

The Two-Higgs Doublet Model in Theory and Practice

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 24 October 2012
 LBNL Higgs Jamboree

HIGGS BOSON H



The **HIGGS BOSON** is the theoretical particle of the Higgs mechanism, which physicists believe will reveal how all matter in the universe gets its mass. Many scientists hope that the Large Hadron Collider in Geneva, Switzerland will detect the elusive Higgs Boson when it begins colliding particles at 99.99% the speed of light.

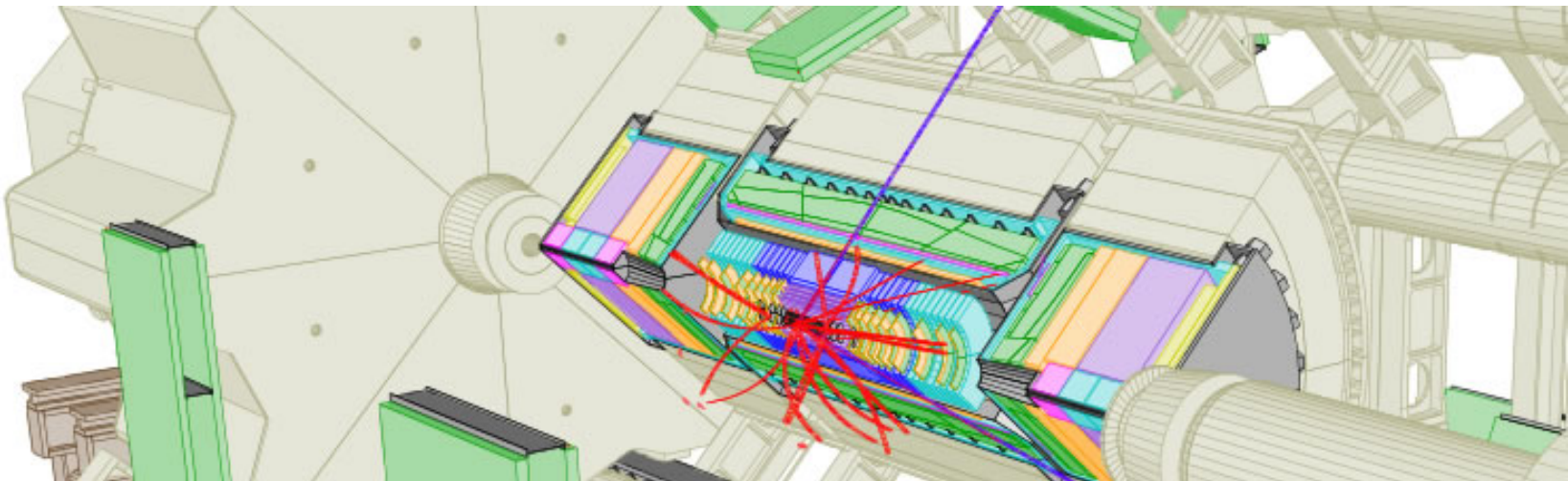
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A brief bibliography

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Outline

- Expanding the Higgs sector beyond the Standard Model
- The Two-Higgs Doublet Model (2HDM)
- The Higgs-fermion Yukawa coupling
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- Dreams of large deviations from SM Higgs behavior
- The Higgs sector of the MSSM
- How radiative corrections saved the MSSM Higgs sector
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Constraints on the non-minimal Higgs sector

Three generations of fermions appear in nature, with each generation possessing the same quantum numbers under the $SU(3) \times SU(2) \times U(1)_Y$ gauge group. So, why should the scalar sector be of minimal form?

For an arbitrary Higgs sector, the tree-level ρ -parameter is given by

$$\rho_0 \equiv \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = \frac{\sum_{T,Y} [4T(T+1) - Y^2] |V_{T,Y}|^2 c_{T,Y}}{\sum_{T,Y} 2Y^2 |V_{T,Y}|^2},$$

where $V_{T,Y} \equiv \langle \phi(T, Y) \rangle$ defines the vacuum expectation values (vevs) of each neutral Higgs field, and T and Y specify the total $SU(2)$ isospin and the hypercharge of the Higgs representation to which it belongs. Y is normalized such that the electric charge of the scalar field is $Q = T_3 + Y/2$, and

$$c_{T,Y} = \begin{cases} 1, & (T, Y) \in \text{complex representation,} \\ \frac{1}{2}, & (T, Y = 0) \in \text{real representation.} \end{cases}$$

For the complex ($c = 1$) Higgs doublet of the Standard Model with $T = 1/2$ and $Y = 1$, it follows that $\rho_0 = 1$ as strongly suggested by the electroweak data. The same result follows from a Higgs sector consisting of multiple complex Higgs doublets (independent of the neutral Higgs vevs). One can also add Higgs singlets ($T = Y = 0$) without changing the value of ρ_0 .

But, one cannot add arbitrary Higgs multiplets in general* unless their corresponding vevs are very small (typically $|V_{T,Y}| \lesssim 0.05v \sim 10 \text{ GeV}$).

Thus, we shall consider non-minimal Higgs sectors consisting of multiple Higgs doublets (and perhaps Higgs singlets), but no higher Higgs representations, in order to avoid the fine-tuning of Higgs vevs.

*To automatically have $\rho_0 = 1$ independently of the Higgs vevs, one must satisfy

$$(2T + 1)^2 - 3Y^2 = 1$$

for integer values of $(2T, Y)$. The smallest nontrivial solution beyond the complex $Y = 1$ Higgs doublet is a Higgs multiplet with $T = 3$ and $Y = 4$.

The Two-Higgs doublet model (2HDM)

The 2HDM, consists of two-complex hypercharge-one scalar doublets Φ_1 and Φ_2 . Of the eight initial degrees of freedom, three are eaten and provide masses for the W^\pm and Z , and the remaining five correspond to physical scalars: a charged Higgs pair, H^\pm , and three neutral scalars h_1 , h_2 and h_3 . In contrast to the SM, where the Higgs-sector is CP-conserving, the 2HDM allows for Higgs-mediated CP-violation.

If CP is conserved, the three scalars can be classified as two CP-even scalars, h and H (where $m_h < m_H$) and a CP-odd scalar A .

Thus, new features of the 2HDM include:

- Charged Higgs bosons
- A CP-odd Higgs boson (if CP is conserved in the Higgs sector)
- Higgs-mediated CP-violation (and neutral Higgs states of indefinite CP)

Start with the most general renormalizable scalar Higgs potential,

$$\begin{aligned} \mathcal{V} = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 \\ & + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\ & + \left\{ \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + [\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\} , \end{aligned}$$

where m_{12}^2 , λ_5 , λ_6 and λ_7 are potentially complex parameters. There is a significant region of the 2HDM parameter space in which the complex vacuum expectation values (vevs) of the two Higgs fields are:

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix} , \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix} ,$$

where $v^2 \equiv |v_1|^2 + |v_2|^2 = (246 \text{ GeV})^2$. The vevs are aligned along the neutral direction, in which case the $SU(2) \times U(1)$ electroweak symmetry is spontaneously broken to $U(1)_{\text{EM}}$ as it is in the Standard Model.

