right\}$$

$$h_4 = \left(\frac{g^2\cot^2\beta}{2m_W^2}\right)H^+H^-$$

$$h_5 = \frac{g}{m_W \sin\beta}\left\{\phi_1 + \frac{g}{4m_W\sin\beta}\left[(\phi_1)^2 + \left(H_3^0\right)^2\cos^2\beta\right]\right\}$$

$$h_6 = \left(\frac{g^2\tan^2\beta}{2m_W^2}\right)H^+H^-$$

$$h_7 = \frac{g}{\sqrt{2}\,m_W\sin\beta\cos\beta}H^-\left\{1 + \frac{g}{2m_W}\left[H_1^0\cos(\beta-\alpha) + H_2^0\sin(\beta-\alpha)\right]\right\}$$

$$h_8 = \frac{g\tan\beta}{\sqrt{2}\,m_W}H^-\left[1 + \frac{g}{2m_W\cos\beta}\left(\phi_2 - iH_3^0\sin\beta\right)\right]$$

$$h_9 = \frac{g\cot\beta}{\sqrt{2}\,m_W}H^-\left[1 + \frac{g}{2m_W\sin\beta}\left(\phi_1 + iH_3^0\cos\beta\right)\right]$$

We list the fields h_i which appear in eqs. (B.22b). The following notation is used: $\phi_1 = H_1^0\sin\alpha + H_2^0\cos\alpha$, $\phi_2 = H_1^0\cos\alpha - H_2^0\sin\alpha$.

Eq. (B.22) can be simplified considerably by making certain model assumptions. Here, we follow the analysis of Duncan [40]. We denote the scalar-quark squared mass matrix (before diagonalization) by $\tilde{X}_{iL}^2$ and $\tilde{X}_{iR}^2$ where $i = 1,2$ corresponds to *up*-type and *down*-type flavors, respectively:

$$-\mathscr{L}_m = \sum_i \tilde{q}_{i0Ra}^* \tilde{X}_{iRab}^2 \tilde{q}_{i0Rb} + \tilde{q}_{i0La}^* \tilde{X}_{iLab}^2 \tilde{q}_{i0Lb}\,. \tag{B.23}$$

In many low-energy supergravity models, one finds that at the Planck scale, $\mathscr{L}_m$ differs from the supersymmetric mass term given by eq. (B.17) by a universal generation independent mass term. However, one must use the renormalization group to evolve down to low-energies. At the low-energy scale, Duncan finds [40]:

$$\tilde{X}_{iR}^2 = m_W^2\mu_{iR}^{(0)} I + \mu_{iR}^{(1)} X_i X_i^\dagger\,, \qquad (i = 1,2)\,, \tag{B.24}$$

$$\tilde{X}_{1L}^2 = m_W^2\mu_{1L}^{(0)} I + \mu_{1L}^{(1)} X_1^\dagger X_1 + \mu_{1L}^{(2)} X_2^\dagger X_2\,, \tag{B.25a}$$

$$\tilde{X}_{2L}^2 = m_W^2\mu_{2L}^{(0)} I + \mu_{2L}^{(1)} X_2^\dagger X_2 + \mu_{2L}^{(2)} X_1^\dagger X_1\,, \tag{B.25b}$$

where I is the identity matrix generation space. The dimensionless numbers $\mu^{(i)}$ are model-dependent, typically of O(1) [40]. The extra term in $\tilde{X}_{i\mathrm{L}}^2$ $(i=1,2)$ as compared to $\tilde{X}_{i\mathrm{R}}^2$ arises due to the difference in $\tilde{q}_{\mathrm{L}}$ and $\tilde{q}_{\mathrm{R}}$ interactions given in eq. (B.22b).

