Hard supersymmetry-breaking "wrong-Higgs" couplings of the MSSM

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In the minimal supersymmetric extension of the standard model (MSSM), if the two-Higgs doublets are lighter than some subset of the superpartners of the standard model particles, then it is possible to integrate out the heavy states to obtain an effective broken-supersymmetric low-energy Lagrangian. This Lagrangian can contain dimension-four gauge-invariant Higgs interactions that violate supersymmetry (SUSY). The *wrong-Higgs* Yukawa couplings generated by one-loop radiative corrections are a well-known example of this phenomenon. In this paper, we examine gauge-invariant gaugino-higgsino-Higgs boson interactions that violate supersymmetry. Such wrong-Higgs gaugino couplings can be generated in models of gauge-mediated SUSY-breaking in which some of the messenger fields couple to the MSSM Higgs bosons. In regions of parameter space where the messenger scale is low and $\tan\beta$ is large, these hard SUSY-breaking operators yield $\tan\beta$-enhanced corrections to tree-level supersymmetric relations in the chargino and neutralino sectors that can be as large as 56%. We demonstrate how physical observables in the chargino sector can be used to isolate the $\tan\beta$-enhanced effects derived from the wrong-Higgs gaugino operators.

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I. INTRODUCTION

The minimal supersymmetric standard model (MSSM) [1–5] is a supersymmetric extension of the standard model, augmented by the most general set of dimension-two and dimension-three supersymmetry (SUSY)-breaking operators (allowed by the rules of [6]). Without additional assumptions, the resulting MSSM is governed by 124 independent parameters [7,8]. These parameters are considered placeholders for the unknown (and simpler) fundamental mechanism of spontaneous SUSY-breaking. Since there are many different models of fundamental SUSY-breaking [9], determining the relations among these soft parameters is an important step towards determining the organizing principle governing the fundamental mechanism for SUSY-breaking. It is unlikely that all 124 parameters can ever be measured in future experiments. Furthermore, additional parameters enter that depend on the mechanism that communicates the fundamental SUSY-breaking to the visible sector of MSSM fields; this often involves hidden sector physics out of the reach of direct detection by colliders because the scale of physics that governs the communication of the SUSY-breaking from the hidden sector to the MSSM is of $\mathcal{O}(100\ \text{TeV})$ or greater. One way to infer information about the soft-SUSY-breaking Lagrangian and the mediation of SUSY-breaking to the MSSM is through the measurement of radiative corrections to supersymmetric relations that are imprinted on the parameters of the theory.

Radiative corrections to supersymmetric relations have been the subject of many studies. It is quite useful to consider the case in which there is a separation of the effective low-energy SUSY-breaking scale ($M_{\rm SUSY}$) and the scale of electroweak symmetry-breaking (characterized by the Higgs vacuum expectation value $v = 246$ GeV). In this case, one can construct an effective Lagrangian [10,11] below the scale of SUSY-breaking in which the effects of the SUSY-breaking one-loop effects appear as corrections to tree-level relations. For example, one can consider decoupling all superpartners, which results in an effective low-energy two-Higgs-doublet model (2HDM) [12] below the SUSY-breaking scale. Although the superpartners do not appear in the effective low-energy 2HDM, their radiative effects do not decouple and yield predictions of modified relations between the tree-level Yukawa couplings and the corresponding quark masses. These effects can be understood as deriving from the radiatively corrected MSSM Higgs-Yukawa couplings and the effects of radiatively generated so-called *wrong-Higgs* Yukawa couplings that violate supersymmetry.

Alternatively, one can consider decoupling only a subset of the superpartner spectrum and looking at the nondecoupling effects in both tree-level 2HDM couplings and in the tree-level relations among the light superpartner couplings. Such a scenario arises in models of split-supersymmetry [13]. In these models, the properties of the squarks can be inferred from deviations of supersymmetric relations between the gauge couplings and the couplings of the light higgsinos and gauginos [14]. Although the separation of scales between decoupled and nondecoupled states is essential for the existence of an effective low-energy local Lagrangian description, such a separation is not required for probes of SUSY-breaking via radiative effects. For example, the deviation of the supersymmetric relations between the gauge and gaugino couplings were analyzed in [15,16] even though the squarks were not decoupled from the low-energy spectrum.

What is remarkable about the examples cited above is the role played by dimension-four hard SUSY-breaking operators. The coefficients of these operators in the effective low-energy Lagrangian below the scale $M_{\rm SUSY}$ are suppressed by a coupling constant and a loop factor. In contrast, the coefficients of dimension-four hard SUSY-breaking operators of the Lagrangian above $M_{\rm SUSY}$ are typically suppressed by one or two powers of F/M^2 [17], where $F^{1/2}$ characterizes the fundamental scale of SUSY-breaking and M is the scale of the physics that transmits SUSY-breaking to the sector of MSSM fields. For example, in cases of gravity-mediated SUSY-breaking where $F/M \sim M_{\rm SUSY}$ and M is the Planck scale, dimension-four hard SUSY-breaking operators are Planck-scale suppressed and hence completely negligible. In models of gauge-mediated supersymmetry-breaking [18–27], $(g^2/16\pi^2)F/M \sim M_{\rm SUSY}$, where g is the relevant gauge coupling constant and M is a typical mass of the messenger fields that in some cases can be as low as a few TeV. In the latter scenario, F/M^2 is a rather mild suppression, in which case the corresponding dimension-four hard SUSY-breaking operators can be phenomenologically relevant.

This paper is organized as follows. In Sec. II, we review the radiative generation of the wrong-Higgs-Yukawa couplings of the effective 2HDM Lagrangian after decoupling the heavy supersymmetric particles. The wrong-Higgs-Yukawa couplings are hard SUSY-breaking dimension-four operators that appear in the effective low-energy theory at the electroweak scale. One notable consequence of the wrong-Higgs-couplings is an enhanced correction to the relation between the bottom-quark Yukawa coupling and the bottom-quark mass in the limit of a large ratio of Higgs vacuum expectation values, $\tan\beta$. This enhancement can yield a radiative correction to the Higgs decay rate to bottom-quark pairs that is significantly larger than the expected size of a one-loop radiative effect. Detection of such a deviation would provide insight into the structure of SUSY-breaking, even while probing interactions at scales below the heavy masses of the MSSM spectrum.

In Sec. III, we examine the possibility of analogous wrong-Higgs interactions that couple gauginos to the higgsinos and Higgs bosons. These gaugino-higgsino-Higgs boson interactions are gauge invariant with respect to the standard model gauge group but are SUSY-breaking, and thus are constrained to be zero at tree-level in the MSSM. Since we are aiming to use an effective Lagrangian description of the chargino/neutralino sector at a scale below the SUSY-breaking scale, we look for regions of MSSM parameter space where threshold corrections from heavy MSSM particles can generate these effective operators at one-loop. We show that a consistent effective Lagrangian treatment of these operators cannot be achieved from decoupling any subset of MSSM fields at some high SUSY-breaking scale. Nevertheless, when we parametrize a simple low-energy gauge-mediated messenger sector with couplings to the Higgs doublets, integrating out the messengers does generate the SUSY-breaking wrong-Higgs operators of interest. Models with such messenger interactions have been suggested in [28]. For our purposes, we note that the quantum numbers of the messenger fields typically allow for supersymmetric and gauge-invariant interactions with the Higgs doublets. In this paper, we have explored in detail some of the detectable nondecoupling effects of such interactions.

After the messenger sector is integrated out and new gaugino couplings are present in the effective Lagrangian, corrections to the off-diagonal elements of the chargino and neutralino mass matrices are generated. In Sec. IV, we focus on the impact of the wrong-Higgs gaugino operators on the chargino mass matrix. These SUSY-breaking interactions will result in deviations in the tree-level supersymmetric relations between the off-diagonal elements of the chargino mass matrix, the W-mass, and $\tan\beta$. We identify one particular correction that is $\tan\beta$-enhanced and dominates over all other one-loop corrections. We briefly indicate how the effects of the $\tan\beta$-enhanced correction can be isolated in precision chargino studies at future colliders. Finally, in Sec. V, we demonstrate that the $\tan\beta$-enhanced effects of the local wrong-Higgs operators are parametrically larger than any nonlocal effects that could in principle wash out such effects. Conclusions and future directions of this work are outlined in Sec. VI.

II. WRONG-HIGGS INTERACTIONS AND THE BOTTOM-QUARK MASS

The tree-level MSSM Lagrangian consists of SUSY-conserving mass and interaction terms, supplemented by soft-SUSY-breaking operators. Following the rules of Ref. [6], the soft-SUSY-breaking operators include arbitrary dimension-two mass terms and holomorphic cubic scalar interactions, consistent with the gauge symmetry of the model.[1] In particular, all tree-level dimension-four gauge-invariant interactions must respect supersymmetry.

When supersymmetry is broken, in principle all SUSY-breaking operators consistent with gauge invariance can be generated in the effective low-energy theory (below the scale of SUSY-breaking). The MSSM Higgs sector provides an especially illuminating example of this phenomenon. The MSSM contains two complex Higgs doublet fields H_u and H_d of hypercharge ± 1, respectively. The

[1]Supersymmetry-breaking mass terms for the fermionic superpartners of scalar fields and nonholomorphic trilinear scalar interactions can potentially destabilize the gauge hierarchy [6] in models with a gauge-singlet superfield. The latter is not present in the MSSM; hence as noted in [29,30], these so-called nonstandard soft-supersymmetry-breaking terms are benign. However, the coefficients of these terms (which have dimensions of mass) are expected to be significantly suppressed compared to the TeV-scale in a fundamental theory of high-scale supersymmetry-breaking.

tree-level Higgs-quark Yukawa Lagrangian is given by:

$$\mathcal{L}_{\rm yuk}^{\rm tree} = -\epsilon_{ij} h_b H_d^i \psi_Q^j \psi_D + \epsilon_{ij} h_t H_u^i \psi_Q^j \psi_U + \text{H.c.}, \quad (1)$$

where we use two-component notation for the quark fields.[2] Note that the supersymmetry restricts the form of the tree-level Yukawa Lagrangian to the so-called Type-II Yukawa interactions [31,32] of the two-Higgs doublet model, in which the neutral component of H_d (H_u) couples exclusively to down-type (up-type) quarks. Two other possible dimension-four gauge-invariant nonholomorphic Higgs-quark interactions terms, the so-called *wrong-Higgs interactions* $H_u^{k*}\psi_d\psi_Q^k$ and $H_d^{k*}\psi_u\psi_Q^k$, are not supersymmetric (since the dimension-four supersymmetric Yukawa interactions must be holomorphic), and hence are absent from the tree-level Yukawa Lagrangian.