It is convenient to define new Higgs doublet fields:

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} \equiv \frac{v_1^* \Phi_1 + v_2^* \Phi_2}{v}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \equiv \frac{-v_2 \Phi_1 + v_1 \Phi_2}{v}.$$

It follows that $\langle H_1^0 \rangle = v/\sqrt{2}$ and $\langle H_2^0 \rangle = 0$. This is the *Higgs basis*, which is uniquely defined up to an overall rephasing, $H_2 \rightarrow e^{i\chi} H_2$. In the Higgs basis, the scalar potential is given by:

$$\begin{aligned} \mathcal{V} = & Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + [Y_3 H_1^\dagger H_2 + \text{h.c.}] + \frac{1}{2} Z_1 (H_1^\dagger H_1)^2 \\ & + \frac{1}{2} Z_2 (H_2^\dagger H_2)^2 + Z_3 (H_1^\dagger H_1) (H_2^\dagger H_2) + Z_4 (H_1^\dagger H_2) (H_2^\dagger H_1) \\ & + \left\{ \frac{1}{2} Z_5 (H_1^\dagger H_2)^2 + [Z_6 (H_1^\dagger H_1) + Z_7 (H_2^\dagger H_2)] H_1^\dagger H_2 + \text{h.c.} \right\}, \end{aligned}$$

where Y_1, Y_2 and Z_1, \dots, Z_4 are real and uniquely defined, whereas Y_3, Z_5, Z_6 and Z_7 are complex and transform under the rephasing of H_2 ,

$$[Y_3, Z_6, Z_7] \rightarrow e^{-i\chi} [Y_3, Z_6, Z_7] \quad \text{and} \quad Z_5 \rightarrow e^{-2i\chi} Z_5.$$

Counting the number of parameter degrees of freedom

The scalar potential in the Higgs basis depends on six real parameters, Y_1 , Y_2 , and $Z_{1,2,3,4}$, and four complex parameters and Y_3 and $Z_{5,6,7}$, for a total of 14 parameter degrees of freedom. After imposing the scalar potential minimum conditions:

$$Y_1 = -\frac{1}{2}Z_1v^2 \quad \text{and} \quad Y_3 = -\frac{1}{2}Z_6v^2,$$

removes three degrees of freedom (not counting $v = 246$ GeV whose value is fixed). Finally, using the rephasing degree of freedom $H_2 \rightarrow e^{i\chi}H_2$ removes one more degree of freedom since

$$[Y_3, Z_6, Z_7] \rightarrow e^{-i\chi}[Y_3, Z_6, Z_7] \quad \text{and} \quad Z_5 \rightarrow e^{-2i\chi}Z_5.$$

can be used to establish a convention where, e.g. Z_5 is real.

Final verdict: the general 2HDM scalar potential is governed by 10 degrees of freedom (and the physical (real) vacuum expectation value v).

The Higgs mass-eigenstate basis

The physical charged Higgs boson is the charged component of the Higgs-basis doublet H_2 , and its mass is given by $m_{H^\pm}^2 = Y_2 + \frac{1}{2}Z_3v^2$.

The three physical neutral Higgs boson mass-eigenstates are determined by diagonalizing a 3×3 real symmetric squared-mass matrix that is defined in the Higgs basis.[†]

$$\mathcal{M}^2 = v^2 \begin{pmatrix} Z_1 & \text{Re}(Z_6) & -\text{Im}(Z_6) \\ \text{Re}(Z_6) & \frac{1}{2}Z_{345} + Y_2/v^2 & -\frac{1}{2}\text{Im}(Z_5) \\ -\text{Im}(Z_6) & -\frac{1}{2}\text{Im}(Z_5) & \frac{1}{2}Z_{345} - \text{Re}(Z_5) + Y_2/v^2 \end{pmatrix},$$

where $Z_{345} \equiv Z_3 + Z_4 + \text{Re}(Z_5)$. The diagonalizing matrix is a 3×3 real orthogonal matrix that depends on three angles: θ_{12} , θ_{13} and θ_{23} . Under the rephasing $H_2 \rightarrow e^{i\chi}H_2$,

$$\theta_{12}, \theta_{13} \text{ are invariant, and } \theta_{23} \rightarrow \theta_{23} - \chi.$$

[†]For details, see H.E. Haber and D. O'Neil, "Basis-independent methods for the two-Higgs-doublet model. II: The significance of $\tan \beta$," *Phys. Rev.* **D74**, 015018 (2006) [hep-ph/0602242].

It is convenient to define the $q_{k\ell}$ which are defined in terms of the invariant angles θ_{12} and θ_{13} , where $c_{ij} \equiv \cos \theta_{ij}$ and $s_{ij} \equiv \sin \theta_{ij}$.

k	q_{k1}	q_{k2}
0	i	0
1	$c_{12}c_{13}$	$-s_{12} - ic_{12}s_{13}$
2	$s_{12}c_{13}$	$c_{12} - is_{12}s_{13}$
3	s_{13}	ic_{13}

The neutral Goldstone boson (h_0) and the physical neutral Higgs states ($h_{1,2,3}$) are given by:

$$h_k = \frac{1}{\sqrt{2}} \left\{ q_{k1}^* \left(H_1^0 - \frac{v}{\sqrt{2}} \right) + q_{k2}^* H_2^0 e^{i\theta_{23}} + \text{h.c.} \right\} .$$

If we also define the physical charged Higgs state by $H^\pm = e^{\pm i\theta_{23}} H_2^\pm$, then all the mass eigenstate fields are invariant under the rephasing $H_2 \rightarrow e^{i\chi} H_2$.

The gauge boson–Higgs boson interactions

$$\mathcal{L}_{VVH} = \left(gm_W W_\mu^+ W^{\mu -} + \frac{g}{2c_W} m_Z Z_\mu Z^\mu \right) \text{Re}(q_{k1}) h_k + em_W A^\mu (W_\mu^+ G^- + W_\mu^- G^+) - gm_Z s_W^2 Z^\mu (W_\mu^+ G^- + W_\mu^- G^+),$$

$$\begin{aligned} \mathcal{L}_{VVHH} = & \left[\frac{1}{4} g^2 W_\mu^+ W^{\mu -} + \frac{g^2}{8c_W^2} Z_\mu Z^\mu \right] \text{Re}(q_{j1}^* q_{k1} + q_{j2}^* q_{k2}) h_j h_k \\ & + \left[\frac{1}{2} g^2 W_\mu^+ W^{\mu -} + e^2 A_\mu A^\mu + \frac{g^2}{c_W^2} \left(\frac{1}{2} - s_W^2 \right)^2 Z_\mu Z^\mu + \frac{2ge}{c_W} \left(\frac{1}{2} - s_W^2 \right) A_\mu Z^\mu \right] (G^+ G^- + H^+ H^-) \\ & + \left\{ \left(\frac{1}{2} eg A^\mu W_\mu^+ - \frac{g^2 s_W^2}{2c_W} Z^\mu W_\mu^+ \right) (q_{k1} G^- + q_{k2} H^-) h_k + \text{h.c.} \right\}, \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{VHH} = & \frac{g}{4c_W} \text{Im}(q_{j1} q_{k1}^* + q_{j2} q_{k2}^*) Z^\mu h_j \overleftrightarrow{\partial}_\mu h_k - \frac{1}{2} g \left\{ iW_\mu^+ \left[q_{k1} G^- \overleftrightarrow{\partial}^\mu h_k + q_{k2} H^- \overleftrightarrow{\partial}^\mu h_k \right] + \text{h.c.} \right\} \\ & + \left[ieA^\mu + \frac{ig}{c_W} \left(\frac{1}{2} - s_W^2 \right) Z^\mu \right] (G^+ \overleftrightarrow{\partial}_\mu G^- + H^+ \overleftrightarrow{\partial}_\mu H^-), \end{aligned}$$

where $s_W \equiv \sin \theta_W$ and $c_W \equiv \cos \theta_W$.

