Can the Mass of the Lightest Higgs Boson of the Minimal Supersymmetric Model be Larger than m_Z?

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In the minimal supersymmetric model (MSSM), the *tree-level* mass of the lightest Higgs scalar h^0 cannot be larger than the mass of the Z boson. We have computed the one-loop radiative correction to the upper bound on m_{h^0} as a function of the free parameters of the MSSM. We find that the dominant correction to $m_{h^0}-m_Z$ is large and positive and grows like m_t^4, where m_t is the top-quark mass. As a result, the MSSM cannot be ruled out if the CERN e^+e^- collider LEP-200 fails to discover the Higgs boson.

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No convincing experimental evidence to date has been found to contradict the standard model. Nevertheless, there are a number of unsolved theoretical puzzles which suggest that new physics beyond the standard model must exist at an energy scale of 1 TeV or below. Supersymmetry is one of the most promising theoretical ideas that may be able to explain the origin of the scale of electroweak interactions. The minimal supersymmetric extension of the standard model (MSSM) is the most economical among models of this type,[1] and deserves close examination as a candidate for a model of physics beyond the standard model.

The Higgs sector of the MSSM is particularly well constrained[2] and may provide the crucial test which could exclude the simplest model. The MSSM possesses five physical Higgs particles: two CP-even scalars (H^0 and h^0, with $m_{h^0} \leq m_{H^0}$), one CP-odd scalar (A^0), and a charged-Higgs-boson pair ($H^\pm$). The parameters of the Higgs sector are fixed at tree level once the ratio of vacuum expectation values (VEVs), $\tan\beta$, and one physical Higgs-boson mass is specified. Here, $\tan\beta = v_2/v_1$, where v_1 (v_2) is the VEV of the Higgs field that couples to down-type (up-type) quarks and leptons. In addition, the lightest Higgs scalar must satisfy the tree-level relation $m_{h^0} \leq m_Z|\cos 2\beta| \leq m_Z$. Thus, in principle, future experiments running at the CERN e^+e^- collider LEP-200 could either discover the Higgs boson or rule out the MSSM. Whether this is possible to do in practice depends on whether these experiments can rule out a Higgs boson with $m_{h^0} \approx m_Z$.[3]

However, the tree-level bound $m_{h^0} \leq m_Z$ need not be respected when radiative corrections are incorporated. In this paper we ask the following question. What is the upper bound for the mass of the lightest Higgs scalar h^0 including the full one-loop radiative corrections of the MSSM? Although there have been some computations of the Higgs-boson mass shifts due to radiative corrections in the MSSM,[4,5] the complete answer to this question has never been given in the literature.[6] In this Letter, we summarize our calculation which answers this question. A more complete presentation will be provided in a longer version of this paper.[7] Complementary work on these issues has also recently been presented by Ellis, Ridolfi, and Zwirner in Ref. 8 and by Okada, Yamaguchi, and Yanagida in Ref. 9.

Our theoretical approach is as follows. We consider the model in which the tree-level bound $m_{h^0} \leq m_Z$ is saturated. To achieve this, we must take $m_A \geq m_Z$ and $\tan\beta = \pi/2$ (or $\tan\beta = 0$), i.e., $v_1 = 0$ (or $v_2 = 0$), in which case all charge $-\frac{1}{3}$ (or $\frac{2}{3}$) quarks would be massless. Since all fermions excluding the top quark are approximately massless (compared to m_Z), we begin by considering the $v_1 = 0$ model. This model is obtained by setting the soft-supersymmetric-breaking mass parameter which mixes the two Higgs doublets (m_{12} in the notation of Ref. 10) to zero. In this model, the tree-level Higgs-boson mass spectrum consists of $m_{h^0} = m_Z$, $m_{H^0} = m_{A^0} \geq m_Z$, and $m_{H^\pm} = (m_W^2 + m_{A^0}^2)^{1/2}$. The mass degeneracy of H^0 and A^0 holds to all orders in perturbation theory due to an unbroken continuous U(1) global symmetry which is present when the Higgs-boson mixing parameter $m_{12} = 0$.