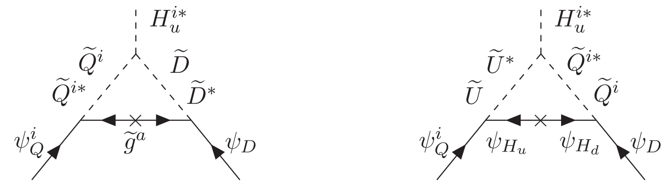
Nevertheless, the wrong-Higgs interactions can be generated in the effective low-energy theory below the scale of SUSY-breaking. In particular, one-loop radiative corrections, in which supersymmetric particles (squarks, higgsinos and gauginos) propagate inside the loop [33–41], can generate the wrong-Higgs interactions as shown in Fig. 1. In constructing the one-loop diagrams that produce the wrong-Higgs interactions, the relevant vertices derive from the following terms of the MSSM Lagrangian. First, we have the three-scalar interactions:

$$\mathcal{L}_{\rm int} = \mu h_t H_d^i \tilde{Q}^{i*} \tilde{U}^* + \mu h_b H_u^i \tilde{Q}^{i*} \tilde{D}^* - \epsilon_{ij}[h_b A_b H_d^i \tilde{Q}^j \tilde{D} - h_t A_t H_u^i \tilde{Q}^j \tilde{U}] + \text{H.c.}, \quad (2)$$

which derive from the μ-term of the superpotential and the soft-SUSY-breaking trilinear scalar interactions (the so-called A-terms). Second, we have the gaugino-quark-squark interactions:

$$\begin{aligned}\mathcal{L}_{\rm int} = &-i\sqrt{2} g_s (\bar{\tilde{g}}^a \bar{\psi}_{Qk}^i T_{kl}^a \tilde{Q}_l^i + \tilde{g}^a \psi_{Uk} T_{k\ell}^a \tilde{U}_\ell^* \\ &+ \tilde{g}^a \psi_{Dk} T_{k\ell}^a \tilde{D}_\ell^* + \text{H.c.}) \\ &- i\sqrt{2} g \left(\bar{\lambda}^a \bar{\psi}_Q^i \frac{1}{2} \tau_{ij}^a \tilde{Q}^j + \text{H.c.} \right) \\ &- i\sqrt{2} g' [y_Q \bar{\lambda}' \bar{\psi}_Q^i \tilde{Q}^i + y_U \bar{\lambda}' \bar{\psi}_U \tilde{U} + y_D \bar{\lambda}' \bar{\psi}_D \tilde{D} \\ &+ \text{H.c.}], \end{aligned} \quad (3)$$

[2]Under SU(3) × SU(2) × U(1), the quantum numbers of the two-component quark fields and Higgs fields are given by: $\psi_Q(\mathbf{3}, \mathbf{2}, 1/3)$, $\psi_U(\mathbf{3}^*, \mathbf{1}, -4/3)$, $\psi_D(\mathbf{3}^*, \mathbf{1}, 2/3)$, $H_d(\mathbf{1}, \mathbf{2}, -1)$ and $H_u(\mathbf{1}, \mathbf{2}, 1)$, where the electric charge Q (in units of e) of the fields are related to the corresponding isospin T_3 and U(1)-hypercharge (Y) by $Q = T_3 + \frac{1}{2}Y$. The two-component spinor product is defined by $\psi\chi \equiv \psi^\alpha \chi_\alpha = \epsilon^{\alpha\beta}\psi_\beta\chi_\alpha$ (α, $\beta = 1, 2$), and $\epsilon^{\alpha\beta}$ is antisymmetric with $\epsilon^{12} = 1$. The antisymmetric tensor ϵ_{ij} (with $\epsilon_{12} = 1$) contracts the gauge SU(2) indices. We denote the Yukawa couplings by h_b and h_t (instead of h_U and h_D) to emphasize that the third-generation Yukawa couplings dominate those of the lighter two generations.



(a) (b)

FIG. 1. One-loop diagrams contributing to the wrong-Higgs-Yukawa effective operators. In (a), the cross (×) corresponds to a factor of the gluino mass M_3. In (b), the cross corresponds to a factor of the higgsino Majorana mass parameter μ. Field labels correspond to annihilation of the corresponding particle at each vertex of the triangle.

which derive from the Kähler term (cf. Eq. (13)). In Eq. (3), g_s, g and g' are the $\text{SU(3)} \times \text{SU(2)} \times \text{U(1)}_\text{Y}$ gauge couplings, k and ℓ are SU(3) color indices and T^a are the SU(3) generators, i and j are the SU(2) gauge indices and τ^a are the Pauli matrices, and $y_Q = 1/3$, $y_U = -4/3$ and $y_D = 2/3$ are the corresponding hypercharges. Finally, the higgsino-quark-squark interactions are the supersymmetric analogs of the Higgs-quark Yukawa couplings:

$$\begin{aligned}\mathcal{L}_{\rm int} = \epsilon_{ij}[&h_b \psi_{H_d}^i (\psi_Q^j \tilde{D} + \psi_D \tilde{Q}^j) \\ &- h_t \psi_{H_u}^i (\psi_Q^j \tilde{U} + \psi_U \tilde{Q}^j) + \text{H.c.}]. \end{aligned} \quad (4)$$

If the squarks are heavy, then one can derive an effective field theory description of the Higgs-quark Yukawa couplings below the scale of the heavy squarks, where one has integrated out the heavy squarks propagating in the loops. The resulting effective Lagrangian is [12,42]:

$$\begin{aligned}\mathcal{L}_{\rm yuk}^{\rm eff} = &-\epsilon_{ij}(h_b + \delta h_b)\psi_b H_d^i \psi_Q^j + \Delta h_b \psi_b H_u^{k*} \psi_Q^k \\ &+ \epsilon_{ij}(h_t + \delta h_t)\psi_t H_u^i \psi_Q^j + \Delta h_t \psi_t H_d^{k*} \psi_Q^k. \end{aligned} \quad (5)$$

Note that in addition to δh_t and δh_b (which renormalize the Type-II Higgs-quark Yukawa interactions), wrong-Higgs-Yukawa interactions, with coefficients denoted by Δh_b and Δh_t, have been generated by the finite loop corrections depicted in Fig. 1. Explicitly, in the limit where the squarks are significantly heavier than the electroweak symmetry-breaking scale [37,42–45],[3]

$$\Delta h_b = h_b \left[\frac{2\alpha_s}{3\pi} \mu M_3 \mathcal{I}(M_{\tilde{b}_1}, M_{\tilde{b}_2}, M_g) + \frac{h_t}{16\pi^2} \mu A_t \mathcal{I}(M_{\tilde{t}_1}, M_{\tilde{t}_2}, \mu) \right], \quad (6)$$

and

[3]We neglect the contribution of the SU(2) × U(1) gauginos to the one-loop graphs of Fig. 1 as these effects are subdominant to the gluino contribution.

$$\mathcal{I}(a,b,c) = \frac{a^2b^2\ln(a^2/b^2) + b^2c^2\ln(b^2/c^2) + c^2a^2\ln(c^2/a^2)}{(a^2-b^2)(b^2-c^2)(a^2-c^2)}. \tag{7}$$

In Eq. (6), M_3 is the Majorana gluino mass, μ is the supersymmetric Higgs-mass parameter, and $\tilde{b}_{1,2}$ and $\tilde{t}_{1,2}$ are the mass-eigenstate bottom squarks and top squarks, respectively, (our notation follows that of Ref. [46]). Note that $\mathcal{I}(a,b,c) \sim 1/\max(a^2,b^2,c^2)$ in the limit where at least one of the arguments of $\mathcal{I}(a,b,c)$ is large. If $a = b = c$, then $\mathcal{I}(a,a,a) = 1/(2a^2)$.

As expected, the coefficients of the nonholomorphic dimension-four operators in Eq. (5) vanish in the supersymmetric limit (i.e., when the SUSY-breaking parameters A_b, A_t, and M_3 vanish). Moreover, it is useful to keep track of the $U(1)_R$-charges of the various operators appearing in the effective Lagrangian [47]. All supersymmetric terms must have total R-charge equal to zero.[4] If we assign the R-charges of the Higgs and quark fields such that $R(H_u) = R(H_d) = 1$ and $R(\psi_Q) = R(\psi_U) = R(\psi_D) = -\frac{1}{2}$ then, all dimension-four Yukawa interactions of the tree-level Lagrangian have R-charge zero. In contrast, the wrong-Higgs-Yukawa interactions are operators with R-charge 2.

We now demonstrate that the effect of the wrong-Higgs couplings is a $\tan\beta$-enhanced modification of a physical observable. The Higgs fields in Eq. (5) can be rewritten in terms of the physical mass-eigenstate neutral and charged Higgs fields and the Goldstone boson fields [48,49]:

$$H_d^1 = \frac{1}{\sqrt{2}}(v\cos\beta + H^0\cos\alpha - h^0\sin\alpha + iA^0\sin\beta - iG^0\cos\beta), \tag{8}$$

$$H_u^2 = \frac{1}{\sqrt{2}}(v\sin\beta + H^0\sin\alpha + h^0\cos\alpha + iA^0\cos\beta + iG^0\sin\beta), \tag{9}$$

$$H_d^2 = H^-\sin\beta - G^-\cos\beta, \tag{10}$$

$$H_u^1 = H^+\cos\beta + G^+\sin\beta, \tag{11}$$

where $v^2 \equiv v_u^2 + v_d^2 = (246\text{ GeV})^2$ and $\tan\beta \equiv v_u/v_d$. Inserting these expressions into Eq. (1), we can identify the bottom-quark mass as

$$m_b = \frac{h_b v}{\sqrt{2}}\cos\beta\left(1 + \frac{\delta h_b}{h_b} + \frac{\Delta h_b \tan\beta}{h_b}\right) \equiv \frac{h_b v}{\sqrt{2}}\cos\beta(1+\Delta_b), \tag{12}$$

which defines the quantity Δ_b. Note that the correction Δ_b is $\tan\beta$-enhanced if $\tan\beta \gg 1$. Typically in the limit of large $\tan\beta$ the term proportional to δh_b can be neglected, in which case, $\Delta_b \simeq (\Delta h_b/h_b)\tan\beta$.

It is especially noteworthy that the contributions of heavy supersymmetric particles propagating in the loops of Fig. 1 do *not* decouple in the limit of very heavy supersymmetric particle masses when the quantities $\mu M_3/M_{\tilde{q}}^2$ and $\mu A_t/M_{\tilde{q}}^2$ ($q = t$ or b) that appear in Eq. (6) are of $\mathcal{O}(1)$. Thus, Δ_b can in principle provide information about the heavy supersymmetric sector even if the supersymmetric particles are too heavy to be directly produced at the LHC. As Δ_b is $\tan\beta$-enhanced, one has the possibility of extracting this quantity from data by measuring the values of the bottom-quark Yukawa coupling, the bottom mass, and $\tan\beta$ at future colliders in a precision Higgs program [50].

We now investigate whether it is possible to implement a similar strategy of probing the heavy sector of supersymmetric models in studies of the gaugino sector.