The cubic and quartic Higgs couplings

$$\begin{aligned}
\mathcal{L}_{3h} = & -\frac{1}{2}v h_j h_k h_\ell \left[q_{j1} q_{k1}^* \text{Re}(q_{\ell 1}) Z_1 + q_{j2} q_{k2}^* \text{Re}(q_{\ell 1}) (Z_3 + Z_4) + \text{Re}(q_{j1}^* q_{k2} q_{\ell 2}) Z_5 e^{-2i\theta_{23}} \right. \\
& \left. + \text{Re}([2q_{j1} + q_{j1}^*] q_{k1}^* q_{\ell 2}) Z_6 e^{-i\theta_{23}} + \text{Re}(q_{j2}^* q_{k2} q_{\ell 2}) Z_7 e^{-i\theta_{23}} \right] \\
& -v h_k G^+ G^- \left[\text{Re}(q_{k1}) Z_1 + \text{Re}(q_{k2} e^{-i\theta_{23}} Z_6) \right] + v h_k H^+ H^- \left[\text{Re}(q_{k1}) Z_3 + \text{Re}(q_{k2} e^{-i\theta_{23}} Z_7) \right] \\
& -\frac{1}{2}v h_k \left\{ G^- H^+ \left[q_{k2}^* Z_4 + q_{k2} e^{-2i\theta_{23}} Z_5 + 2\text{Re}(q_{k1}) Z_6 e^{-i\theta_{23}} \right] + \text{h.c.} \right\}, \\
\mathcal{L}_{4h} = & -\frac{1}{8}h_j h_k h_l h_m \left[q_{j1} q_{k1} q_{\ell 1}^* q_{m1}^* Z_1 + q_{j2} q_{k2} q_{\ell 2}^* q_{m2}^* Z_2 + 2q_{j1} q_{k1}^* q_{\ell 2} q_{m2}^* (Z_3 + Z_4) \right. \\
& \left. + 2\text{Re}(q_{j1}^* q_{k1} q_{\ell 2} q_{m2}) Z_5 e^{-2i\theta_{23}} + 4\text{Re}(q_{j1} q_{k1}^* q_{\ell 1}^* q_{m2}) Z_6 e^{-i\theta_{23}} + 4\text{Re}(q_{j1}^* q_{k2} q_{\ell 2} q_{m2}^*) Z_7 e^{-i\theta_{23}} \right] \\
& -\frac{1}{2}h_j h_k G^+ G^- \left[q_{j1} q_{k1}^* Z_1 + q_{j2} q_{k2}^* Z_3 + 2\text{Re}(q_{j1} q_{k2}) Z_6 e^{-i\theta_{23}} \right] \\
& -\frac{1}{2}h_j h_k H^+ H^- \left[q_{j2} q_{k2}^* Z_2 + q_{j1} q_{k1}^* Z_3 + 2\text{Re}(q_{j1} q_{k2}) Z_7 e^{-i\theta_{23}} \right] \\
& -\frac{1}{2}h_j h_k \left\{ G^- H^+ \left[q_{j1} q_{k2}^* Z_4 + q_{j1}^* q_{k2} Z_5 e^{-2i\theta_{23}} + q_{j1} q_{k1}^* Z_6 e^{-i\theta_{23}} + q_{j2} q_{k2}^* Z_7 e^{-i\theta_{23}} \right] + \text{h.c.} \right\} \\
& -\frac{1}{2}Z_1 G^+ G^- G^+ G^- - \frac{1}{2}Z_2 H^+ H^- H^+ H^- - (Z_3 + Z_4) G^+ G^- H^+ H^- \\
& -\frac{1}{2}(Z_5 e^{-2i\theta_{23}} H^+ H^+ G^- G^- + \text{h.c.}) - G^+ G^- (Z_6 e^{-i\theta_{23}} H^+ G^- + \text{h.c.}) - H^+ H^- (Z_7 e^{-i\theta_{23}} H^+ G^- + \text{h.c.}).
\end{aligned}$$

Higgs-fermion Yukawa couplings in the 2HDM

In the Higgs basis, $\kappa^{U,D}$ and $\rho^{U,D}$, are the 3×3 Yukawa coupling matrices,

$$-\mathcal{L}_Y = \bar{U}_L(\kappa^U H_1^{0\dagger} + \rho^U H_2^{0\dagger})U_R - \bar{D}_L K^\dagger(\kappa^U H_1^- + \rho^U H_2^-)U_R \\ + \bar{U}_L K(\kappa^D H_1^+ + \rho^D H_2^+)D_R + \bar{D}_L(\kappa^D H_1^0 + \rho^D H_2^0)D_R + \text{h.c.},$$

where $U = (u, c, t)$ and $D = (d, s, b)$ are the physical quark fields and K is the CKM mixing matrix. (Repeat for the leptons.)

By setting $H_1^0 = v/\sqrt{2}$ and $H_2^0 = 0$, one obtains the quark mass terms. Hence, κ^U and κ^D are proportional to the diagonal quark mass matrices M_U and M_D , respectively,

$$M_U = \frac{v}{\sqrt{2}}\kappa^U = \text{diag}(m_u, m_c, m_t), \quad M_D = \frac{v}{\sqrt{2}}\kappa^{D\dagger} = \text{diag}(m_d, m_s, m_b).$$

Note that $\rho^Q \rightarrow e^{-i\chi}\rho^Q$ under the rephasing $H_2 \rightarrow e^{i\chi}H_2$, (for $Q = U, D$).

It follows that the Yukawa couplings of the mass-eigenstate Higgs bosons and the Goldstone bosons to the quarks are:

$$\begin{aligned}
-\mathcal{L}_Y = & \frac{1}{v} \overline{D} \sum_k \left\{ M_D (q_{k1} P_R + q_{k1}^* P_L) + \frac{v}{\sqrt{2}} \left[q_{k2} [e^{i\theta_{23}} \rho^D]^\dagger P_R + q_{k2}^* e^{i\theta_{23}} \rho^D P_L \right] \right\} D h_k \\
& + \frac{1}{v} \overline{U} \sum_k \left\{ M_U (q_{k1} P_L + q_{k1}^* P_R) + \frac{v}{\sqrt{2}} \left[q_{k2}^* e^{i\theta_{23}} \rho^U P_R + q_{k2} [e^{i\theta_{23}} \rho^U]^\dagger P_L \right] \right\} U h_k \\
& + \left\{ \overline{U} \left[K [e^{i\theta_{23}} \rho^D]^\dagger P_R - [e^{i\theta_{23}} \rho^U]^\dagger K P_L \right] D H^+ + \frac{\sqrt{2}}{v} \overline{U} [K M_D P_R - M_U K P_L] D G^+ + \text{h.c.} \right\},
\end{aligned}$$

where $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ are left and right-handed projection operators.

- The combinations $e^{i\theta_{23}} \rho^U$ and $e^{i\theta_{23}} \rho^D$ that appear in the interactions above are invariant under the rephasing of H_2 .
- Note that no $\tan \beta$ parameter appears above! This is because $\tan \beta$ is an unphysical parameter in the general 2HDM.
- If ρ^U and ρ^D are complex non-diagonal 3×3 matrices, then the 2HDM exhibits (tree-level) flavor changing neutral currents (FCNCs) mediated by neutral Higgs exchange and new sources of CP-violation.

How to avoid tree-level Higgs-mediated FCNCs

- Arbitrarily declare ρ^U and ρ^D to be diagonal matrices. This is an unnaturally fine-tuned solution.
- Impose a discrete symmetry or supersymmetry (e.g. “Type-I” or “Type-II” Higgs-fermion interactions), which selects out a special basis of the 2HDM scalar fields. In this case, ρ^Q is automatically proportional to M_Q (for $Q = U, D, L$), and is hence diagonal.
- Impose alignment without a symmetry: $\rho^Q = \alpha^Q \kappa^Q$, ($Q = U, D, L$), where the α^Q are complex scalar parameters [e.g. see Pich and Tuzon (2009)].
- Impose the decoupling limit. Tree-level Higgs-mediated FCNCs will be suppressed by factors of squared-masses of heavy Higgs states. (How heavy is sufficient?)