We find that $\tilde{X}_{i\mathrm{R}}^2$ is easily diagonalized: using eq. (B.10), it follows from eqs. (B.21) and eq. (B.24) that:

$$\tilde{U}_i = U_i\,, \qquad (i=1,2)\,, \tag{B.26a}$$

$$B_i = I\,. \tag{B.26b}$$

In order to diagonalize $\tilde{X}_{i\mathrm{L}}^2$, we consider two special cases:

Case I: $\mu^{(2)}=0$. In this case, we see that $\tilde{V}_i = V_i$ $(i=1,2)$, which implies that $\tilde{K}=K$ and $\Gamma_i = I$. That is, there is only one CKM matrix for W-interactions with quarks and scalar-quarks. The scalar-quarks–Higgs-boson interactions [eq. (B.22b)] simplify significantly since $\Gamma_i = B_i = I$. Tree-level flavor-changing neutral currents due to $H^0\tilde{q}_{i\mathrm{L}}\tilde{q}_{j\mathrm{L}}$ $(i\neq j)$ vertices do exist (e.g. $KM_{\mathrm{d}}^2K^\dagger$ is not diagonal), although they tend to be suppressed by small mixing angles and quark mass differences. (Note that in this case, $\tilde{\mathrm{q}}_{\mathrm{L}} - \tilde{\mathrm{q}}_{\mathrm{R}}$ mixing can be easily treated since it decouples from the intergenerational scalar-quark mixing.)

Case II: neglect terms proportional to $X_2^\dagger X_2$ in eq. (B.25). This is suggested in supergravity models where a large top-quark mass is responsible for the SU(2) × U(1) breaking in the low-energy effective theory. Then $\tilde{V}_1 = \tilde{V}_2 = V_1$ since both up and down flavors of $\tilde{q}_{\mathrm{L}}$ are now diagonalized by the *same* unitary matrix which diagonalizes the (left-handed) up-quark mass matrix. In this case, $\tilde{K} = \Gamma_1 = I$ and $\Gamma_2 = K$.

Our final task is to see the effect of generational mixing on the $\mathrm{q}\tilde{\mathrm{q}}\tilde{\chi}^+$ and $\mathrm{q}\tilde{\mathrm{q}}\tilde{\chi}^0$ interactions (see sect. 5). We first focus on the pieces of these interactions which arise from the second term of eq. (B.2). This simply requires us to put the generational indices correctly in the terms proportional to quark masses in eqs. (5.1) and (5.4).

As an example, one term which appears in eq. (5.1) is $\bar{u}_0 P_{\mathrm{L}} X_1 \tilde{d}_{0\mathrm{L}} \tilde{H}$ [where we have used eq. (B.9)]. Using the results summarized in table 5, it is simple to verify that

$$\bar{u}_0 P_{\mathrm{L}} X_1 \tilde{d}_{0\mathrm{L}} = \bar{u} P_{\mathrm{L}} M_{\mathrm{u}} K \Gamma_2^\dagger \tilde{d}_{\mathrm{L}}\,. \tag{B.27}$$

The remaining terms are calculated in a similar manner. The terms proportional to g and g' in eqs. (5.1) and (5.4) are generation-diagonal using the "interaction" eigenstates. The correct generalization of the $\mathrm{q}\tilde{\mathrm{q}}\tilde{\chi}^+$ [eq. (5.3)] and $\mathrm{q}\tilde{\mathrm{q}}\tilde{\chi}^0$ [eq. (5.5)] interactions is summarized in table 6, which exhibits a few noteworthy features.

First, in general there are exactly five independent generational matrices which arise when describing interactions of quarks and scalar-quarks: K, $\Gamma_i^\dagger$ and $B_i^\dagger$