III. WRONG-HIGGS INTERACTIONS IN THE GAUGINO SECTOR

In the MSSM, the supersymmetric partners of the gauge interactions of charged matter fields (either scalars or fermions) are dimension-four interactions that couple gauginos to fermions and the scalar superpartners (the sfermions). As in the case of the Yukawa Higgs-fermion interactions, only a subset of all possible dimension-four gauge-invariant gaugino-fermion-sfermion interactions are supersymmetric. Thus, we address the following question: in the low-energy effective theory below the scale that characterizes SUSY-breaking, are nonsupersymmetric dimension-four gauge-invariant gaugino-fermion-sfermion interactions generated with appreciable coefficients that can be probed by precision measurements of low-energy observables?

A. SUSY-violating dimension-four gauge-invariant gaugino-higgsino-Higgs boson interactions

In a supersymmetric field theory, the tree-level supersymmetric gaugino-fermion-sfermion interactions originate from the Kähler term [51,52]:

$$\mathcal{L}_K = \int d^4\theta \Phi_i^\dagger (e^{2gV})_{ij}\Phi_j \ni -i\sqrt{2}g_a(\bar{\lambda}^a\bar{\psi}_i T^a_{ij}A_j - A_i^* T^a_{ij}\psi_j\lambda^a), \tag{13}$$

where the Φ_i are chiral superfields (with physical scalar and two-component fermion components A_i and ψ_i) and V is the gauge vector superfield (with gaugino component λ). We denote the gauge group generators by T^a and allow for

[4] Note that the dimension-four terms of the tree-level Lagrangian of a spontaneously broken supersymmetric model respect the supersymmetry, and consequently these terms must have zero R-charge.

a product group structure for the gauge group by labeling the gauge coupling with the index a such that g_a is constant within each simple or U(1) factor of the full gauge group.

The tree-level MSSM chargino and neutralino mass matrices derive from three sources: (1) a supersymmetric higgsino Majorana mass term that is proportional to the μ term,

$$\mathcal{L}_{\mu} = \mu \int d^2\theta \epsilon_{ij} \hat{H}_u^i \hat{H}_d^j + \text{H.c.}, \quad (14)$$

where $\hat{H}_u$ and $\hat{H}_d$ are the Higgs superfields whose scalar and fermionic components are (H_u, ψ_{H_u}) and (H_d, ψ_{H_d}), respectively; (2) soft SUSY-breaking Majorana gaugino masses:

$$\mathcal{L}_{\text{soft}} = -M\lambda^a\lambda^a - M'\lambda'\lambda' + \text{H.c.}; \quad (15)$$

and (3) the gaugino-higgsino-Higgs boson interactions that arise from Eq. (13). Contributions to the chargino and neutralino masses are generated from the latter when the neutral Higgs fields acquire vacuum expectation values.

Summarizing, after including soft-SUSY-breaking terms, the gaugino-higgsino-Higgs boson sector of the MSSM Lagrangian (including mass terms) is given by

$$\begin{aligned}\mathcal{L}^{\text{gaugino}} = {}& \frac{ig_u}{\sqrt{2}}\lambda^a\tau^a_{ij}\psi^j_{H_u}H_u^{*i} + \frac{ig_d}{\sqrt{2}}\lambda^a\tau^a_{ij}\psi^j_{H_d}H_d^{*i} \\ & + \frac{ig'_u}{\sqrt{2}}\lambda'\psi^i_{H_u}H_u^{*i} - \frac{ig'_d}{\sqrt{2}}\lambda'\psi^i_{H_d}H_d^{*i} - M\lambda^a\lambda^a \\ & - M'\lambda'\lambda' - \mu\epsilon_{ij}\psi^i_{H_u}\psi^j_{H_d} + \text{H.c.} \end{aligned} \quad (16)$$

where

$$g_u = g_d = g, \qquad g'_u = g'_d = g'. \quad (17)$$

Following the strategy of Sec. II, we catalog all possible dimension-four gauge-invariant operators in the gaugino-higgsino-Higgs boson sector that violate supersymmetry. One class of operators of this type are given by

$$\begin{aligned} & \frac{ig_u}{\sqrt{2}}\lambda^a\tau^a_{ij}\psi^j_{H_u}H_u^{*i} + \frac{ig_d}{\sqrt{2}}\lambda^a\tau^a_{ij}\psi^j_{H_d}H_d^{*i} + \frac{ig'_u}{\sqrt{2}}\lambda'\psi^i_{H_u}H_u^{*i} \\ & - \frac{ig'_d}{\sqrt{2}}\lambda'\psi^i_{H_d}H_d^{*i} + \text{H.c.}, \end{aligned} \quad (18)$$

where the coupling g_u, g_d, g'_u and g'_d deviate from their supersymmetric values given in Eq. (17). Such effects are generated in the one-loop corrections to these interactions. They have been studied in detail in Refs. [15,16]. In this paper, we focus on the following gauge-invariant four-dimensional operators that are not present in the supersymmetric Lagrangian:

$$ik_1\lambda^a\tau^a_{ij}\psi^j_{H_u}\epsilon_{ki}H_d^k, \quad (19)$$

$$ik_2\lambda'\psi^k_{H_u}\epsilon_{ki}H_d^i, \quad (20)$$

$$ik_3\lambda^a\tau^a_{ij}\psi^j_{H_d}\epsilon_{ki}H_u^k, \quad (21)$$

$$ik_4\lambda'\psi^i_{H_d}\epsilon_{ki}H_u^k. \quad (22)$$

It is straightforward to verify that these are SUSY-breaking operators. For example, if we assign R-charges to the Higgs superfields so that $R(\hat{H}_u) = R(\hat{H}_d) = 1$ as before, then the component Higgs fields possess the same R-charges as their superfield parents, whereas corresponding higgsino fields have R-charges $R(\psi_{H_u}) = R(\psi_{H_d}) = 0$. The vector superfield V has R-charge equal to zero, which implies that R-charges of the gaugino fields are given by $R(\lambda) = R(\lambda') = 1$. Consequently, the operators in Eq. (18) all have total R-charge equal to zero, whereas the operators listed in Eqs. (19)–(22) have R-charge equal to 2. Hence, these hard-breaking operators do not appear in the tree-level MSSM. Nevertheless, these operators could be generated radiatively by the threshold effects of integrating out heavy fields just as the wrong-Higgs-Yukawa couplings to the quarks were generated after integrating out the superpartners. We now investigate whether these operators are generated in the low-energy effective theory at energies below the scale of SUSY-breaking.

B. Generating wrong-Higgs gaugino operators from a partially decoupled MSSM

In the case of the radiative corrections to the bottom-quark-Higgs-Yukawa interactions, the effective Lagrangian description was successful because the one-loop Feynman graphs with heavy supersymmetric particles propagating in the loops yielded effective local operators after integrating out the heavy states. Because of SUSY-breaking effects that generate large mass splitting between particles and their superpartners, the resulting dimension-four local operators that survive in the effective low-energy theory can violate supersymmetry; hence the origin of the wrong-Higgs-Yukawa couplings. In the case of gaugino interactions, one cannot usefully integrate out all the superpartners (in the limit where all superpartners are heavy), as this would remove the gaugino interaction terms of interest from the effective low-energy theory. Instead, one must consider a different limit where a subset of superpartners (*not* including the Higgs doublets, the gauginos, and the higgsinos) are integrated out. In this limit, we take μ, M, and M' in Eq. (16) small compared to squark and slepton masses. In particular, we assume that the soft-SUSY-breaking scalar mass parameters and A-terms (that govern the holomorphic trilinear scalar couplings) are of $\mathcal{O}(M_{\text{SUSY}})$, which we shall take to be significantly larger than the scale of electroweak symmetry-breaking.

In constructing the one-loop diagrams that produce the wrong-Higgs gaugino operators, the relevant vertices again derive from the interaction terms of the MSSM Lagrangian exhibited in Eqs. (2)–(4). We first attempt to construct graphs analogous to those of Fig. 1. Two possible graphs

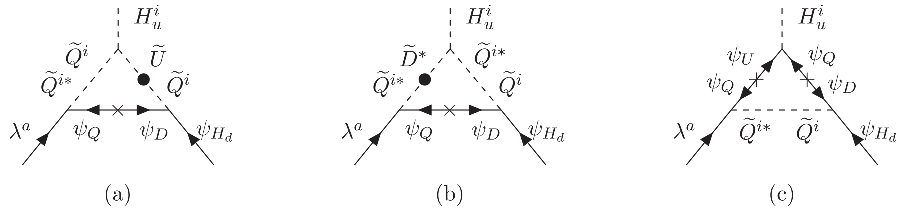


FIG. 2. One-loop diagrams contributing to the wrong-Higgs gaugino effective operators. The cross (×) indicates the two-component fermion propagator that is proportional to the corresponding Dirac mass. In (a) and (b) the solid dot indicates an insertion of the Higgs vacuum expectation value. Field labels correspond to annihilation at each vertex of the triangle.

that contribute to the wrong-Higgs gaugino operator that is proportional to k_3 (Eq. (21)) are exhibited in Figs. 2(a) and 2(b). However, these graphs must contain an insertion of a Higgs vacuum expectation value at the location of the solid dot on the squark lines (corresponding to $\tilde{q}_L$–$\tilde{q}_R$ mixing). Thus, simple power counting, under the assumption that the squark masses and A-terms are of order $M_{\rm SUSY} \gg v$, implies that the contributions of Figs. 2(a) and 2(b) are of $\mathcal{O}(m_b m_t/M_{\rm SUSY}^2)$ and $\mathcal{O}(m_b^2/M_{\rm SUSY}^2)$, respectively. Hence, these contributions decouple in the limit of $M_{\rm SUSY} \gg v$.

There is another vertex correction with an internal squark line, shown in Fig. 2(c) that can potentially contribute to the wrong-Higgs gaugino operators. Simple power counting again implies that the contribution of Fig. 2(c) is of $\mathcal{O}(m_b m_t/M_{\rm SUSY}^2)$ and hence decouples. The decoupling properties of Fig. 2 could have been anticipated due to the insertion of two vacuum expectation values in each diagram (either via the Dirac mass for the bottom and/or top quark or the $\tilde{Q}$–$\tilde{U}$ and/or $\tilde{Q}$–$\tilde{D}$ squark mixing). Hence, replacing the vacuum expectation value by the appropriate Higgs field, we see that the contributions of Fig. 2 actually correspond to dimension-six operators with the expected decoupling behavior.

Similar conclusions also apply to the contributions to the three other wrong-Higgs gaugino operators (Eqs. (19), (20), and (22)) introduced above. Consequently, we conclude that there are no nondecoupling one-loop contributions to the effective operators in Eqs. (19)–(22) from heavy MSSM fields.