The CP-conserving limit

In the generic 2HDM, new sources of CP-violation arise due to the fact that

- $Z_{5,6,7}$ are complex, and cannot be made real by rephasing $H_2 \rightarrow e^{i\chi}H_2$.
- CP-violating neutral Higgs–fermion couplings due to complex ρ^U and ρ^D .

Imposing CP-violation in the neutral Higgs sector, the q_{ki} are given by:

k	q_{k1}	q_{k2}
1	$\sin(\beta - \alpha)$	$\cos(\beta - \alpha)$
2	$-\cos(\beta - \alpha)$	$\sin(\beta - \alpha)$
3	0	i

where $\tan \beta \equiv \langle \Phi_2^0 \rangle / \langle \Phi_1^0 \rangle$ and α is the CP-even Higgs mixing angle in the generic basis. Note that the quantity $\theta_{12} \equiv \beta - \alpha$ does not depend on the choice of basis. The other angles are $\theta_{13} = \theta_{23} = 0$; the latter fixes the phase of H_2 .

We shall also take ρ^U and ρ^D to be real 3×3 matrices.

The resulting Higgs-fermion Yukawa couplings are:

$$\begin{aligned}
-\mathcal{L}_Y = & \frac{i}{v} \bar{D} M_D \gamma_5 D G^0 + \bar{D} \left[\frac{M_D}{v} \sin(\beta - \alpha) + \frac{\rho^D}{\sqrt{2}} \cos(\beta - \alpha) \right] D h^0 \\
& + \bar{D} \left[\frac{M_D}{v} \cos(\beta - \alpha) - \frac{\rho^D}{\sqrt{2}} \sin(\beta - \alpha) \right] D H^0 + \frac{i}{\sqrt{2}} \rho^D \bar{D} \gamma_5 D A^0 \\
& - \frac{i}{v} \bar{U} M_U \gamma_5 U G^0 + \bar{U} \left[\frac{M_U}{v} \sin(\beta - \alpha) + \frac{\rho^U}{\sqrt{2}} \cos(\beta - \alpha) \right] U h^0 \\
& + \bar{U} \left[\frac{M_U}{v} \cos(\beta - \alpha) - \frac{\rho^U}{\sqrt{2}} \sin(\beta - \alpha) \right] U H^0 - \frac{i}{\sqrt{2}} \rho^U \bar{U} \gamma_5 U A^0 \\
& + \left\{ \bar{U} \left[K \rho^D P_R - \rho^U K P_L \right] D H^+ + \frac{\sqrt{2}}{v} \bar{U} \left[K M_D P_R - M_U K P_L \right] D G^+ + \text{h.c.} \right\} ,
\end{aligned}$$

where h^0 , H^0 are the CP-even neutral Higgs bosons and A^0 is the CP-odd neutral Higgs boson. Of course, tree-level Higgs-mediated FCNCs still remain.

Type I and II Higgs-fermion Yukawa couplings in the 2HDM

Glashow and Weinberg showed that a sufficient condition for the absence of tree-level Higgs-mediated FCNCs is to require that at most one neutral Higgs field couple to fermions of a given electric charge. To avoid FCNCs in the 2HDM, one can impose a discrete symmetry to appropriately restrict the structure of the Higgs-fermion interactions in a specific basis for the Higgs fields. The corresponding 2HDM Yukawa Lagrangian is:

$$-\mathcal{L}_Y = \bar{U}_L \Phi_a^0 h_a^U U_R - \bar{D}_L K^\dagger \Phi_a^- h_a^U U_R + \bar{U}_L K \Phi_a^+ h_a^D \dagger D_R + \bar{D}_L \Phi_a^0 h_a^D \dagger D_R + \text{h.c.},$$

where $h^{U,D}$ are 3×3 Yukawa coupling matrices, and there is an implicit sum over $a = 1, 2$.

Different choices for the discrete symmetry yield:

- Type-I Yukawa couplings: $h_1^U = h_1^D = 0$,
- Type-II Yukawa couplings: $h_1^U = h_2^D = 0$.

The parameter $\tan\beta = \langle\Phi_2^0\rangle/\langle\Phi_1^0\rangle$ is now meaningful since it refers to vacuum expectation values with respect to the basis of scalar fields where the discrete symmetry has been imposed. (Typical discrete symmetries involve requiring invariance of the Lagrangian under a sign change of one of the scalar fields and some of the fermion fields.)

Notes:

- By imposing the discrete symmetry (or supersymmetry), one finds that the neutral Higgs boson sector is CP-conserving.
- Type-II Yukawa couplings arise in the MSSM due to supersymmetry.
- The matrix ρ^Q is constrained to be diagonal and proportional to M_Q . The proportionality constant depends on $\tan\beta$.

The Higgs-fermion couplings in in Type-I and Type-II models

- Type-I Yukawa couplings: $h_1^U = h_1^D = 0$,

$$\rho^D = \frac{\sqrt{2}M_d \cot \beta}{v}, \quad \rho^U = \frac{\sqrt{2}M_u \cot \beta}{v}.$$

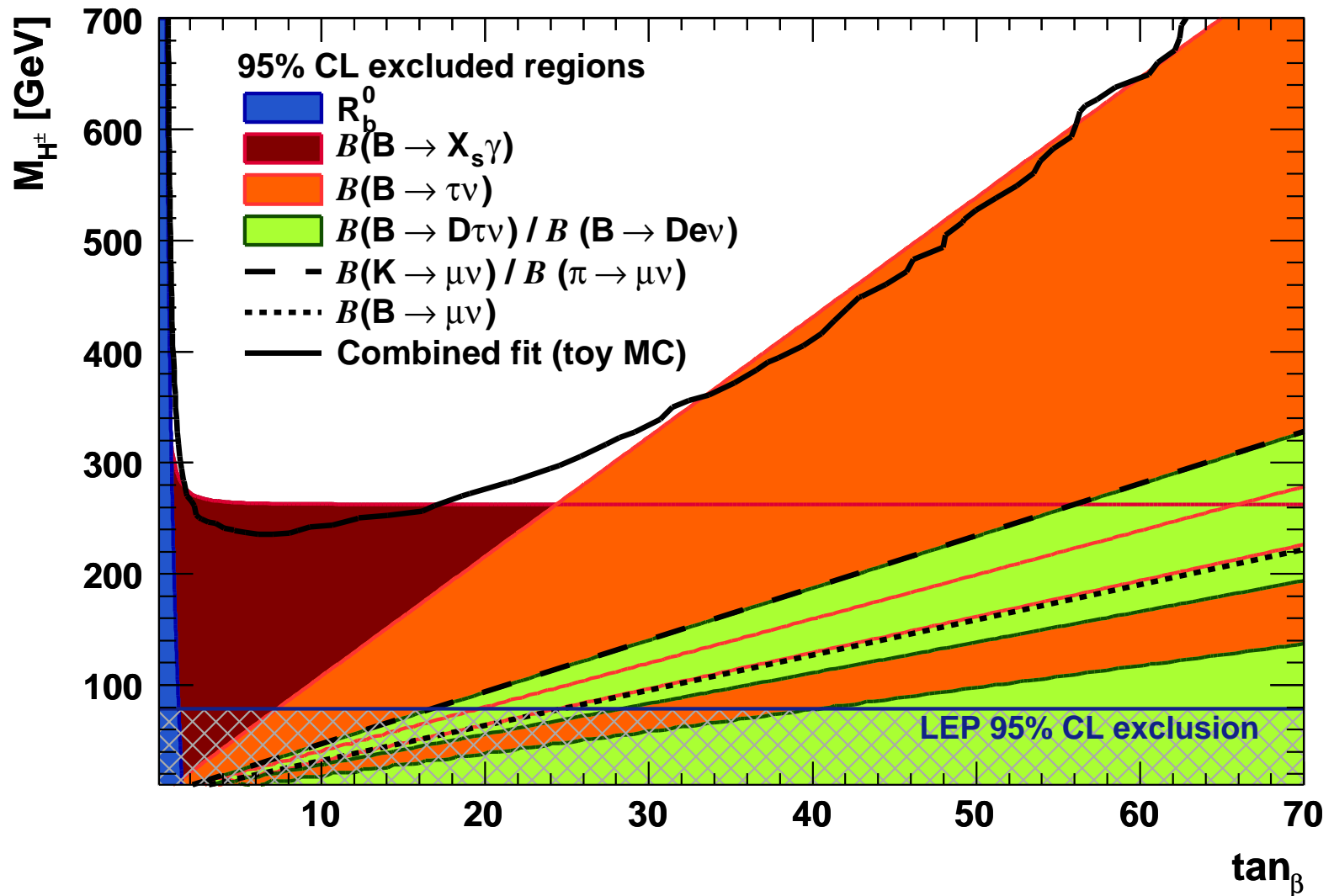
Type-I couplings	h^0	A^0	H^0
Up-type quarks	$\cos \alpha / \sin \beta$	$\cot \beta$	$\sin \alpha / \sin \beta$
Down-type quarks and charged leptons	$\cos \alpha / \sin \beta$	$-\cot \beta$	$\sin \alpha / \sin \beta$