In computing the corrections to the tree-level value of m_{h^0}, we will derive an expression for $\Delta m_h^2 \equiv m_{h^0}^2 - m_Z^2$. First, we compute the one-loop radiative corrections to the model specified above. This will be denoted by $(\Delta m_{h^0}^2)_{\beta=\pi/2}$, where the subscript emphasizes that we have computed this quantity in the model where $v_1 = 0$ (i.e., $\beta = \pi/2$). Second, we compute the shift in m_{h^0} due to the fact that any realistic model must have two nonvanishing VEVs. We incorporate this correction *at tree level* by employing the exact tree-level mass formula. The final result for the squared mass shift is then

$$\Delta m_h^2 = (\Delta m_h^2)_{\beta=\pi/2} - \tfrac{1}{2}\{[(m_{A^0}^2 - m_Z^2)^2 + 4m_{A^0}^2 m_Z^2 \sin^2 2\beta]^{1/2} - (m_{A^0}^2 - m_Z^2)\}\,. \quad (1)$$

As long as $\tan\beta$ is not close to 1, the correction due to the second term above will be small, and we are consistent in ignoring new one-loop corrections which arise when $\beta \neq \pi/2$.

We now turn to the computation of $(\Delta m_h^2)_{\beta=\pi/2}$. The tree-level potential of the $v_1=0$ model is

$$\mathcal{V}_h = m_0^2\left(\frac{h}{\sqrt{2}}+v_0\right)^2+\frac{1}{8}(g_0^2+g_0'^2)\left(\frac{h}{\sqrt{2}}+v_0\right)^4, \quad (2)$$

where $v \equiv v_2$ and the 0 subscripts indicate the bare parameters. In addition, the Z mass term arises from

$$\mathcal{V}_Z = \tfrac{1}{2} m_{Z0}^2 Z_\mu Z^\mu , \quad (3)$$

where $m_{Z0}^2 = \frac{1}{2}(g_0^2+g_0'^2)v_0^2$. We do not need to renormalize the fields, so bare fields will not be indicated explicitly. Minimizing $\mathcal{V}_h$, it follows that $m_0^2=-m_{Z0}^2/2$. We now introduce the renormalized parameters by shifting the corresponding bare parameters: $m_0^2 \equiv m^2-\delta m^2$, $v_0 = v-\delta v$, etc. Then we find

$$\mathcal{V}_h = (t-\delta t)h+\tfrac{1}{2}(m_h^2-\delta m_h^2)h^2+O(h^3), \quad (4)$$

$$\mathcal{V}_Z = \tfrac{1}{2}(m_Z^2-\delta m_Z^2)Z_\mu Z^\mu , \quad (5)$$

where $m_Z^2=\frac{1}{2}(g^2+g'^2)v^2$ and

$$t = v\sqrt{2}(\tfrac{1}{2}m_Z^2+m^2), \quad (6)$$

$$m_h^2 = \tfrac{3}{2}m_Z^2+m^2 . \quad (7)$$

We then obtain (after eliminating δv by using the tree-level minimum condition)

$$\delta m_h^2 = \delta m_Z^2 + (g/2m_W)\delta t , \quad (8)$$

where we have used $m_W = gv/\sqrt{2}$. The physical masses of h^0 and Z (indicated below with a subscript P) are identified in the usual way as the poles in the corresponding propagators. In our notation, the sum of all one-loop Feynman graphs contributing to the Z-boson and h^0 two-point functions are denoted by $iA_{ZZ}(q^2)g^{\mu\nu}+iB_{ZZ}(q^2)q^\mu q^\nu$ and $-iA_{hh}(q^2)$, respectively, where q is the four-momentum of one of the external legs. The physical masses are then given by

$$m_{ZP}^2 = m_Z^2+\mathrm{Re}\,A_{ZZ}(m_Z^2)-\delta m_Z^2 , \quad (9)$$

$$m_{hP}^2 = m_h^2+\mathrm{Re}\,A_{hh}(m_h^2)-\delta m_h^2 . \quad (10)$$

We now demand that v is the true vacuum expectation value at the one-loop level. This implies that the full tadpole vanishes, so that $t=0$ and $\delta t = A_h(0)$, where $-iA_h(0)$ is the sum of all one-loop Feynman graphs contributing to the h^0 one-point function. We find this choice convenient, since there will be no tadpole contributions to A_{ZZ} and A_{hh}. It follows that $m_h=m_Z$, and therefore

$$(\Delta m_h^2)_{\beta=\pi/2} \equiv m_{hP}^2-m_{ZP}^2 = \mathrm{Re}[A_{hh}(m_Z^2)-A_{ZZ}(m_Z^2)] - \frac{g}{2m_W}A_h(0). \quad (11)$$