C. Generating wrong-Higgs gaugino operators in a model of gauge-mediated supersymmetry-breaking

The MSSM is an effective low-energy theory of broken supersymmetry. One expects that the soft-SUSY-breaking dimension-two and dimension-three terms of the MSSM Lagrangian are generated by a new sector of heavy states. In models of gauge-mediated supersymmetry-breaking (GMSB), supersymmetry-breaking is transmitted to the MSSM via gauge forces [18–27]. A typical structure of such models involves a hidden sector where supersymmetry is broken, a messenger sector consisting of particles (messengers) with SU(3) × SU(2) × U(1) quantum numbers, and the visible sector consisting of the fields of the MSSM. The direct coupling of the messengers to the hidden sector generates a SUSY-breaking spectrum in the messenger sector. Finally, supersymmetry-breaking is transmitted to the MSSM via the virtual exchange of the messenger fields.

In order to maintain the unification of gauge coupling constants the messengers and the Higgs doublets are taken to be members of complete irreducible representations of SU(5). Moreover, for appropriate choices of gauge quantum numbers for the messenger fields, it is possible to construct gauge-invariant supersymmetric direct Yukawa couplings between the Higgs and messenger fields. Here we consider a model of such interactions and show that one-loop corrections involving messenger fields in the loop can generate the wrong-Higgs gaugino operators that survive in the low-energy theory below the scale of SUSY-breaking.

We begin by parametrizing a simple hidden and messenger sector that couples to the Higgs doublet. All the hidden sector dynamics will be described by a chiral superfield $\hat{Z}$ whose scalar component (Z) and F-term component (F_Z) acquire vacuum expectation values. $\hat{Z}$ couples to four messenger superfields, $\hat{M}_1$, $\hat{\bar{M}}_1$, $\hat{M}_2$, and $\hat{\bar{M}}_2$, whose quantum numbers under SU(3) × SU(2) × U(1)$_{\rm Y}$ are listed in Table I.

In GMSB models, it is typically difficult to generate a μ and B term of the same order of magnitude at the scale of low-energy SUSY-breaking. Addressing this problem is beyond the scope of this paper. Here we simply note that messenger loops will generate both μ and B parameters, as

TABLE I. Gauge quantum numbers of the Higgs and messenger superfields.

Superfield	SU(3)	SU(2)	U(1)$_{\rm Y}$
$\hat{H}_d$	1	2	−1
$\hat{H}_u$	1	2	1
$\hat{M}_1$	1	2	1
$\hat{\bar{M}}_1$	1	2	−1
$\hat{M}_2$	1	1	−2
$\hat{\bar{M}}_2$	1	1	2

explained in [28] and such messenger loops might play a role in a solution to the μ and B problem as recently discussed in [53,54]. Here, we shall focus on the interactions of the messenger sector and determine the phenomenological implications of messenger interactions with the Higgs fields. A simple superpotential that communicates SUSY-breaking through gauge mediation with Higgs interactions and is consistent with the symmetries exhibited in Table I is given by[5]:

$$W = \gamma_1 \epsilon_{ij} \hat{Z} \hat{M}_1^i \hat{\bar{M}}_1^j + \gamma_2 \hat{Z} \hat{M}_2 \hat{\bar{M}}_2 + \alpha \epsilon_{ij} \hat{H}_u^i \hat{M}_1^j \hat{M}_2 + \beta \epsilon_{ij} \hat{H}_d^i \hat{\bar{M}}_1^j \hat{\bar{M}}_2. \tag{23}$$

After taking into account the vacuum expectation values of the superfield $\hat{Z}$, this superpotential yields masses and interaction terms for the messenger scalar and fermionic component fields. For the computations presented in this section, we record the relevant mass and interaction terms:

$$\begin{aligned} -\mathcal{L}_{\rm mass} = {}& |\gamma_1 \langle Z\rangle|^2 |M_1|^2 + |\gamma_1 \langle Z\rangle|^2 |\bar{M}_1|^2 \\ &+ |\gamma_2 \langle Z\rangle|^2 |M_2|^2 + |\gamma^2 \langle Z\rangle|^2 |\bar{M}_2|^2 \\ &+ \gamma_1 F_Z \epsilon_{ij} M_1^i \bar{M}_1^j + \gamma_2 F_Z M_2 \bar{M}_2 \\ &+ \gamma_1 \langle Z\rangle \epsilon_{ij} \psi_{M_1}^i \psi_{\bar{M}_1}^j + \gamma_2 \langle Z\rangle \psi_{M_2} \psi_{\bar{M}_2} + \text{H.c.}, \end{aligned} \tag{24}$$

and

$$\begin{aligned} \mathcal{L}_{\rm int} = {}& -\epsilon_{ij}[\alpha(H_u^i \psi_{M_1}^j \psi_{M_2} + M_1^j \psi_{H_u}^i \psi_{M_2} + M_2 \psi_{H_u}^i \psi_{M_1}^j) \\ &+ \beta(H_d^i \psi_{\bar{M}_1}^j \psi_{\bar{M}_2} + \bar{M}_1^j \psi_{H_d}^i \psi_{\bar{M}_2} + \bar{M}_2 \psi_{H_d}^i \psi_{\bar{M}_1}^j)] \\ &- \gamma_2 \epsilon_{ij} \langle Z\rangle [\alpha H_u^i M_1^j \bar{M}_2^* + \beta H_d^i \bar{M}_1^j M_2^*] \\ &+ \gamma_1 \langle Z\rangle [\alpha H_u^i \bar{M}_1^{i*} M_2 - \beta H_d^i M_1^{i*} \bar{M}_2] + \text{H.c.} \end{aligned} \tag{25}$$

We also record the relevant gaugino-particle-sparticle interactions involving the messenger scalars and their fermionic superpartners. From Eq. (13), we obtain

$$\begin{aligned} \mathcal{L}_{\rm int} = {}& \frac{ig}{\sqrt{2}} \lambda^a \tau_{ij}^a [\psi_{M_1}^j M_1^{i*} + \psi_{\bar{M}_1}^j \bar{M}_1^{i*}] - \frac{ig'}{\sqrt{2}} \lambda' [\psi_{M_1}^i M_1^{i*} \\ &- \psi_{\bar{M}_1}^i \bar{M}_1^{i*} - 2\psi_{M_2} M_2^* + 2\psi_{\bar{M}_2} \bar{M}_2] + \text{H.c.} \end{aligned} \tag{26}$$

[5]Note that this superpotential could in principle be embedded in a grand unified theory, where the various superfields live within the following SU(5) multiplets: $\hat{M}_1 \subset \mathbf{5}$, $\hat{\bar{M}}_1 \subset \mathbf{5}^*$, $\hat{M}_2 \subset \mathbf{10}^*$, $\hat{\bar{M}}_2 \subset \mathbf{10}$, $\hat{H}_u \subset \mathbf{5}$, and $\hat{H}_d \subset \mathbf{5}^*$. In this case, the Higgs/messenger couplings would originate as subsets of the $\mathbf{5}^* \times \mathbf{5}^* \times \mathbf{10}$ and $\mathbf{5} \times \mathbf{5} \times \mathbf{10}^*$ couplings.

In typical GMSB models, soft-SUSY-breaking masses for the gauginos are generated at one-loop and soft-SUSY-breaking squared-masses for the scalars (squarks, sleptons, and Higgs bosons) are generated at two-loops. Consequently, the soft-SUSY-breaking masses of the gauginos and scalars are of the same order of magnitude. For example, in order to ensure that $M \sim M' \sim \mu \sim$ 100–500 GeV, one must choose $F_Z/\langle Z\rangle \sim 100$ TeV. However, if the Higgs bosons couple directly to the messengers as in Eq. (23), soft-SUSY-breaking masses for the Higgs fields and a B-term will be generated at *one* loop order. In this case, an unnatural fine-tuning is required to keep these Higgs soft-masses $\lesssim \mathcal{O}(M_{\rm SUSY})$. In order to reduce the amount of fine-tuning,[6] we shall take $F_Z/\langle Z\rangle \sim$ 20 TeV. In such a model, the contributions of the messenger superfields $\hat{M}_1$ and $\hat{M}_2$ to slepton and gaugino masses are phenomenologically too small. One must then add an additional source of SUSY-breaking to the theory. An extra pair of weak doublet messenger fields (and corresponding color-triplet messenger fields) coupling to a different spurion $\hat{X}$, where $\hat{X} = \langle X\rangle + \theta^2 F_X$ and $F_X/\langle X\rangle \sim 100$ TeV is sufficient to raise the masses of the sleptons, squarks, and gauginos above the current experimental bounds. Henceforth, we shall focus exclusively on the radiative effects of the messenger fields that couple to the spurion $\hat{Z}$ (and in what follows, the term "messengers" will always refer to these fields).

Since $\langle Z\rangle$ sets the scale for the average messenger masses, the consistency of the model[7] requires that $F_Z \lesssim \langle Z\rangle^2$. Under our model assumption, $F_Z/\langle Z\rangle \sim 20$ TeV, we can write $\langle Z\rangle \sim 20\ \text{TeV}/(F_Z/\langle Z\rangle^2)$. There are then two regimes of possible interest. If $F_Z \sim \langle Z\rangle^2$, then the messengers are rather "light," with an average mass of order 20 TeV. In contrast, if $F_Z \ll \langle Z\rangle^2$, then the messenger masses are significantly heavier.

Consider first the case of $F_Z \ll \langle Z\rangle^2$. In this case, the mass splittings of M_1, $\bar{M}_1$, M_2, and $\bar{M}_2$ can be treated as perturbations about the average mass $\langle Z\rangle$. Let us examine the contributions to the SUSY-breaking wrong-Higgs gaugino interactions generated by integrating out the messenger fields. In this case we can evaluate the diagrams of Fig. 3 in the mass-insertion approximation, with messengers running in the loops and mass insertions of F_Z on the scalar propagator lines.

The graphs with two scalar propagators enter with the opposite sign compared to the graph with the two fermion propagators. We find the following leading contribution to k_3 in the mass-insertion approximation:

[6]In this context, we accept the order 1–10% fine-tuning associated with the so-called little hierarchy problem [55–57].

[7]For $F_Z \gtrsim \langle Z\rangle^2$, large splittings of squared-masses in the messenger sector would drive some scalar squared-masses negative. In practice, one requires the masses of all messengers to lie above the masses of the superpartners of the standard model particles. This sets an upper bound on $F_Z/\langle Z\rangle^2$ of $\mathcal{O}(1)$.