- Type-II Yukawa couplings: $h_1^U = h_2^D = 0$,

$$\rho^D = -\frac{\sqrt{2}M_d \tan \beta}{v}, \quad \rho^U = \frac{\sqrt{2}M_u \cot \beta}{v}.$$

Type-II couplings	h^0	A^0	H^0
Up-type quarks	$\cos \alpha / \sin \beta$	$\cot \beta$	$\sin \alpha / \sin \beta$
Down-type quarks and charged leptons	$-\sin \alpha / \cos \beta$	$\tan \beta$	$\cos \alpha / \cos \beta$

There are interesting experimental constraints on the Type-II 2HDM. The GFITTER result below was obtained in 2009, so it should be updated.



2HDM constraints in the m_{H^+} vs. $\tan \beta$ plane, from F. Mahmoudi and O. Stal, *Flavor constraints on the two-Higgs-doublet model with general Yukawa couplings*, Phys. Rev. **D81**, 035016 (2010).

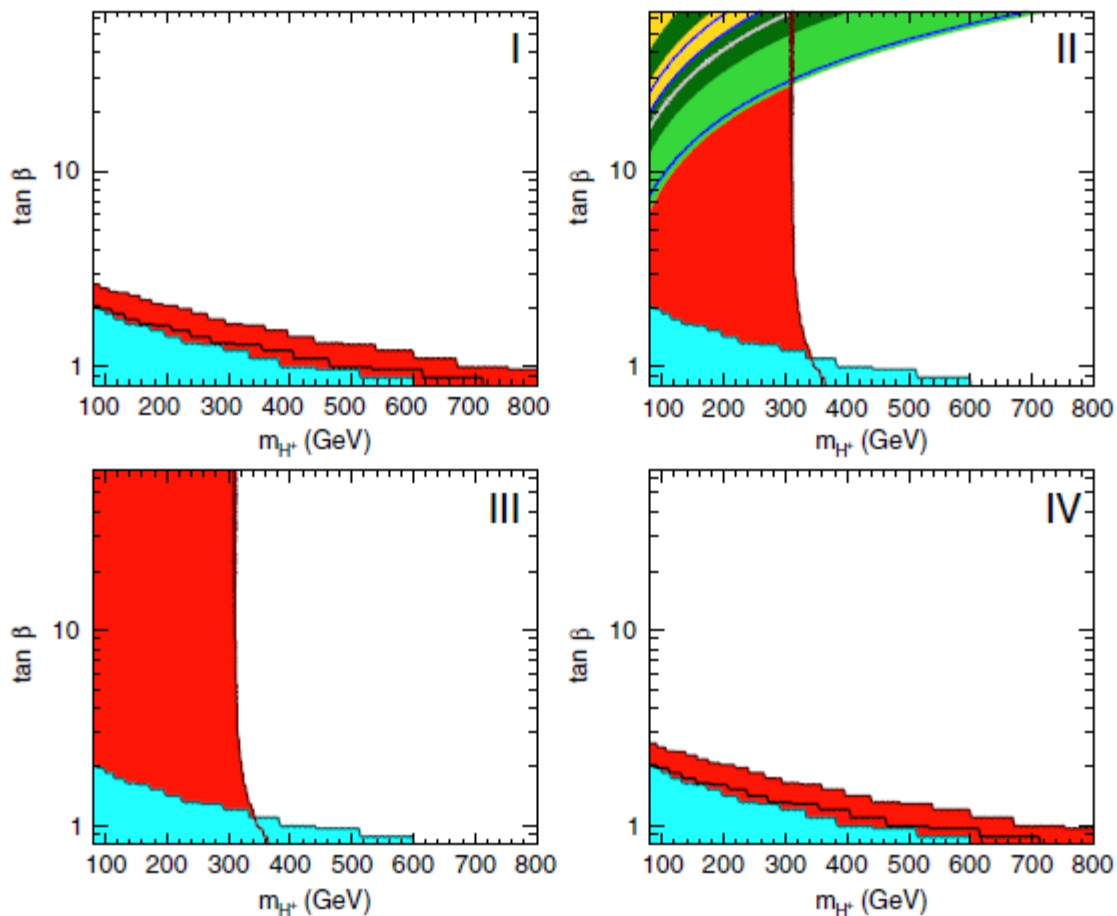


FIG. 10 (color online). Excluded regions of the $(m_{H^+}, \tan\beta)$ parameter space for Z_2 -symmetric 2HDM types. The color coding is as follows: $\text{BR}(B \rightarrow X_s \gamma)$ (red), Δ_{0-} (black contour), ΔM_{B_d} (cyan), $B_u \rightarrow \tau \nu_\tau$ (blue), $B \rightarrow D \tau \nu_\tau$ (yellow), $K \rightarrow \mu \nu_\mu$ (gray contour), $D_s \rightarrow \tau \nu_\tau$ (light green), and $D_s \rightarrow \mu \nu_\mu$ (dark green). The white region is not excluded by any of these constraints.

Is $\tan \beta$ a physical observable?

In a generic 2HDM, $\tan \beta$ is meaningless, unless there is some additional symmetry, which picks out a special basis for the scalar fields. But, if you do not know the symmetry *a priori*, how should you proceed?

- You can test for specific models, such as a Type-I or II model.
- You can measure observables that are basis-independent, and determine whether additional symmetries are present.

Example: For simplicity, ignore the first two generations of quarks and assume CP conservation. Then, you should measure ρ^D and ρ^U .

$$\begin{aligned}\rho^D / \rho^U &= m_b / m_t, && \text{for Type-I couplings,} \\ \rho^D \rho^U &= -2m_t m_b / v^2, && \text{for Type-II couplings.}\end{aligned}$$

Thus, measuring ρ^D and ρ^U would help determine the underlying structure of the Higgs-fermion interaction.

Other 2HDMs

- The inert 2HDM

In this model, one imposes a symmetry $H_2 \rightarrow -H_2$ in the Higgs basis. As a result, The Higgs sector is CP-conserving. In the CP-conserving neutral Higgs-fermion Lagrangian, this model corresponds to $\sin(\beta - \alpha) = 1$, $\cos(\beta - \alpha) = 0$ and $\rho^U = \rho^D = 0$. The CP-even h^0 is identical to the SM Higgs boson, and H^0 , A^0 and H^\pm appear only quadratically in interactions and so the lightest among these states is absolutely stable.