This is the basic result of this paper. Although it has been derived using a specific convention for the tadpoles, it is easy to see that the result is independent of this choice. For example, another possible convention would be to simply define $\delta t=0$. (In this case v would not be the true VEV, but this does not matter.) Then one would obtain $(\Delta m_h^2)_{\beta=\pi/2}=\mathrm{Re}[A_{hh}(m_Z^2)-A_{ZZ}(m_Z^2)]$. However, since $\delta t=0$, one would have to include the tadpole contributions to both A_{hh} and A_{ZZ}. It is a simple exercise to check that these additional terms simply reproduce the term $-(g/2m_W)A_h(0)$ in Eq. (11).

Note that each term in Eq. (11) is separately divergent. The divergences will cancel only when one sums over a complete supersymmetric multiplet, since m_{h^0} is calculable in the MSSM whereas in the standard model it is an infinitely renormalized parameter. The largest contribution to the Higgs-boson mass shift comes from the quark and squark loop contributions, so we first focus on this sector of the MSSM. The parameters of the squark sector include common soft-supersymmetry-breaking masses: $M_{\tilde Q}$, $M_{\tilde U}$, and $M_{\tilde D}$, corresponding to $\tilde q_L \equiv (\tilde u_L, \tilde d_L)$, $\tilde u_R$, and $\tilde d_R$, respectively. (Generation labels will be suppressed. For the sleptons, the definitions are similar, except that there is no $\tilde\nu_R$.) In addition, A is the $\tilde q_L$-$\tilde q_R$ mixing parameter. The exact expressions for the quark/squark and lepton/slepton contributions to the h^0 mass shift will be presented in Ref. 7. Here, we simply quote a convenient approximate formula which is valid in the limit where $m_Z < m_t \ll M_{\tilde Q}$. We sum over six flavors of quarks/squarks and leptons/sleptons, and we assume that the common soft-supersymmetry-breaking squark and slepton masses are all equal to $M_{\tilde Q}$. Finally, we neglect $\tilde q_L$-$\tilde q_R$ mixing. The resulting formula is

$$(\Delta m_h^2)_{\beta=\pi/2} = \frac{3g^2 m_Z^4}{16\pi^2 m_W^2}\left\{\ln\left(\frac{M_{\tilde Q}^2}{m_t^2}\right)\left[\frac{2m_t^4-m_t^2m_Z^2}{m_Z^4}+\tfrac{1}{6}(1-\tfrac{8}{3}s_W^2+\tfrac{32}{9}s_W^4)\right] + \ln\left(\frac{M_{\tilde Q}^2}{m_Z^2}\right)\left[\tfrac{1}{3}(1-\tfrac{8}{3}s_W^2+\tfrac{32}{9}s_W^4)+\tfrac{1}{2}(1-\tfrac{4}{3}s_W^2+\tfrac{8}{9}s_W^4)+\tfrac{1}{3}(1-2s_W^2+4s_W^4)\right]+\frac{m_t^2}{3m_Z^2}\right\}, \quad (12)$$

where $s_W \equiv \sin\theta_W$. The most dramatic feature of this result is the m_t^4 growth, which arises due to the top-quark/squark loops.