TABLE 5
Summary of quark and scalar-quark mixing and mass matrices

I. Quark sector	
$X_1^{\mathrm{T}} = v_2 f_2$	f_i ($i = 1,2$) are the Yukawa couplings of quarks
$X_2^{\mathrm{T}} = v_1 f_1$	to the Higgs bosons H_i, where $\langle H_i \rangle = v_i$.
$P_{\mathrm{L}} q_{i0} = P_{\mathrm{L}} V_i^{\dagger} q_i$	$P_{\mathrm{L}} = \frac{1}{2}(1 - \gamma_5)$
$P_{\mathrm{R}} q_{i0} = P_{\mathrm{R}} U_i^{\mathrm{T}} q_i$	$P_{\mathrm{R}} = \frac{1}{2}(1 + \gamma_5)$
$U_i^* X_i V_i^{-1} = M_i$	
$M_{\mathrm{u}} = M_1 = \mathrm{diag}(m_{\mathrm{u}}, m_{\mathrm{c}}, m_{\mathrm{t}}, \dots)$	
$M_{\mathrm{d}} = M_2 = \mathrm{diag}(m_{\mathrm{d}}, m_{\mathrm{s}}, m_{\mathrm{b}}, \dots)$	
$K = V_1 V_2^{\dagger}$	Kobayashi-Maskawa matrix
II. Scalar-quark sector	
$\tilde{X}_{i\mathrm{L}}^2$	mass matrix of $\tilde{q}_{i\mathrm{L}}$ in interaction basis
$\tilde{X}_{i\mathrm{R}}^2$	mass matrix of $\tilde{q}_{i\mathrm{R}}$ in interaction basis
$\tilde{q}_{i0\mathrm{L}} = \tilde{V}_i^{\dagger} \tilde{q}_{i\mathrm{L}}$	
$\tilde{q}_{i0\mathrm{R}} = \tilde{U}_i^{\mathrm{T}} \tilde{q}_{i\mathrm{R}}$	
$\tilde{V}_i \tilde{X}_{i\mathrm{L}}^2 \tilde{V}_i^{\dagger} = \tilde{M}_{q_i\mathrm{L}}^2$	diagonal $\tilde{q}_{\mathrm{L}}$ mass-matrix
$\tilde{U}_i^* \tilde{X}_{i\mathrm{R}}^2 \tilde{U}_i^{\mathrm{T}} = \tilde{M}_{q_i\mathrm{R}}^2$	diagonal $\tilde{q}_{\mathrm{R}}$ mass-matrix
$\tilde{K} = \tilde{V}_1 \tilde{V}_2^{\dagger}$	super-Kobayashi-Maskawa matrix
III. Other mixing matrices	
$\Gamma_i = \tilde{V}_i V_i^{\dagger}$	note that $\tilde{K} = \Gamma_1 K \Gamma_2^{\dagger}$
$B_i = \tilde{U}_i^* U_i^{\mathrm{T}}$	

We denote the interaction-eigenstate quarks and scalar-quarks by q_{i0} and $\tilde{q}_{i0}$ respectively, where $i = 1, 2$ corresponds to up-type and down-type flavors respectively. The corresponding mass eigenstates are q_i and $\tilde{q}_i$. If N is the number of generations, then the symbols q, $\tilde{q}$ above all are N-vectors. All other symbols above are $N \times N$ matrices. We neglect $\tilde{\mathrm{q}}_{\mathrm{L}} - \tilde{\mathrm{q}}_{\mathrm{R}}$ mixing here so that $\tilde{q}_{i\mathrm{L}}$ and $\tilde{q}_{i\mathrm{R}}$ are the appropriate scalar-quark mass eigenstates. In the expressions above, do *not* sum over the repeated index i.

($i = 1,2$). As argued above, eq. (B.26), we expect to find $B_i = I$ which reduces the number of independent matrices to three. These remarks are also true for the other interactions previously studied, since the super-CKM matrix is not independent but can be written as $\tilde{K} = \Gamma_1 K \Gamma_2^{\dagger}$. If we make further simplifications (e.g. cases I and II above), then all generational matrices are related to the CKM matrix,* K.