- Fermiophobic 2HDM

This is a model that purports to invent a Higgs scalar that couples only to gauge bosons and is decoupled from fermions. This can be realized in a Type-I 2HDM where $\cos \alpha = 0$. In this case, h^0 has no tree-level couplings to fermions. However, the h^0 couplings to W^+W^- and ZZ are suppressed by a factor of $\cos \beta$.

The decoupling limit of the 2HDM

In the decoupling limit, one of the two Higgs doublets of the 2HDM receives a very large mass which then decouples from the theory. This is achieved when $Y_2 \gg v^2$ and $|Z_i| \lesssim \mathcal{O}(1)$ [for all i]. The effective low energy theory is a one-Higgs-doublet model, which yields the SM Higgs boson.

We order the neutral scalar masses according to $m_1 < m_{2,3}$ and define the Higgs mixing angles accordingly. The conditions for the decoupling limit are:

$$|\sin \theta_{12}| \lesssim \mathcal{O}\left(\frac{v^2}{m_2^2}\right) \ll 1, \quad |\sin \theta_{13}| \lesssim \mathcal{O}\left(\frac{v^2}{m_3^2}\right) \ll 1,$$
$$\text{Im}(Z_5 e^{-2i\theta_{23}}) \lesssim \mathcal{O}\left(\frac{v^2}{m_3^2}\right) \ll 1.$$

In the decoupling limit, $m_1 \ll m_2, m_3, m_{H^\pm}$. In particular, the properties of h_1 coincide with the SM Higgs boson with $m_1^2 = Z_1 v^2$ up to corrections of $\mathcal{O}(v^4/m_{2,3}^2)$, and $m_2 \simeq m_3 \simeq m_{H^\pm}$ with squared mass splittings of $\mathcal{O}(v^2)$.

In the exact decoupling limit, where $s_{12} = s_{13} = \text{Im}(Z_5 e^{-2i\theta_{23}}) = 0$, the interactions of h_1 are precisely those of the SM Higgs boson. In particular, the interactions of the h_1 in the decoupling limit are CP-conserving and diagonal in quark flavor space.

In the most general 2HDM, CP-violating and neutral Higgs-mediated FCNCs are suppressed by factors of $\mathcal{O}(v^2/m_{2,3}^2)$ in the decoupling limit. In contrast, the interactions of the heavy neutral Higgs bosons (h_2 and h_3) and the charged Higgs bosons (H^\pm) in the decoupling limit can exhibit both CP-violating and quark flavor non-diagonal couplings (proportional to the ρ^Q).

The decoupling limit is a generic feature of extended Higgs sectors.

- The observation of a SM-like Higgs boson does not rule out the possibility of an extended Higgs sector in the decoupling regime.

Sources for deviations of the properties of h_1^0 from the SM Higgs boson

In the decoupling limit of the 2HDM, we can identify a scale $\Lambda_H \gg v$ which controls the masses of the heavier Higgs states.

In addition, additional new physics beyond the Standard Model (BSM) may exist. Denote the energy scale of this new physics by Λ_{BSM} .

- Deviations from SM Higgs behavior can be due to corrections to tree-level Higgs couplings of $\mathcal{O}(v^2/\Lambda_H^2)$ arising from non-minimal Higgs physics [Λ_H characterizes the scale of the heavy Higgs states].
- Additional deviations of order $\mathcal{O}(v^2/\Lambda_{\text{BSM}}^2)$ can arise in loop-induced Higgs couplings due to BSM particles in the loops. In principle, the scales Λ_H and Λ_{BSM} are unconnected. For example, in the MSSM, $\Lambda_H \sim m_A$ and $\Lambda_{\text{BSM}} \sim \Lambda_{\text{SUSY}}$ (the supersymmetry-breaking scale).

Deviations from SM Higgs behavior would provide clues to the structure of the extended Higgs sector and/or the BSM physics.

Dreams of large departures from SM Higgs behavior

Suppose the $\gamma\gamma$ excess in the Higgs data persists, whereas the $ZZ^* \rightarrow 4$ leptons signal conforms to Standard Model expectations. Can this be explained within the 2HDM?

Here is one possible scenario. For simplicity, assume CP conservation. Suppose that h^0 and A^0 were approximately degenerate in mass, and the properties of h^0 were close to those of the SM Higgs boson. Then, the A^0 could be produced in gluon-gluon fusion, and could subsequently decay into $\gamma\gamma$. However, the A^0 does not couple at tree level to ZZ . Thus, the $\gamma\gamma$ signal could be enhanced due to simultaneous contributions from both h^0 and A^0 .

Pedro Ferreira, Rui Santos, Joao Silva, and I have performed parameter scans for both the Type-I and Type-II 2HDM, to see whether an enhanced $\gamma\gamma$ signal is plausible.

Enhanced final state Higgs channels

We define

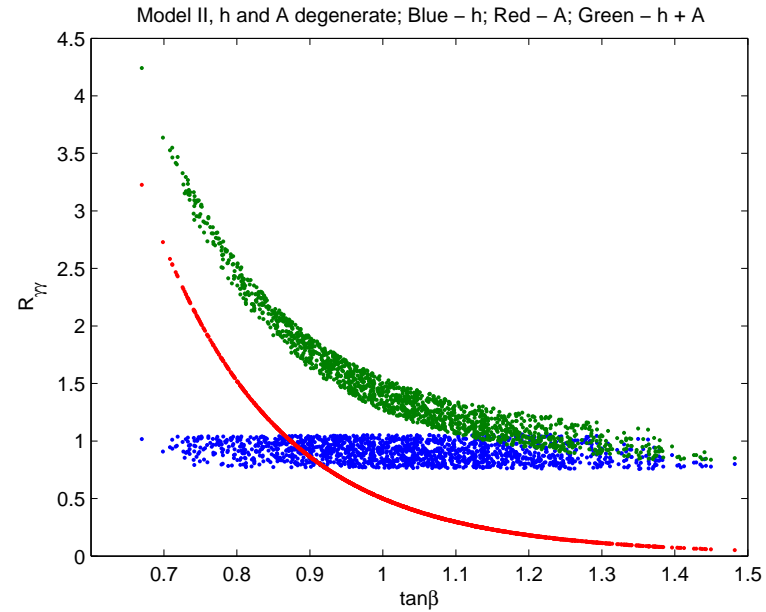
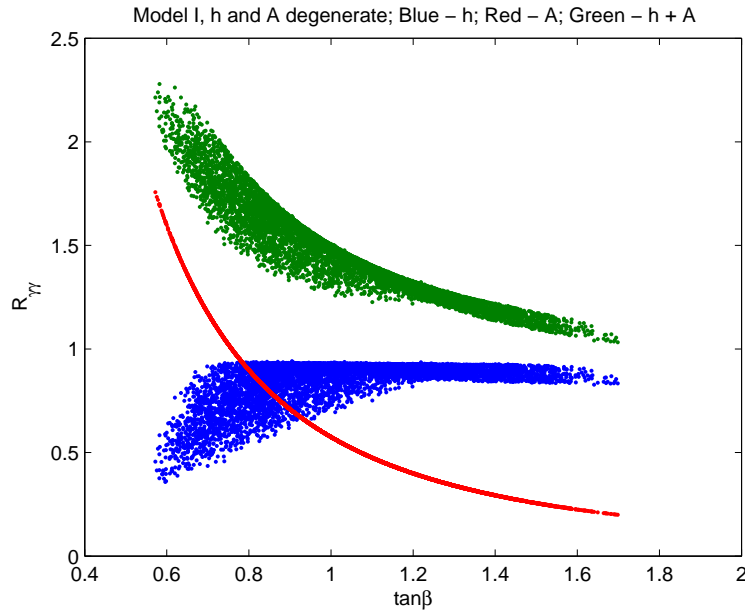
$$R_f^H = \frac{\sigma(pp \rightarrow H)_{2\text{HDM}} \times \text{BR}(H \rightarrow f)_{2\text{HDM}}}{\sigma(pp \rightarrow h_{\text{SM}}) \times \text{BR}(h_{\text{SM}} \rightarrow f)},$$

where f is the final state of interest, and H is one of the two 125 GeV mass-degenerate scalars. The observed ratio of f production relative to the SM expectation is

$$R_f \equiv \sum_H R_f^H.$$

In obtaining $\sigma(pp \rightarrow S)$, we include the two main Higgs production mechanisms: gg fusion and vector boson (W^+W^- and ZZ) fusion. The final states of interest are $f = \gamma\gamma, ZZ^*, WW^*$ and $\tau^+\tau^-$. Note that the LHC is (eventually) sensitive to the $b\bar{b}$ final state primarily in associated $V + H$ production, which is less relevant to our analysis.