We find it convenient to present numerical results for the linear mass shift:

$$\Delta m_h \equiv m_{h^0} - m_Z = (\Delta m_h^2 + m_Z^2)^{1/2} - m_Z . \qquad (13)$$

In Fig. 1, we plot the contribution of the quarks and leptons and their supersymmetric scalar partners to Δm_h for $M_{\tilde{Q}} = 1$ TeV, and we confirm that the asymptotic formula given by Eq. (12) (illustrated by the dash-dotted line) is a rather good approximation to the exact result when $M_{\tilde{Q}}$ is large. Clearly, the mass shift can be very significant as m_t becomes large. In fact, for $m_t \gtrsim 175$ GeV, we see that the one-loop correction Δm_h^2 is larger than the tree-level value $m_{h^0}^2 = m_Z^2$. However, to ascertain the region of validity of the perturbation expansion, one must determine the relevant expansion parameter. By examining the largest possible contribution to the two-loop Feynman graphs, one finds that the necessary criterion for the validity of the perturbation expansion is

$$\frac{3g^2 m_t^2}{16\pi^2 m_W^2} \ln\left(\frac{M_{\tilde{Q}}^2}{m_t^2}\right) < 1 , \qquad (14)$$

which translates roughly into $m_t \lesssim 6 m_W$ (for $M_{\tilde{Q}} = 1$ TeV). This condition is satisfied for the range of top quark masses considered here.

It is evident from Eq. (12) that the dependence of Δm_h^2 on $M_{\tilde{Q}}$ is logarithmic. Thus, even if $M_{\tilde{Q}}$ is significantly smaller than 1 TeV, the Higgs-boson mass shift can be appreciable if m_t is sufficiently large. For example, if $M_{\tilde{Q}} = 400$ GeV, an exact numerical computation yields $\Delta m_h = 4$ GeV for $m_t = 100$ GeV and $\Delta m_h = 30$ GeV for $m_t = 200$ GeV.

We now consider briefly the consequences of relaxing two assumptions made above. We could take nondegenerate soft-supersymmetry-breaking masses for the squarks and sleptons. (In fact, it is sufficient to consider the effect on the top-squark sector alone.) If we continue to ignore $\tilde{t}_L$-$\tilde{t}_R$ mixing, the results remain qualitatively unchanged from above. If we include the effects of squark mixing, Δm_h will initially increase. As the mixing becomes very large, Δm_h becomes negative if $M_{\tilde{Q}} \gg m_t$, allowing m_{h^0} to become considerably smaller than m_Z. If we parametrize the mixing by an off-diagonal square mass squared equal to $A m_t$, then for values of $A \lesssim M_{\tilde{Q}}$, the effect of the mixing results in a mild increase of Δm_h. In our numerical analysis, we find that a mixing parameter $A \gtrsim 3.5 M_{\tilde{Q}}$ is required in order to generate substantial negative corrections to Eq. (12). Such large mixing is disfavored in supersymmetric-model building.

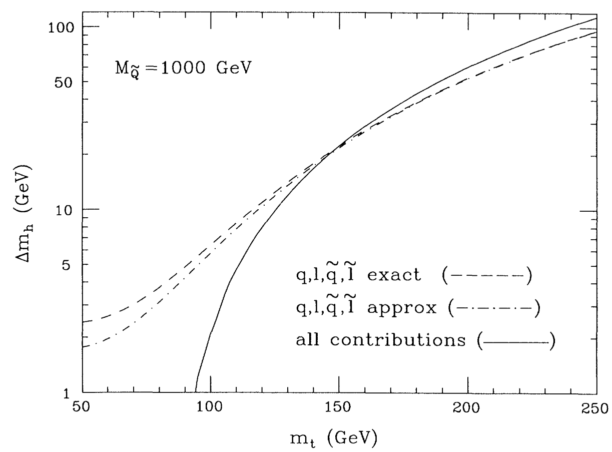

FIG. 1. Higgs mass shift due to one-loop radiative corrections. The dashed line denotes the contribution to Δm_h due to three generations of quarks, leptons, and their supersymmetric scalar partners. The squarks and sleptons are taken to have a common soft-supersymmetry-breaking mass of $M_{\tilde{Q}} = 1$ TeV and $\tilde{t}_L$-$\tilde{t}_R$ mixing is neglected (i.e., $A = 0$). The dash-dotted line is a plot of Eq. (12), and provides a good approximation to the dashed line. The solid line represents a sum of *all* contributions to the exact one-loop calculation of Δm_h for a choice of supersymmetric parameters: $\tan\beta = 20$ and $M_{\tilde{Q}} = A = m_{A^0} = \mu = M = 1$ TeV (where M and μ determine the neutralino/chargino spectrum).