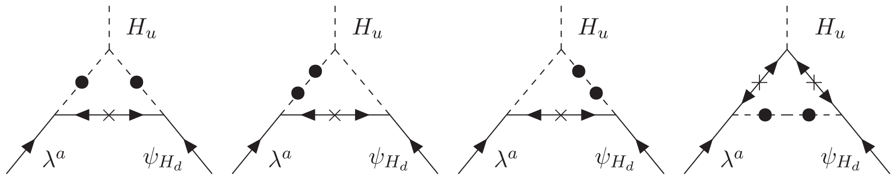


FIG. 3. One-loop diagrams with internal lines consisting of scalar and fermionic messenger fields. The cross ($\times$) indicates the two-component fermion propagator that is proportional to the corresponding Dirac mass. The solid dot indicates an F_Z mass-insertion on the scalar messenger line.

$$k_3 \sim \frac{g}{16\pi^2}\left(\frac{F_Z}{\langle Z\rangle^2}\right)^2, \qquad (27)$$

where we have suppressed an overall numerical coefficient of $\mathcal{O}(1)$. The contribution to k_3 (and similarly for the other k_i) decouples in the limit of $F_Z \ll \langle Z\rangle^2$, in agreement with the expectations of [17]. In Sec. IV, we will exhibit a correction to a physical quantity that is proportional to $k_3 \tan\beta$. However, even with a $\tan\beta$-enhancement as large as 50, the ultimate effect of such corrections is too small to be observed.

If $F_Z \sim \langle Z\rangle^2$, the mass-insertion approximation employed above is no longer valid. The results of Eq. (27) are suggestive of the possibility that the effect of integrating out the messenger fields could produce nondecoupling contributions to the k_i in the low-energy effective theory below the messenger scale. In order to evaluate the one-loop contributions to k_i for $F_Z \sim \langle Z\rangle^2$, we must employ mass-eigenstates for the scalar messengers that appear in the loops of Fig. 3. Scalar messengers must be reexpressed in terms of their mass-eigenstates, and the diagrams must be evaluated with internal line mass-eigenstates. In this paper, we present the explicit computation for k_3, as this coefficient is the only one that governs unsuppressed corrections to the chargino mass matrix and interactions.

From Eq. (24), it follows that the fermionic messenger fields organize themselves into two Dirac fermions $\Psi_1 \equiv (\psi_{M_1}, \bar{\psi}_{\bar{M}_1})$ and $\Psi_2 \equiv (\psi_{M_2}, \bar{\psi}_{\bar{M}_2})$ with corresponding Dirac masses $m_1 \equiv \gamma_1\langle Z\rangle$ and $m_2 \equiv \gamma_2\langle Z\rangle$. Moreover, we can express the mass-eigenstate scalar messengers in terms of the corresponding interaction-eigenstate fields. We work in the limit of exact $SU(2)\times U(1)$ and ignore small corrections to the messenger masses of order the electroweak scale that are generated by the neutral Higgs vacuum expectation values. It is convenient to rewrite the complex scalar messenger fields in terms of their real and imaginary parts:

$$M_1^i = \frac{1}{\sqrt{2}}(M_{1R}^i + M_{1I}^i), \qquad M_2 = \frac{1}{\sqrt{2}}(M_{2R} + M_{2I}^i), \qquad (28)$$

and similarly for the barred fields $\bar{M}_1^i$ and $\bar{M}_2$. From Eq. (24), the scalar messenger mass-eigenstates are determined from:

$$\begin{aligned} -\mathcal{L}_{\rm mass} = {} & \frac{1}{2}(M_{1R}^i\epsilon_{ij}\bar{M}_{1R}^j)\mathbf{\Gamma}_+^{(1)}\begin{pmatrix} M_{1R}^i \\ \epsilon_{ik}\bar{M}_{1R}^k\end{pmatrix} \\ & + \frac{1}{2}(M_{1I}^i\epsilon_{ij}\bar{M}_{1I}^j)\mathbf{\Gamma}_-^{(1)}\begin{pmatrix} M_{1I}^i \\ \epsilon_{ik}\bar{M}_{1I}^k\end{pmatrix} \\ & + \frac{1}{2}(M_{2R}\bar{M}_{2R})\mathbf{\Gamma}_+^{(2)}\begin{pmatrix} M_{2R} \\ \bar{M}_{2R}\end{pmatrix} \\ & + \frac{1}{2}(M_{2I}\bar{M}_{2I})\mathbf{\Gamma}_-^{(2)}\begin{pmatrix} M_{2I} \\ \bar{M}_{2I}\end{pmatrix}, \end{aligned} \qquad (29)$$

where

$$\mathbf{\Gamma}_\pm^{(n)} = \begin{pmatrix} \gamma_n^2\langle Z\rangle^2 & \pm\gamma_n F_Z \\ \pm\gamma_n F_Z & \gamma_n^2\langle Z\rangle^2\end{pmatrix}, \qquad (n = 1, 2). \qquad (30)$$

The scalar messenger mass-eigenstates are

$$\begin{aligned} M_{\pm 1R,I}^i &\equiv \frac{1}{\sqrt{2}}(M_{1R,I}^i \pm \epsilon_{ij}\bar{M}_{1R,I}^j), \\ M_{\pm 2R,I} &\equiv \frac{1}{\sqrt{2}}(M_{2R,I} \pm \bar{M}_{2R,I}), \end{aligned} \qquad (31)$$

with corresponding masses $m_{nR,I}^\pm$ given by (for $n = 1, 2$)

$$\begin{aligned} (m_{1I}^-)^2 = (m_{1R}^+)^2 &= \gamma_1^2\langle Z\rangle + \gamma_1 F_Z, \\ (m_{2I}^-)^2 = (m_{2R}^+)^2 &= \gamma_2^2\langle Z\rangle + \gamma_2 F_Z, \end{aligned} \qquad (32)$$

$$\begin{aligned} (m_{1I}^+)^2 = (m_{1R}^-)^2 &= \gamma_1^2\langle Z\rangle - \gamma_1 F_Z, \\ (m_{2I}^+)^2 = (m_{2R}^-)^2 &= \gamma_2^2\langle Z\rangle - \gamma_2 F_Z. \end{aligned} \qquad (33)$$

We can then evaluate the exact one-loop threshold corrections contributing to k_3 by employing Feynman rules with messenger mass-eigenstates. In the limit where the internal particle masses are much greater than the external momenta,

$$\begin{aligned}\frac{k_3}{g} = {} & \frac{\alpha\beta(\gamma_2+\gamma_1)}{128\sqrt{2}\pi^2} m_1\langle Z\rangle[\mathcal{I}(m_1, m_{1R}^+, m_{2R}^+) + \mathcal{I}(m_1, m_{1I}^+, m_{2I}^+) + \mathcal{I}(m_1, m_{1R}^+, m_{2I}^-) + \mathcal{I}(m_1, m_{1I}^-, m_{2R}^+) + \mathcal{I}(m_1, m_{1R}^-, m_{2R}^-) \\ & + \mathcal{I}(m_1, m_{1I}^-, m_{2I}^-) + \mathcal{I}(m_1, m_{1I}^+, m_{2R}^-) + \mathcal{I}(m_1, m_{1R}^-, m_{2I}^+)] \\ & + \frac{\alpha\beta(\gamma_2-\gamma_1)}{128\sqrt{2}\pi^2} m_1\langle Z\rangle[\mathcal{I}(m_1, m_{1R}^-, m_{2R}^+) + \mathcal{I}(m_1, m_{1R}^+, m_{2R}^-) + \mathcal{I}(m_1, m_{1I}^+, m_{2R}^+) + \mathcal{I}(m_1, m_{1I}^-, m_{2R}^-) \\ & + \mathcal{I}(m_1, m_{1R}^+, m_{2I}^+) + \mathcal{I}(m_1, m_{1R}^-, m_{2I}^-) + \mathcal{I}(m_1, m_{1I}^-, m_{2I}^+) + \mathcal{I}(m_1, m_{1I}^+, m_{2I}^-)] \\ & - \frac{\alpha\beta m_1 m_2}{32\sqrt{2}\pi^2}[\mathcal{I}(m_1, m_2, m_{1R}^+) + \mathcal{I}(m_1, m_2, m_{1I}^+) + \mathcal{I}(m_1, m_2, m_{1R}^-) + \mathcal{I}(m_1, m_2, m_{1I}^-)] \end{aligned} \tag{34}$$

where the triangle integral $\mathcal{I}(a, b, c)$ is defined in Eq. (7). In the limit of $\gamma_1 = \gamma_2 \equiv \gamma$, the above results simplify significantly, and the messenger masses are given by

$$m_1^2 = m_2^2 = \gamma^2\langle Z\rangle^2, \tag{35}$$

$$(m_{1I}^+)^2 = (m_{1R}^-)^2 = (m_{2I}^+)^2 = (m_{2R}^-)^2 = \gamma^2\langle Z\rangle^2 - \gamma F_Z, \tag{36}$$

$$(m_{1I}^-)^2 = (m_{1R}^+)^2 = (m_{2R}^+)^2 = (m_{2I}^-)^2 = \gamma^2\langle Z\rangle^2 + \gamma F_Z, \tag{37}$$

in which case Eq. (34) simplifies to:

$$\begin{aligned}\frac{k_3}{g} = {} & \frac{\sqrt{2}\alpha\beta\gamma^2\langle Z\rangle^2}{32\pi^2}[\mathcal{I}((\gamma^2\langle Z\rangle^2 + \gamma F_Z)^{1/2}, \gamma\langle Z\rangle) \\ & + \mathcal{I}((\gamma^2\langle Z\rangle^2 - \gamma F_Z)^{1/2}, \gamma\langle Z\rangle) \\ & - \mathcal{I}(\gamma\langle Z\rangle, (\gamma^2\langle Z\rangle^2 + \gamma F_Z)^{1/2}) \\ & - \mathcal{I}(\gamma\langle Z\rangle, (\gamma^2\langle Z\rangle^2 - \gamma F_Z)^{1/2})], \end{aligned} \tag{38}$$

where

$$\begin{aligned}\mathcal{I}(a, b) \equiv \mathcal{I}(a, a, b) = \mathcal{I}(b, a, a) \\ = \frac{a^2(a^2-b^2) + a^2b^2\ln(b^2/a^2)}{a^2(a^2-b^2)^2}. \end{aligned} \tag{39}$$

An explicit evaluation of Eq. (38) yields

$$\frac{k_3}{g} = \frac{\sqrt{2}\alpha\beta}{32\pi^2} f(x), \qquad x \equiv \frac{F_Z}{\gamma\langle Z\rangle^2}, \tag{40}$$

where

$$f(x) \equiv \frac{(x-2)\ln(1-x) - (x+2)\ln(1+x)}{x^2}. \tag{41}$$

The small x expansion of $f(x)$ gives

$$f(x) = \frac{x^2}{3} + \frac{4x^4}{15} + \mathcal{O}(x^6), \tag{42}$$

which confirms the behavior of k_3 for $x \ll 1$ given in Eq. (27). Note that $f(x) \to \infty$ as $x \to 1$, which reflects the fact that one of the messenger masses is approaching zero. Thus, we cannot take x as large as 1.