Second, in the most general case, the $\mathrm{q}\tilde{\mathrm{q}}\tilde{\chi}^0$ interaction terms are flavor nondiagonal. One must therefore be careful lest ones model predict flavor changing neutral current processes at too large a rate. In case I the $\mathrm{q}\tilde{\mathrm{q}}\tilde{\chi}^0$ interaction is exactly flavor diagonal. However, case II probably represents a more realistic supergravity model. In such a model, the $u\tilde{u}\tilde{\chi}^0$ vertex is flavor-diagonal, but the $d\tilde{d}\tilde{\chi}^0$ vertex is flavor nondiagonal (as emphasized in ref. [40]). Note that these arguments can also be

* Note in particular that a "right-handed CKM matrix" ($U_1 U_2^{\dagger}$) never appears in the theory.

TABLE 6
Effect of generational mixing on $q\tilde{q}\tilde{\chi}^+$ and $q\tilde{q}\tilde{\chi}^0$ vertices

$q\tilde{q}\tilde{\chi}^+$

Interaction term	Mixing matrices	Case I	Case II
$\bar{u}_0 P_R \tilde{d}_{0L}$	$K\Gamma_2^\dagger$	K	I
$\bar{d}_0 P_R \tilde{u}_{0L}$	$K^\dagger \Gamma_1^\dagger$	$K^\dagger$	$K^\dagger$
$\bar{u}_0 P_L X_1 \tilde{d}_{0L}$	$M_u K \Gamma_2^\dagger$	$M_u K$	$M_u I$
$\bar{u}_0 P_R X_2^\dagger \tilde{d}_{0R}$	$K M_d B_2^\dagger$	$K M_d$	$K M_d$
$\bar{d}_0 P_L X_2 \tilde{u}_{0L}$	$M_d K^\dagger \Gamma_1^\dagger$	$M_d K^\dagger$	$M_d K^\dagger$
$\bar{d}_0 P_R X_1^\dagger \tilde{u}_{0R}$	$K^\dagger M_u B_1^\dagger$	$K^\dagger M_u$	$K^\dagger M_u$

$q\tilde{q}\tilde{\chi}^0$

Interaction term	Mixing matrices	Case I	Case II	
			$i=1$	$i=2$
$\bar{q}_{i0} P_R \tilde{q}_{i0L}$	$\Gamma_i^\dagger$	I	I	$K^\dagger$
$\bar{q}_{i0} P_L \tilde{q}_{i0R}$	$B_i^\dagger$	I	I	I
$\bar{q}_{i0} P_L X_i \tilde{q}_{i0L}$	$M_i \Gamma_i^\dagger$	M_i	M_u	$M_d K^\dagger$
$\bar{q}_{i0} P_R X_i^\dagger \tilde{q}_{i0R}$	$M_i B_i^\dagger$	M_i	M_u	M_d

The unmixed terms obtained from eqs. (5.3) and (5.5) respectively are listed in column 1. Column 2 lists the appropriate combination of mixing matrices which will appear if "interaction eigenstates" are replaced by mass eigenstates. Columns 3 and 4 list two interesting special cases of column 2: case I: $\tilde{U}_i = U_i$, $\tilde{V}_i = V_i$ and case II: $\tilde{U}_i = U_i$, $\tilde{V}_1 = \tilde{V}_2 = V_1$. We denote the *diagonal* quark mass matrices by M_u and M_d. Definitions of mixing and mass matrices are summarized in table 5.

extended to the q$\tilde{\text{q}}\tilde{\text{g}}$ interaction which is given by:

$$\mathscr{L}_{q\tilde{q}\tilde{g}} = -\sqrt{2}\, g_s T^c_{jk} \left(\bar{q}^j_{ia} P_R \tilde{g}_c \Gamma^\dagger_{iab} \tilde{q}^k_{iLb} - \bar{q}^j_{ia} P_L \tilde{g}_c B^\dagger_{iab} \tilde{q}^k_{iRb} + \text{h.c.} \right), \qquad \text{(B.29)}$$

where i sums over u and d-type quarks, j and k are quark color indices, c is the gluino color index and a and b are generational labels.