Next, we turn to the contributions to the Higgs-boson mass shift from the gauge and Higgs bosons and their supersymmetric partners (the charginos and neutralinos). The exact expressions will be given in Ref. 7. Here, we provide a formula for the leading logarithmic term of the correction, assuming that $m_Z \ll m_{A^0}, M_{\tilde{\chi}}$. If we assume that all four neutralinos and two charginos are very heavy and the mass splittings among these states are small (i.e., we take $M \sim M' \sim |\mu| \gg m_Z$, where M and M' are gaugino Majorana masses and μ is the supersymmetric Higgs-boson mass parameter), then we obtain

$$(\Delta m_h^2)_{\beta=\pi/2} = -\frac{g^2 m_Z^4}{48\pi^2 m_W^2}\left[(5 - 10c_W^2 + 32c_W^4)\ln\left(\frac{M_{\tilde{\chi}}^2}{m_Z^2}\right) - (1 - 2c_W^2 + 2c_W^4)\ln\left(\frac{m_{A^0}^2}{m_Z^2}\right)\right], \qquad (15)$$

where $c_W \equiv \cos\theta_W$. The contribution of the charginos and neutralinos to the mass shift is negative, while the contribution of the other Higgs bosons (H^0, A^0, and $H^\pm$) is positive but somewhat smaller in magnitude. Moreover, by varying the supersymmetric parameters over all possible values (keeping supersymmetric masses less than 1 TeV), we find the contribution to Δm_h^2 is quite stable and lies between 0 and $-(5\text{ GeV})^2$. This agrees qualitatively with the results obtained previously in Ref. 4. Comparing Eqs. (12) and (15), it is clear that the contributions of the top quark and top squark will dominate the one-loop correction to Δm_h^2.

Finally, we can combine all our results. In addition, we can account for a value of $\beta \neq \pi/2$ by using Eq. (1). We have computed Δm_h as a function of m_t, with all contributions to the exact one-loop calculation included. In Fig. 1, the solid line is a plot of Δm_h for the following choice of supersymmetric parameters: $\tan\beta = 20$, $M_{\tilde{Q}} = A = m_{A^0} = \mu = M = 1$ TeV (where A parametrizes $\tilde{t}_L$-$\tilde{t}_R$ mixing), and $M' \simeq M/2$. We believe that this result provides a realistic indication of the true upper bound for the mass of h^0 in the MSSM. Note that the effect of including the gauge and Higgs bosons (and their supersymmetric partners) and the effect of a finite $\tan\beta$ is to reduce the Higgs-boson mass shift slightly. This can be seen in Fig. 1 for the smaller values of m_t shown (where the relative effect is the greatest). For larger values of m_t, the solid line in Fig. 1 lies slightly above the dashed line because we have included $\tilde{t}_L$-$\tilde{t}_R$ mixing by taking $A = M_{\tilde{Q}}$.

Although the calculations described in this Letter were performed specifically in the MSSM, the results are more general. The tree-level bound $m_{h^0} \leq m_Z$ holds in a supersymmetric model which contains an arbitrary (even) number of Higgs doublets[11] (but no other types of Higgs multiplets are allowed). The calculation of the contributions from the top-quark and top-squark loops presented above is valid in this class of nonminimal supersymmetric models. Moreover, these contributions will dominate for the same reasons as discussed above.

In conclusion, until the top quark is discovered and its mass determined, the upper bound of the mass of the lightest Higgs boson of the MSSM remains in doubt. For example, if $m_t \gtrsim 145$ GeV, then the upper bound on m_{h^0} is above 110 GeV. If this upper bound were saturated, then h^0 would not be kinematically accessible to the CERN e^+e^- collider LEP-200. As a result, the MSSM could not be ruled out if the CERN e^+e^- collider LEP-200 fails to discover the Higgs boson.

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