We shall choose x such that the lightest messenger mass lies above 1 TeV. With this bound, x can assume values quite close to 1. As an example, consider the case of $\gamma = 1$ and $F_z/\langle Z\rangle \sim 20$ TeV in order that squark and gaugino masses lie in the appropriate mass range. If x is close to 1, then $F_Z \sim \langle Z\rangle^2$, in which case, $\langle Z\rangle = 20$ TeV. If $x \simeq 0.98$, then for $\gamma = 1$ the lightest messenger has a mass of 2.8 TeV. This is as large an x value that one could sensibly allow. At this particular point in parameter space $f(0.98) = 2.0$. This yields a value for the effective contribution to k_3/g of

$$\frac{k_3}{g} = 2.0\frac{\sqrt{2}\alpha\beta}{32\pi^2}. \tag{43}$$

This is roughly a maximum possible value for these threshold effects. Using the formulae given above, we can compute k_3/g for a variety of sample points in the parameter space. In Table II, we provide four representative points. The conclusion we draw from this small sample is that with Yukawa couplings of messengers to Higgs boson of $\mathcal{O}(1)$, and the lightest messenger mass above 2 TeV, it is typical to find values of $k_3/g \sim (0.1\text{–}1.4)/(16\pi^2)$. Such corrections are of one-loop order in size—small but not too small.

We have established that in a theory with a low SUSY-breaking scale in a simple gauge-mediated scenario in which messenger fields couple to the Higgs bosons of the MSSM, there are dimension-four SUSY-breaking wrong-Higgs gaugino operators operators (cf. Eqs. (19)–(22)) generated as threshold corrections at one-loop order. Therefore, one should include these effects in the char-

TABLE II. Sample points in the messenger parameter space. We have fixed $\langle Z\rangle = 20$ TeV and $\alpha = \beta = 1$. The mass of the lightest messenger state is denoted by M_-.

γ_1	γ_2	F_Z	M_-	$16\pi^2 k_3/g$
1	1	$(19.8\ \text{TeV})^2$	2.8 TeV	1.44
0.9	1	$(18.8\ \text{TeV})^2$	2.4 TeV	1.38
1	1	$(16.8\ \text{TeV})^2$	10.9 TeV	0.19
0.75	1	$(14\ \text{TeV})^2$	8.8 TeV	0.15

gino/neutralino sector of the effective Lagrangian of the MSSM below the fundamental SUSY-breaking scale. The usual supersymmetric relations between the parameters of the chargino/neutralino sector and the gauge sector of the MSSM will then be modified. In Sec. IV, we shall demonstrate that such effects can be considerably enhanced if the parameter $\tan\beta$ is large.

D. Renormalization group improvement

The effective Lagrangian describing the gaugino sector for the MSSM just below the scale of fundamental SUSY-breaking is given by

$$\mathcal{L}^{\rm eff}_{\rm gaugino} = \frac{ig_u}{\sqrt{2}}\lambda^a\tau^a_{ij}\psi^j_{H_u}H^{*i}_u + \frac{ig_d}{\sqrt{2}}\lambda^a\tau^a_{ij}\psi^j_{H_d}H^{*i}_d + \frac{ig'_u}{\sqrt{2}}\lambda'\psi^i_{H_u}H^{*i}_u - \frac{ig'_d}{\sqrt{2}}\lambda'\psi^i_{H_d}H^{*i}_d - M\lambda^a\lambda^a - M'\lambda'\lambda' - \mu\epsilon_{ij}\psi^i_{H_u}\psi^j_{H_d} + ik_1\lambda^a\tau^a_{ij}\psi^j_{H_u}\epsilon_{ki}H^k_d + ik_2\lambda'\psi^k_{H_u}\epsilon_{ki}H^i_d + ik_3\lambda^a\tau^a_{ij}\psi^j_{H_d}\epsilon_{ki}H^k_u + ik_4\lambda'\psi^i_{H_d}\epsilon_{ki}H^k_u + \text{H.c.}, \tag{44}$$

where we have added the dimension-four wrong-Higgs gaugino operators given by Eqs. (19)–(22) to the tree-level gaugino Lagrangian (Eq. (16)). The effective Lagrangian displayed in Eq. (44) is defined at the threshold scale of the messengers. We then use the renormalization group (RG) to run down to the electroweak scale. In general the messengers decouple in two stages: once at the scale $[\langle Z\rangle^2 + F]^{1/2}$ and once at the scale $[\langle Z\rangle^2 - F]^{1/2}$. For simplicity, we will estimate the effects of the RG by decoupling the messengers at the scale $\mu_M = \langle Z\rangle$. However, in the limit where the lightest messenger state is extremely light, two stages of decoupling must be used. Our goal here is to estimate the effects of the RG analysis on the results from the threshold loops obtained in the previous section.

We begin with $\mathcal{L}^{\rm MSSM}_{\rm eff}(\mu_M)$ as given in Eq. (44). The parameters that appear in this Lagrangian are effective parameters. For example, $g_u = g + \delta g_u$, $g_d = g + \delta g_d$, $g'_u = g' + \delta g'_u$, and $g'_d = g' + \delta g'_d$, where the $\delta g's$ include threshold and renormalization group effects from SUSY-breaking below the fundamental SUSY-breaking scale. For M', M, and μ we simply absorb renormalization and threshold corrections into these coefficients. In the previous section, we presented an explicit calculation for k_3/g. The other coefficients k_1/g, k_2/g', and k_4/g' are also generated with the same order of magnitude.[8]

Because SUSY is broken by dimension-four hard-breaking operators, the theory below μ_M is nonsupersymmetric and the RG for all couplings must be evolved independently [58–60]. For all supersymmetric tree-level couplings, it is a very good approximation to neglect the presence of the new couplings k_i, as these new couplings are one-loop-suppressed. Moreover, the wrong-Higgs gaugino operators break the R-symmetry by two units of R-charge (with the standard R-charge assignments to the regular MSSM superfields). Therefore, the supersymmetric RG equations for the MSSM couplings are always modified by terms proportional to the square of k_i (corresponding diagrammatically to a change of R-charge by ± 2 at the two wrong-Higgs interaction vertices, respectively). Therefore the resulting contribution to the effective supersymmetric coupling is always suppressed by a factor of $\mathcal{O}(1/(16\pi^2)^3)$, which is negligible. The k_i also evolve according to the RG, and the R-charge analysis implies that they satisfy RG equations that are linear and trilinear in the k_i. The RG equations for the couplings k_i (neglecting the deviation of the couplings g_u, g_d [and g'_u, g'_d] from their supersymmetric values g [and g'], respectively) are given by

$$16\pi^2\frac{dk_1}{dt} = \frac{1}{2}k_1(6h_t^2 + 6h_b^2 + 2h_l^2 + 11k_1^2 + 3k_2^2 + 2k_3^2) + (g^2 - g'^2)k_3 - g'gk_2, \tag{45}$$

$$16\pi^2\frac{dk_2}{dt} = \frac{1}{2}k_2(6h_t^2 + 6h_b^2 + 2h_l^2 + 2k_4^2 + 9k_1^2 + 5k_2^2) + 3g^2k_4 - 3g'gk_1 + g'^2(k_4 + 12k_2), \tag{46}$$

$$16\pi^2\frac{dk_3}{dt} = \frac{1}{2}k_3(6h_t^2 + 6h_b^2 + 2h_l^2 + 11k_3^2 + 3k_4^2 + 2k_1^2) + (g^2 - g'^2)k_1 + g'gk_4, \tag{47}$$

$$16\pi^2\frac{dk_4}{dt} = \frac{1}{2}k_4(6h_t^2 + 6h_b^2 + 2h_l^2 + 2k_2^2 + 9k_3^2 + 5k_4^2) + 3g^2k_2 + 3g'gk_3 + g'^2(k_2 + 12k_4). \tag{48}$$

If one keeps only the largest terms in the RG equation for k_3, then Eq. (47) reduces to

[8] We do not present an explicit calculation of k_1, k_2, and k_4 here. Instead, we narrow our focus to the chargino sector and in particular the chargino mass matrix. The coefficients k_2 and k_4 only affect the neutralino mass matrix so for the subsequent analysis we do not need the coefficients of these operators. In Sec. IV, we demonstrate that the effects of the wrong-Higgs gaugino operator proportional to k_1 are suppressed at large $\tan\beta$, and can likewise be neglected.

$$16\pi^2 \frac{dk_3}{dt} = k_3(3h_t^2 + 3h_b^2). \tag{49}$$

As an example, for $\tan\beta = 50$, we obtain $h_t = 0.95$ and $h_b = 1.16$. This provides a first estimate of the RG correction to k_3,

$$0.86k_3(\mu_M = 20\ \text{TeV}) = k_3(\mu = 500\ \text{GeV}). \tag{50}$$

That is, RG-evolution has reduced the size of k_3 (obtained in Sec. III C) by roughly 14%. More generally, we expect modifications of the threshold values of the k_i to be of order 10% by RG running in the parameter regime of interest.

IV. EFFECTS OF WRONG-HIGGS CHARGINO OPERATORS ON THE CHARGINO MASS MATRIX

A. Dimension-four hard SUSY-breaking corrections to the chargino mass matrix

After the neutral Higgs bosons acquire their vacuum expectation values, $\langle H_u^0\rangle = v_u/\sqrt{2}$ and $\langle H_d^0\rangle = v_d/\sqrt{2}$, the quadratic terms of the effective gaugino Lagrangian (Eq. (44)) are given by

$$\begin{aligned}\mathcal{L}_{\text{mass}} = {} & \frac{ig_u v_u}{2}\lambda^a\tau^a_{2j}\psi^j_{H_u} + \frac{ig_d v_d}{2}\lambda^a\tau^a_{1j}\psi^j_{H_d} + \frac{ig'_u v_u}{2}\lambda'\psi^2_{H_u} \\ & - \frac{ig'_d v_d}{2}\lambda'\psi^1_{H_d} - M\lambda^a\lambda^a - M'\lambda'\lambda' \\ & - \mu\epsilon_{ij}\psi^i_{H_u}\psi^j_{H_d} + \frac{ik_1 v_d}{\sqrt{2}}\lambda^a\tau^a_{2j}\psi^j_{H_u} - \frac{ik_2 v_d}{\sqrt{2}}\lambda'\psi^2_{H_u} \\ & - \frac{ik_3 v_u}{\sqrt{2}}\lambda^a\tau^a_{1j}\psi^j_{H_d} - \frac{ik_4 v_u}{\sqrt{2}}\lambda'\psi^1_{H_d} + \text{H.c.}\end{aligned} \tag{51}$$

Isolating the terms that contribute to the chargino matrix, we introduce

$$\psi_i^+ = \begin{pmatrix} -i\lambda^+ \\ \psi^1_{H_u}\end{pmatrix}, \qquad \psi_i^- = \begin{pmatrix} -i\lambda^- \\ \psi^2_{H_d}\end{pmatrix}, \tag{52}$$

where $\lambda^\pm = \frac{1}{\sqrt{2}}(\lambda^1 \mp i\lambda^2)$. Then, the chargino mass terms are given by

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2}(\psi^+ \quad \psi^-)\begin{pmatrix} 0 & (X^{\text{eff}})^T \\ X^{\text{eff}} & 0\end{pmatrix}\begin{pmatrix}\psi^+ \\ \psi^-\end{pmatrix} + \text{H.c.}, \tag{53}$$

where

$$X^{\text{eff}} \equiv \begin{pmatrix} X^{\text{eff}}_{11} & X^{\text{eff}}_{12} \\ X^{\text{eff}}_{21} & X^{\text{eff}}_{22}\end{pmatrix} = \begin{pmatrix} M & (g+\delta g_u)\frac{v_u}{\sqrt{2}}\left(1 - \frac{\sqrt{2}k_1\cot\beta}{g+\delta g_u}\right) \\ (g+\delta g_d)\frac{v_d}{\sqrt{2}}\left(1 + \frac{\sqrt{2}k_3\tan\beta}{g+\delta g_d}\right) & \mu\end{pmatrix} \tag{54}$$

with $v_u \equiv v\sin\beta$ and $v_d \equiv v\cos\beta$.