The entire discussion of this appendix can be equally well applied to leptons. Since neutrinos are massless in the standard model, there is no CKM matrix for leptons and we may set $X_1 = M_u = 0$ and $K = I$ in the above formulas when applying them to leptons. Furthermore, X_2 can be chosen diagonal because all interactions involving leptons and scalar-leptons conserve individual lepton numbers (one for each generation). Using eqs. (B.24), (B.25), we see that by choosing X_2 diagonal, one automatically obtains diagonal scalar-lepton mass matrices (again, a consequence of lepton number conservation). This is so, despite the fact that both charged and neutral scalar-leptons of different generations differ in mass-squared (proportional to the difference of the corresponding charged-lepton squared masses).

Unlike in the scalar-quark sector, the inclusion of $\tilde{\ell}_L - \tilde{\ell}_R$ mixing is straightforward since there is no communication among different generations.

References

[1] G. Arnison et al., Phys. Lett. 122B (1983) 103; 126B (1983) 398; 129B (1983) 273;
G. Banner et al., Phys. Lett. 122B (1983) 476;
P. Bagnaia et al., Phys. Lett. 129B (1983) 130

[2] S.L. Glashow, Nucl. Phys. B22 (1961) 579;
S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264;
A. Salam, Proc. 8th Nobel Symposium, Stockholm 1968, ed. N. Svartholm (Almquist and Wiksells, Stockholm, 1968) p. 367

[3] M. Veltman, Acta Phys. Pol. B8 (1977) 475; *in* Proc. Int. Europhysics Conf. High-energy physics, Brighton, ed. J. Guy and C. Costain (Rutherford Appleton Laboratory, 1983) p. 880

[4] H. Georgi, H.R. Quinn and S. Weinberg, Phys. Rev. Lett. 33 (1974) 451;
E. Gildener, Phys. Rev. D14 (1976) 1667;
A.J. Buras, J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Nucl. Phys. B135 (1978) 66;
C. Sachrajda, Phys. Lett. 98B (1981) 74;
C.H. Llewellyn-Smith and G.G. Ross, Phys. Lett. 105B (1981) 38;
Y. Kazama and Y.-P. Yao, Phys. Rev. D25 (1982) 1650

[5] K. Wilson, Phys. Rev. D3 (1971) 1818;
H. Georgi and A. Pais, Phys. Rev. D10 (1974) 539;
G. 't Hooft, *in* Recent developments in gauge theories, Proc. NATO Advanced Summer Institute, Cargèse, 1979, ed. G. 't Hooft et al. (Plenum, New York, 1980) p. 135

[6] E. Witten, Nucl. Phys. B188 (1981) 513;
S. Dimopoulos and H. Georgi, Nucl. Phys. B193 (1981) 150;
N. Sakai, Z. Phys. C11 (1981) 153

[7] L.J. Hall and M. Suzuki, Nucl. Phys. B231 (1984) 419;
I.-H. Lee, Phys. Lett. 138B (1984) 121; Nucl. Phys. B246 (1984) 120;
J. Ellis, G. Gelmini, C. Jarlskog, G.G. Ross and J.W.F. Vale, Phys. Lett. 150B (1985) 142;
G.G. Ross and J.W.F. Valle, Phys. Lett. 151B (1985) 375

[8] K. Inoue, A. Komatsu and S. Takeshita, Prog. Theor. Phys. 67 (1982) 927; (E)70 (1983) 330; 71 (1984) 413

[9] L. Girardello and M.T. Grisaru, Nucl. Phys. B194 (1982) 65

[10] P. Nath, R. Arnowitt and A.H. Chamseddine, Applied $N=1$ supergravity, The ICTP Series in Theoretical Physics, vol. 1 (World Scientific, Singapore, 1984); Harvard preprint HUTP 83/A077 (1983)