An enhanced $\gamma\gamma$ signal due to mass-degenerate h^0 and A^0 :

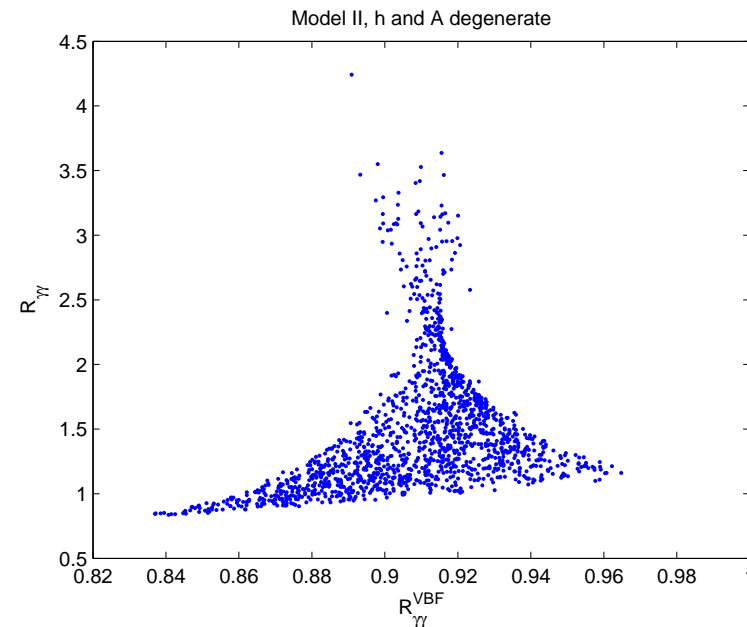
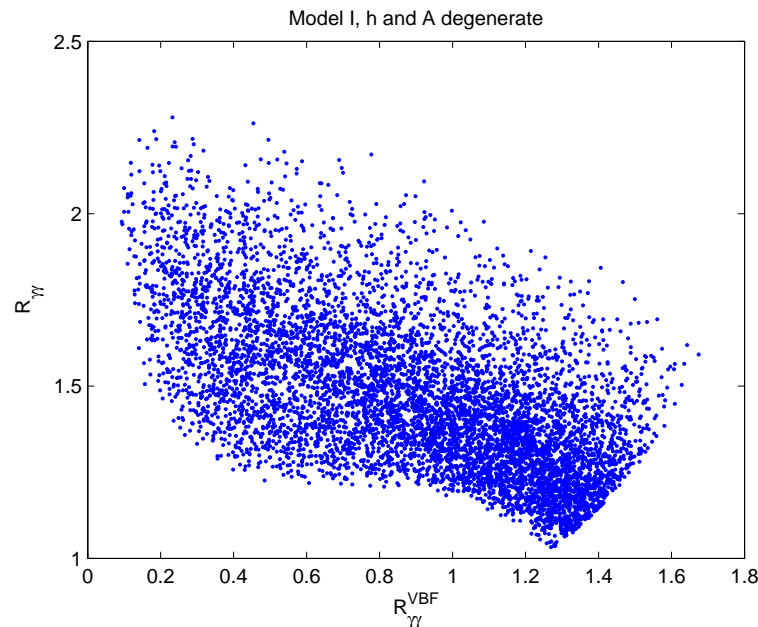


The enhancement occurs in the parameter regime of $\tan\beta \lesssim 1.5$ and $\sin(\beta - \alpha)$ near 1.

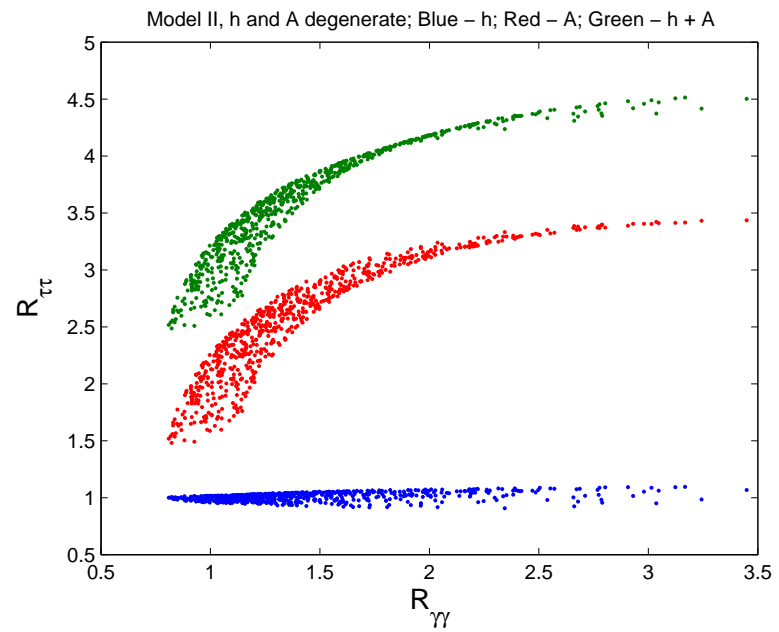
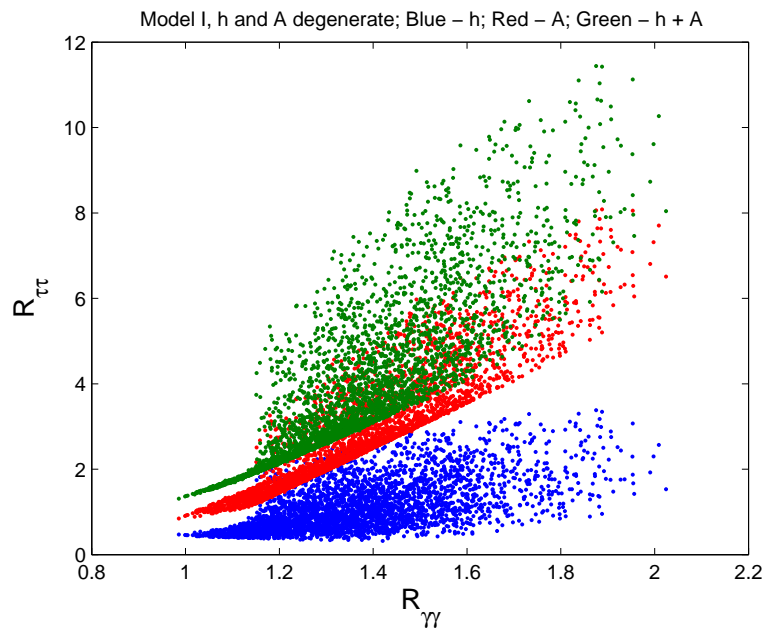
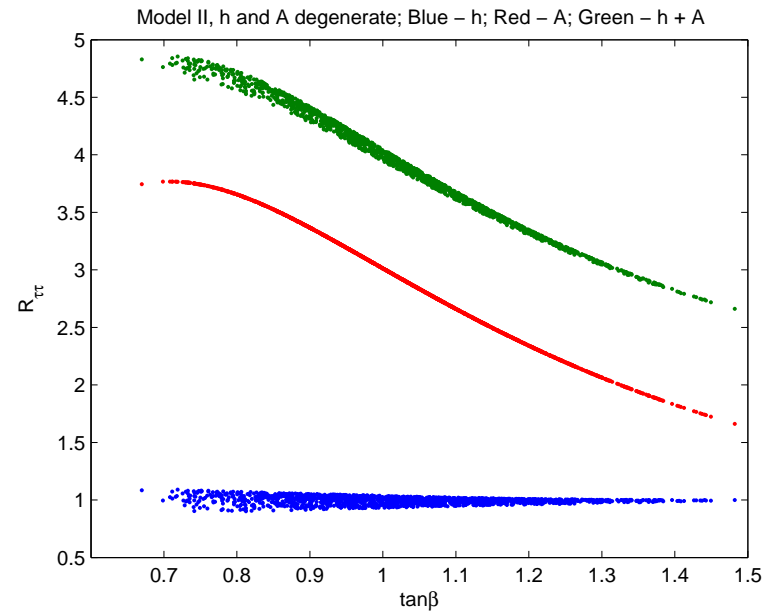
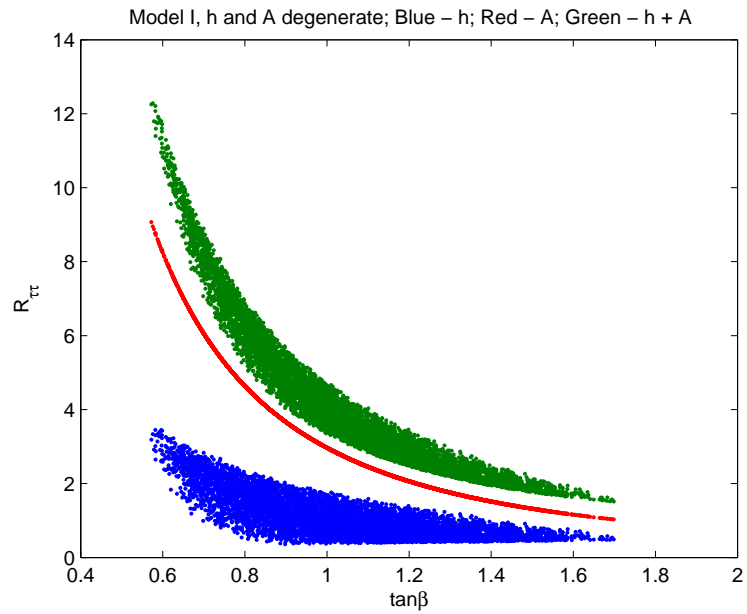
Indeed, we see that the scenario of a mass-degenerate h^0 and A^0 (and more generally any mass-degenerate Higgs pair) that yields an enhanced $\gamma\gamma$ signal is *incompatible* with the MSSM Higgs sector, since such low values of $\tan\beta$ in the MSSM are ruled out by LEP data.