In the limit of large $\tan\beta$, the correction to the supersymmetric relation, $X_{21} = gv\cos\beta/\sqrt{2}$, is significant. Including effects from the improved renormalization group running of the parameters of Table II, this correction can be as large as 7%–56% for $\tan\beta = 50$ as F_Z varies between 14–19 TeV. In this estimate we have neglected the effects of δg_d as these are one-loop effects with no $\tan\beta$-enhancements.

In [61–64], it was shown in detail how to extract the parameters of the chargino sector from polarized e^+e^- experiments. By employing these techniques, it should be possible to detect deviations from the standard MSSM expectations. We have seen above that the effect of the wrong-Higgs chargino operators is to generate a potentially significant $\tan\beta$-enhanced correction to the supersymmetric value of X_{21}. Hence, we focus on the perturbation of the chargino mass matrix due to a shift in the value of X_{21}.

B. A perturbative analysis of the contribution of the wrong-Higgs gaugino couplings to the chargino mass matrix

Given the effective chargino mass matrix of Eq. (54) (henceforth denoted as X), we can compute the chargino eigenvalues and corresponding diagonalization matrices. Any complex matrix possesses a singular value decomposition [65] of the form

$$U^* X V^{-1} = M_D \equiv \text{diag}(m_{\chi_1^+}, m_{\chi_2^+}), \tag{55}$$

for some suitably chosen unitary matrices U and V, where the elements of the diagonal matrix M_D are real and nonnegative. Note that Eq. (55) implies that

$$VX^\dagger XV^{-1} = U^* XX^\dagger U^T = M_D^2. \tag{56}$$

Thus, the chargino masses are determined by solving the eigenvalue problem for either $X^\dagger X$ (or equivalently, for $XX^\dagger$). Moreover, to compute the unitary matrices U and V, one can first determine the matrix U by diagonalizing $XX^\dagger$ and then compute V from Eq. (55) (or equivalently, one can first determine the matrix V by diagonalizing $X^\dagger X$ and then compute U from Eq. (17)). In the former procedure, U is unique up to multiplication on the right by a diagonal matrix of phases (assuming that the elements of M_D are nondegenerate). We shall use this phase freedom to reduce the number of parameters of the unitary matrix U from four to two; that is, we can parametrize U as follows [61–63]:

$$U = \begin{pmatrix} \cos\theta_L & e^{-i\beta_L}\sin\theta_L \\ -e^{i\beta_L}\sin\theta_L & \cos\theta_L\end{pmatrix}. \tag{57}$$

Once U has been fixed, then V is uniquely determined by Eq. (55). The unitary matrix V depends on four parameters, which we parametrize as

$$V = \begin{pmatrix} e^{i\zeta_1} & 0 \\ 0 & e^{i\zeta_2} \end{pmatrix} \begin{pmatrix} \cos\theta_R & e^{-i\beta_R}\sin\theta_R \\ -e^{i\beta_R}\sin\theta_R & \cos\theta_R \end{pmatrix}. \quad (58)$$

In the MSSM (with the hard-breaking SUSY contributions set to zero), the only nontrivial phase is the relative phase between M and μ. One can always absorb the phase of M by rephasing the SU(2) gaugino field, in which case the only remaining potentially complex parameter is $\mu \equiv |\mu| e^{i\Phi}$. Thus, without loss of generality, we take M real and positive. New phases can also enter due to the complexity of the parameters γ_1, γ_2, α, and β that parametrize the messenger superpotential (cf. Eq. (23)). Consequently, in the chargino mass matrix, two new independent phases can appear in X_{12} and X_{21}. For simplicity, we assume in what follows that these phases are either absent or negligible. We shall address the implication of non-negligible CP-violating phases in X_{12} and X_{21} in a future publication.

For $M > 0$ and X_{12} and X_{21} real, the chargino squared-masses and mixing angles θ_L and θ_R are easily obtained:

$$m^2_{\chi^\pm_{1,2}} = \frac{1}{2}(M^2 + |\mu|^2 + X_{12}^2 + X_{21}^2 \mp \Delta), \quad (59)$$

$$\cos 2\theta_{R,L} = \frac{|\mu|^2 - M^2 \pm (X_{12}^2 - X_{21}^2)}{\Delta}, \quad (60)$$

where the quantity Δ is defined by

$$\Delta \equiv [(M^2 - |\mu|^2 - X_{12}^2 + X_{21}^2)^2 + 4(M^2 X_{12}^2 + |\mu|^2 X_{21}^2 + 2M|\mu| X_{12} X_{21} \cos\Phi)]^{1/2}. \quad (61)$$

The four phase angles β_L, β_R, ζ_1, and ζ_2 are given by

$$\tan\beta_L = \frac{-X_{12}|\mu|\sin\Phi}{X_{21}M + X_{12}|\mu|\cos\Phi}, \quad (62)$$

$$\tan\beta_R = \frac{X_{21}|\mu|\sin\Phi}{X_{12}M + X_{21}|\mu|\cos\Phi}, \quad (63)$$

$$\tan\zeta_1 = \frac{X_{12}X_{21}|\mu|\sin\Phi}{X_{12}X_{21}|\mu|\cos\Phi + M(m^2_{\chi_1^\pm} - |\mu|^2)}, \quad (64)$$

$$\tan\zeta_2 = \frac{-(m^2_{\chi_2^\pm} - M^2)|\mu|\sin\Phi}{(m^2_{\chi_2^\pm} - M^2)|\mu|\cos\Phi + X_{12}X_{21}M}. \quad (65)$$

Equations (59)–(65) are a simple extension of the MSSM results obtained in [61]. It is convenient to define the following quantities:

$$C^+_{RL} \equiv -(\cos 2\theta_R + \cos 2\theta_L), \qquad C^-_{RL} \equiv \cos 2\theta_R - \cos 2\theta_L. \quad (66)$$

Then, Eqs. (59) and (60) are equivalent to the following four relations:

$$C^+_{RL}(m^2_{\chi_2^\pm} - m^2_{\chi_1^\pm}) = 2(M^2 - \mu^2), \quad (67)$$

$$C^-_{RL}(m^2_{\chi_2^\pm} - m^2_{\chi_1^\pm}) = 2(X_{12}^2 - X_{21}^2), \quad (68)$$

$$m^2_{\chi_2^\pm} + m^2_{\chi_1^\pm} = M^2 + \mu^2 + X_{12}^2 + X_{21}^2, \quad (69)$$

$$\Delta = m^2_{\chi_2^\pm} - m^2_{\chi_1^\pm}. \quad (70)$$

In the absence of dimension-four hard-SUSY-breaking operators, the tree-level values of the off-diagonal elements of X are given by $X_{12} = \sqrt{2} m_W \sin\beta$ and $X_{21} = \sqrt{2} m_W \cos\beta$. Including corrections due to the hard SUSY-breaking operators, the chargino matrix is modified by small corrections (of one-loop order), which can be treated perturbatively. That is, we write:

$$X_{12} = \sqrt{2} m_W \sin\beta(1 + \delta_{12}), \quad (71)$$

$$X_{21} = \sqrt{2} m_W \cos\beta(1 + \delta_{21}), \quad (72)$$

where δ_{12} and δ_{21} are small, and we work to first order in these small quantities. Our ultimate goal is to express δ_{12} and δ_{21} in terms of the chargino masses $m_{\chi^\pm_{1,2}}$, the ratio of Higgs vacuum expectation values, $\tan\beta$, the mixing angles θ_L and θ_R, and the phase of μ (denoted above by Φ). In principle, these quantities can be determined by precision measurements of the chargino system as described in [63].

We first rewrite Eq. (68) as

$$s_\beta^2 \delta_{12} - c_\beta^2 \delta_{21} = \frac{1}{2} c_{2\beta} + \frac{C^-_{RL}(m^2_{\chi_2^\pm} - m^2_{\chi_1^\pm})}{8 m_W^2}, \quad (73)$$

where $s_\beta \equiv \sin\beta$, $c_\beta \equiv \cos\beta$, etc. We next use Eqs. (67) and (69) to solve for M and $|\mu|$. Inserting these results into Eq. (61) yields a second linear equation for δ_{12} and δ_{21} (after dropping higher-order terms in the δ's) of the following form:

$$g\delta_{21} + h\delta_{12} = 2f^{1/2}(\Delta - f^{1/2}), \quad (74)$$

where

$$f = \left(\frac{1}{2} C^+_{RL}\Delta + 2m_W^2 c_{2\beta}\right)^2 + 4m_W^2(m^2_{\chi_2^\pm} + m^2_{\chi_1^\pm} - 2m_W^2) - 2m_W^2 C^+_{RL}\Delta c_{2\beta} + 4m_W^2 \Gamma s_{2\beta}\cos\Phi, \quad (75)$$

$$g = 2m_W^2 c_\beta^2 \Big[4(m^2_{\chi_2^\pm} + m^2_{\chi_1^\pm}) + 4m_W^2 c_{2\beta} - 16 m_W^2 - C^+_{RL}\Delta + 4\Gamma\tan\beta\cos\Phi - \frac{8m_W^2}{\Gamma}(m^2_{\chi_2^\pm} + m^2_{\chi_1^\pm} - 2m_W^2) s_{2\beta}\cos\Phi \Big], \quad (76)$$

$$h = 2m_W^2 s_\beta^2 \bigg[4(m^2_{\chi_2^\pm} + m^2_{\chi_1^\pm}) - 4m_W^2 c_{2\beta} - 16m_W^2 + C_{RL}^+ \Delta + 4\Gamma \tan\beta \cos\Phi - \frac{8m_W^2}{\Gamma}(m^2_{\chi_2^\pm} + m^2_{\chi_1^\pm} - 2m_W^2) s_{2\beta} \cos\Phi \bigg], \quad (77)$$

with

$$\Gamma \equiv \bigg[(m^2_{\chi_1^\pm} + m^2_{\chi_2^\pm} - 2m_W^2)^2 - \frac{1}{4}(C_{RL}^+ \Delta)^2 \bigg]^{1/2}. \quad (78)$$

Equations (73) and (74) provide two equations for the unknowns δ_{12} and δ_{21}. Solving for δ_{21}, we find

$$\delta_{21} = \frac{2s_\beta^2 f^{1/2}(\Delta - f^{1/2}) - \frac{1}{2}h[c_{2\beta} + \frac{C_{RL}^-(m^2_{\chi_2^\pm} - m^2_{\chi_1^\pm})}{4m_W^2}]}{hc_\beta^2 + gs_\beta^2}. \quad (79)$$

As a check, consider the supersymmetric limit where $X_{12}/X_{21} = \tan\beta$ and $X_{12}X_{21} = m_W^2 s_{2\beta}$. In this limit, a straightforward computation yields $f = \Delta^2$ and $C_{RL}^-(m^2_{\chi_2^\pm} - m^2_{\chi_1^\pm}) = -4m_W^2 c_{2\beta}$. Hence, $\delta_{21} = 0$ in the limit of exact supersymmetry as expected.