[11] H.P. Nilles, Phys. Reports 110 (1984) 1

[12] L. Alvarez-Gaumé, J. Polchinski and M.B. Wise, Nucl. Phys. B221 (1983) 495

[13] J. Ellis, J.S. Hagelin, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. 125B (1983) 275;
L. Ibãnez and C. Lopez, Phys. Lett. 126B (1983) 54; Nucl. Phys. B233 (1984) 511;
L. Ibãnez, Nucl. Phys. B218 (1983) 514;
B.A. Ovrut and S. Raby, Phys. Lett. 130B (1983) 277;
C. Kounnas, A.B. Lahanas, D.V. Nanopoulos and M. Quiros, Phys. Lett. 132B (1983) 95; Nucl. Phys. B236 (1984) 438

[14] P. Fayet, Nucl. Phys. B90 (1975) 104; B237 (1984) 367

[15] A.H. Chamseddine, R. Arnowitt and P. Nath, Phys. Rev. Lett. 49 (1982) 970;
R. Barbreri, S. Ferrara and C.A. Savoy, Phys. Lett. 119B (1982) 343

[16] J.-P. Derendinger and C.A. Savoy, Nucl. Phys. B237 (1984) 307

[17] H.E. Haber, R.M. Barnett and K.S. Lackner, Phys. Rev. D29 (1984) 1990

[18] H.E. Haber and G.L. Kane, Phys. Reports 117 (1985) 75

[19] J. Gunion and H.E. Haber, UC Davis preprint UCD-86-07 (1986)