We have achieved our goal of expressing δ_{21} in terms of the chargino masses, $\tan\beta$, the mixing angles θ_L and θ_R, and $\Phi = \arg\mu$. In [63], it is shown how to extract the values of the chargino masses and the mixing angles θ_L and θ_R and Φ from precision chargino data at the International Linear Collider (ILC) in a model-independent way, using measurements of the total production cross sections for $e^+e^- \to \tilde{\chi}_i^\pm \tilde{\chi}_j^\mp$ and asymmetries with polarized beams. (A similar proposal for measuring the chargino masses and mixing angles in a CP-conserving scenario was put forward in [64].) If $\tan\beta$ and $\Phi = \arg\mu$ are known independently, then Eq. (79) provides a prediction for δ_{21}. For example, $\tan\beta$ can be determined from precision Higgs measurements (if the heavy Higgs states are observed). An independent determination of Φ is more problematical. Within the context of the MSSM chargino sector, it is shown in [63] that one can also extract values for $\tan\beta$ and Φ from the precision chargino data. But, this determination relies on the standard MSSM chargino mass matrix where $\delta_{12} = \delta_{21} = 0$. This procedure must be generalized if the δ's are nonzero. In principle, it should be possible to solve for all the unknown quantities if the appropriate linear combinations of the phases β_L, β_R, ζ_1, and ζ_2 can be determined experimentally.[9]

Thus, a measurement of the effective chargino mass matrix in this way can signal an effect of SUSY-breaking physics beyond the MSSM. These conclusions depend on the assertion that the tree-level effects of the dimension-four hard-SUSY-breaking operators dominate the more generic loop corrections of pole masses and interactions that cannot be described by terms in a local effective Lagrangian. We discuss the validity of this assumption in the next section.

V. LOCAL VERSUS NON-LOCAL EFFECTS

In this paper, we have analyzed the local effective Lagrangian generated by including a low-scale messenger sector that couples via Yukawa interactions to the Higgs doublets. As a consequence, dimension-four wrong-Higgs gaugino interactions are generated with a strength proportional to the product of messenger-Higgs-Yukawa couplings, $\alpha\beta$. These couplings then enter the chargino mass matrix, thereby perturbing the standard MSSM relations satisfied by chargino mass matrix elements. However, the chargino masses and mixing angles are also modified at one-loop due to momentum-dependent radiative corrections in which the MSSM fields propagate in the loop. Such effects have been thoroughly investigated in various regions of the MSSM parameter space [66–68]. These "nonlocal" effects can compete with the local effects of the hard-SUSY-breaking operators in certain regimes of parameter space, and have not been included in Eq. (79). Here, we shall argue that in our GMSB scenario, it is possible for the local effects to dominate if the messenger-Higgs-Yukawa couplings are larger than the electroweak gauge couplings.

First, consider the one-loop corrections to the off-diagonal elements of the chargino mass matrix arising from squark exchange. Examples of such corrections are depicted in Fig. 2, after the Higgs field acquires a vacuum expectation value. In Sec. III B, we demonstrated that these corrections decouple at large squark masses[10] and are additionally suppressed by a factor of the bottom-quark Yukawa coupling. Indeed, in GMSB scenarios, the squark masses are expected to be rather large, as one generically expects mass relations of the form

$$\frac{m_{\tilde{e}_R^\pm}}{m_{\tilde{q}}} \sim \frac{g'^2}{g_3^2}. \quad (80)$$

Using the current lower bound on the selectron mass of $m_{\tilde{e}_R^\pm} > 73$ GeV [69], it follows that squarks should be quite heavy, $m_{\tilde{q}} > 800$ GeV. As a result, we do not expect the squark-exchange contributions to be significant.

Next, we consider the effects of virtual slepton, chargino, and neutralino exchange at one-loop that can contribute competing nonlocal effects to the wrong-Higgs operators of interest. Here we note that any loop correction

[9] A similar analysis of the neutralino sector can provide important cross checks of the SUSY parameter determination. We will address these points in more detail in a future publication.

[10] As discussed in Sec. III C, in the limit where squarks decouple, the one-loop wrong-Higgs gaugino interaction actually arises from a local dimension-six operator. The corresponding local wrong-Higgs gaugino operators are generated when the neutral Higgs bosons take on their vacuum expectation values.

to chargino/neutralino masses and interactions with charginos/neutralinos or sleptons/leptons propagating on the internal lines will enter with at least two factors of the electroweak couplings g and g', for the chargino/neutralino contributions or the lepton Yukawa couplings for the lepton/slepton contributions. As a result, the only important nonlocal effects are $\sim g'^2$ and $\sim g^2$ competing against effects $\sim \alpha\beta$ from the messenger sector. As long as $\alpha\beta > g^2, g'^2$, the messenger effects will always be parametrically larger than the nonlocal corrections. Thus, with the assumption that $\alpha\beta > g^2, g'^2$, a measurement of a significant deviation of δ_{21} from zero means that the measured deviation is coming from effects beyond the MSSM. The gauge-mediated model with messenger-Higgs-Yukawa couplings provides a plausible scenario in which non-negligible effects in the chargino sector due to the messenger sector are possible.

VI. CONCLUSIONS AND OUTLOOK

In models of low-energy supersymmetry, there is often a hierarchy of scales that governs the structure of the model. At scales above 2.5 TeV, messenger fields can provide an avenue for the communication of the fundamental SUSY-breaking from the hidden sector to the visible sector of the MSSM fields. The scale of the superpartner masses of the MSSM is roughly determined by the scale of low-energy SUSY-breaking, which we take to be $M_{\rm SUSY} \sim \mathcal{O}(1\ \text{TeV})$. Finally, the electroweak symmetry-breaking scale $v \sim 246$ GeV provides the masses for the electroweak gauge bosons and one or more of the Higgs bosons. At each of the two higher scales, one can integrate out the heavy states to obtain an effective low-energy Lagrangian, valid at the electroweak scale. Some of the physics of SUSY-breaking is then encoded in dimension-four hard supersymmetry-breaking operators that appear in the low-energy effective Lagrangian.

In this paper, we have focused on the so-called wrong-Higgs couplings of the MSSM. These are gauge-invariant dimension-four couplings of the Higgs bosons to other standard model and/or MSSM fields that violate supersymmetry. If the low-energy effective Lagrangian describes the two-Higgs-doublet extension of the standard model, then the wrong-Higgs couplings are dimension-four Higgs-fermion Yukawa couplings that violate supersymmetry. These couplings are generated in one-loop corrections to the Yukawa interactions due to the exchange of heavy superpartners in the loops. The effects of the heavy superpartners do not decouple if all supersymmetry mass parameters are simultaneously taken large. The implication of these wrong-Higgs interactions include some $\tan\beta$-enhanced corrections to certain tree-level relations that can be phenomenologically important.

If the low-energy Lagrangian includes the charginos and neutralinos of the MSSM, then the wrong-Higgs couplings are dimension-four gaugino-higgsino-Higgs boson couplings that violate supersymmetry. We have demonstrated that such couplings do *not* arise from one-loop corrections with heavy squarks in the loop. The latter effects decouple as the squark mass is taken heavy, and are derivable from a dimension-six operator with a coefficient that behaves inversely with the square of the heavy squark mass. In models of gauge-mediated supersymmetry-breaking with a low messenger scale, the messenger fields can have direct couplings to the Higgs bosons. Consequently, one must also evaluate one-loop corrections to gaugino-higgsino-Higgs boson couplings with the messenger fields in the loop. Integrating out the messenger fields yields an effective low-energy Lagrangian with wrong-Higgs gaugino interactions. The wrong-Higgs gaugino interactions modify the tree-level chargino and neutralino mass matrix.

In this paper, we have focused on the effect of the wrong-Higgs gaugino operators on the chargino mass matrix. The off-diagonal elements of this mass matrix are modified from their supersymmetric values. For one of the two off-diagonal elements, this deviation is enhanced at large $\tan\beta$, and can range from a few percent to as much as 56% for $\tan\beta = 50$. To detect such a deviation in experimental data, one would need to initiate a program of precision chargino measurements in order to reconstruct the underlying parameters that govern the chargino mass matrix. Such a program would begin at the LHC, but the required precision would most likely require chargino production at the ILC. A strategy for the reconstruction of the chargino mass matrix at the ILC has been given in [61–63], in the case where the wrong-Higgs gaugino couplings are absent. We have derived a relation between observable chargino parameters and the coefficient of the $\tan\beta$-enhanced wrong-Higgs coupling. Whether future ILC chargino data can provide statistically significant evidence for the wrong-Higgs couplings under realistic experimental conditions requires further study.

In this paper, we have focused on the implications of the wrong-Higgs gaugino couplings for the chargino sector. Similar $\tan\beta$-enhanced effects due to wrong-Higgs gaugino couplings also modify the off-diagonal elements of the neutralino mass matrix. The analysis of these effects and their phenomenological implications are somewhat more complicated and will be postponed to a future investigation.

Finally, we note that the existence of the wrong-Higgs gaugino couplings derived in this paper was a consequence of a very specific Higgs-messenger interaction which need not be generic in the class of gauge-mediated supersymmetry-breaking models. It would be interesting to classify extensions of the MSSM that yield similar conclusions. Such an extension would have to possess a field that experiences SUSY-breaking, is charged under the electroweak gauge group, and couples to the Higgs bosons. On the other hand, one can also take a purely phenomenological point of view. Having established that the wrong-

Higgs gaugino couplings do arise in some class of models, one can simply assume their existence and classify all possible phenomenological consequences of such operators for supersymmetric events at future colliders. Ultimately, if experimental evidence for such wrong-Higgs operators can be confirmed, such a result would have a profound impact on the search for the fundamental principles that govern supersymmetry-breaking.

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