[20] H. Georgi, Hadronic Jour. 1 (1978) 155
[21] H.E. Haber, G.L. Kane and T. Sterling, Nucl. Phys. B161 (1979) 493
[22] N.G. Deshpande and E. Ma, Phys. Rev. D18 (1978) 2574;
J.F. Donoghue and L.-F. Li, Phys. Rev. D19 (1979) 945;
L.F. Abbott, P. Sikivie and M.B. Wise, Phys. Rev. D21 (1980) 1393;
B. McWilliams and L.-F. Li, Nucl. Phys. B179 (1981) 62;
L.J. Hall and M.B. Wise, Nucl. Phys B187 (1981) 397
[23] R. Flores and M. Sher, Ann. of Phys. (NY) 148 (1983) 95;
H.P. Nilles and M. Nusbaumer, Phys. Lett. 145B (1984) 73;
P. Majumdar and P. Roy, Phys. Rev. D30 (1984) 2432;
M. Drees, M. Gluck and K. Grassie, Phys. Lett. 159B (1985) 32
[24] J. Wess and J. Bagger, Supersymmetry and supergravity (Princeton Univ. Press, Princeton, NJ, 1983)
[25] P. Fayet and J. Iliopoulos, Phys. Lett. 51B (1974) 461
[26] H. Huffel and G. Pocsik, Z. Phys. C8 (1981) 13;
G. Pocsik and G. Zsigmond, Z. Phys. C10 (1981) 367; Phys. Lett. 112B (1982) 157;
G. Zsigmond, Acta Phys. Hung. 56 (1984) 73
[27] J.A. Grifols and A. Mendez, Phys. Rev. D22 (1980) 1725
[28] S.L. Glashow and S. Weinberg, Phys. Rev. D15 (1977) 1958
[29] J. Ellis and S. Rudaz, Phys. Lett. 128B (1983) 248
[30] J. Ellis and G.G. Ross, Phys. Lett. 117B (1982) 397
[31] B. Grinstein, J. Polchinski and M.B. Wise, Phys. Lett. 130B (1983) 285
[32] J.-M. Frere and G.L. Kane, Nucl. Phys. B223 (1983) 331;
J. Ellis, J.S. Hagelin, D.V. Nanopoulos and M. Srednicki, Phys. Lett. 127B (1983) 233;
A.H. Chamseddine, P. Nath and R. Arnowitt, Phys. Lett. 129B (1983) 445;
D.A. Dicus, S. Nandi and X. Tata, Phys. Lett. 129B (1983) 451;
V. Barger, R.W. Robinett, W.Y. Keung and R.J.N. Phillips, Phys. Lett. 131B (1983) 372;
J. Ellis, J.-M. Frere, J.S. Hagelin, G.L. Kane and S.T. Petcov, Phys. Lett. 132B (1983) 436
[33] S.K. Jones and C.H. Llewellyn-Smith, Nucl. Phys. B217 (1983) 145
[34] M. Claudson, L.J. Hall and I. Hinchliffe, Nucl. Phys. B228 (1983) 501;
J. Ellis, A.B. Lahanas, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. 134B (1984) 429
[35] J. Ellis, S. Ferrara and D.V. Nanopoulos, Phys. Lett. B114 (1982) 231;
F. del Aguila, J. Grifols, A. Mendez, D. Nanopoulos and M. Srednicki, Phys. Lett. 129B (1983) 77;
J.-M. Frere and M.B. Gavela, Phys. Lett. 132B (1983) 107;
E. Franco and M. Mangano, Phys. Lett. 135B (1984) 445;
M. Dugan, B. Grinstein and L. Hall, Nucl. Phys. B255 (1985) 413
[36] W. Buchmüller and D. Wyler, Phys. Lett. 121B (1983) 321;
J. Polchinski and M.B. Wise, Phys. Lett. 125B (1983) 393;
F. del Aguila, M. Gavela, J. Grifols and A. Mendez, Phys. Lett. 126B (1983) 71; (E) 129B (1983) 473;
D.V. Nanopoulos and M. Srednicki, Phys. Lett. 128B (1983) 61;
M.J. Duncan, Nucl. Phys. B224 (1983) 289;
K. Enqvist, D.V. Nanopoulos and A. Raychaudhuri, Phys. Lett. 151B (1985) 210
[37] J.M. Gerard, W. Grimus, A. Raychaudhuri and G. Zoupanos, Phys. Lett. 140B (1984) 349;
J.M. Gerard, W. Grimes, A. Masiero, D.V. Nanopoulos and A. Raychaudhuri, Phys. Lett. 141B (1984) 79;
J.M. Gerard, W. Grimus and A. Raychaudhuri, Phys. Lett. 145B (1984) 400
[38] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49 (1973) 65
[39] J. Ellis and D.V. Nanopoulos, Phys. Lett. 110B (1982) 44;
R. Barbieri and R. Gatto, Phys. Lett. 110B (1982) 211;
T. Inami and C.S. Lim, Nucl. Phys. B207 (1982) 533;
B.A. Campbell, Phys. Rev. D28 (1983) 203;
M.J. Duncan and J. Trampetic, Phys. Lett. 134B (1984) 439;
J.C. Romão, A. Barroso, M.C. Bento and G.C. Branco, Nucl. Phys. B250 (1985) 295
[40] J.F. Donoghue, H.P. Nilles and D. Wyler, Phys. Lett. 128B (1983) 55;

M.J. Duncan, Nucl. Phys. B221 (1983) 285;
P. Langacker and B. Sathiapalan, Phys. Lett. 144B (1984) 395, 401;
A. Bouquet, J. Kaplan and C.A. Savoy, Phys. Lett. 148B (1984) 69
[41] R.N. Cahn and S. Dawson, Phys. Lett. 136B (1984) 196; (E) 138B (1984) 464;
G.L. Kane, W.W. Repko and W.B. Rolnick, Phys. Lett. 148B (1984) 367;
S. Dawson, Nucl. Phys. B249 (1985) 42
[42] H.E. Haber and G.L. Kane, Nucl. Phys. B250 (1985) 716
[43] S. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D2 (1970) 1285
[44] H.-S. Tsao, *in* Proc. 1980 Guangzhou Conf. on Theoretical particle physics, ed. H. Ning and T Hung-yuan (Van-Nostrand, New York, 1980) p. 1240.