But an enhanced $\gamma\gamma$ signal in the mass-degenerate scenario yields two associated predictions that must be confirmed by experiment if this framework is to be consistent.

1. The $\gamma\gamma$ signal produced in vector boson fusion events must be less than the SM rate in 2HDM-II (but is not as constrained in 2HDM-I).



2. The inclusive $\tau^+\tau^-$ signal is enhanced with respect to the SM due to the production of A via gg fusion.



The Higgs sector of the MSSM

The Higgs sector of the MSSM is a 2HDM, whose Yukawa couplings and Higgs potential are constrained by SUSY. Instead of employing two hypercharge-one scalar doublets $\Phi_{1,2}$, it is more convenient to introduce a $Y = -1$ doublet $H_d \equiv i\sigma_2\Phi_1^*$ and a $Y = +1$ doublet $H_u \equiv \Phi_2$:

$$H_d = \begin{pmatrix} H_d^1 \\ H_d^2 \end{pmatrix} = \begin{pmatrix} \Phi_1^{0*} \\ -\Phi_1^- \end{pmatrix}, \quad H_u = \begin{pmatrix} H_u^1 \\ H_u^2 \end{pmatrix} = \begin{pmatrix} \Phi_2^+ \\ \Phi_2^0 \end{pmatrix}.$$

The origin of the notation originates from the Higgs Yukawa Lagrangian:

$$\mathcal{L}_{\text{Yukawa}} = -h_u^{ij} (\bar{u}_R^i u_L^j H_u^2 - \bar{u}_R^i d_L^j H_u^1) - h_d^{ij} (\bar{d}_R^i d_L^j H_d^1 - \bar{d}_R^i u_L^j H_d^2) + \text{h.c.}$$

Note that the neutral Higgs field H_u^2 couples exclusively to up-type quarks and the neutral Higgs field H_d^1 couples exclusively to down-type quarks. This is a Type-II 2HDM.

The Higgs potential of the MSSM is:

$$V = \left(m_d^2 + |\mu|^2\right) H_d^{i*} H_d^i + \left(m_u^2 + |\mu|^2\right) H_u^{i*} H_u^i - m_{ud}^2 \left(\epsilon^{ij} H_d^i H_u^j + \text{h.c.}\right) \\ + \frac{1}{8} \left(g^2 + g'^2\right) \left[H_d^{i*} H_d^i - H_u^{j*} H_u^j\right]^2 + \frac{1}{2} g^2 |H_d^{i*} H_u^i|^2,$$

where $\epsilon^{12} = -\epsilon^{21} = 1$ and $\epsilon^{11} = \epsilon^{22} = 0$, and the sum over repeated indices is implicit. Above, μ is a supersymmetric Higgsino mass parameter and m_d^2 , m_u^2 , m_{ud}^2 are soft-supersymmetry-breaking masses. The quartic Higgs couplings are related to the SU(2) and U(1)_Y gauge couplings as a consequence of SUSY.

Minimizing the Higgs potential, the neutral components of the Higgs fields acquire vevs: $\langle H_d^0 \rangle = v_d$ and $\langle H_u^0 \rangle = v_u$, where $v^2 \equiv v_d^2 + v_u^2 = 4m_W^2/g^2 = (246 \text{ GeV})^2$. The ratio of the two vevs is

$$\tan \beta \equiv \frac{v_u}{v_d}, \quad 0 \leq \beta \leq \frac{1}{2}\pi.$$

In the Higgs basis, the phase of H_2 can be chosen such that Z_5 , Z_6 and Z_7 are real:

$$Z_1 = Z_2 = \frac{1}{4}(g^2 + g'^2) \cos^2 2\beta, \quad Z_3 = Z_5 + \frac{1}{4}(g^2 - g'^2), \quad Z_4 = Z_5 - \frac{1}{2}g^2, \\ Z_5 = \frac{1}{4}(g^2 + g'^2) \sin^2 2\beta, \quad Z_7 = -Z_6 = \frac{1}{4}(g^2 + g'^2) \sin 2\beta \cos 2\beta.$$

The five physical Higgs particles consist of a charged Higgs pair

$$H^\pm = H_d^\pm \sin \beta + H_u^\pm \cos \beta ,$$

one CP-odd scalar

$$A^0 = \sqrt{2} \left(\text{Im } H_d^0 \sin \beta + \text{Im } H_u^0 \cos \beta \right) ,$$

and two CP-even scalars

$$h^0 = -(\sqrt{2} \text{Re } H_d^0 - v_d) \sin \alpha + (\sqrt{2} \text{Re } H_u^0 - v_u) \cos \alpha ,$$
$$H^0 = (\sqrt{2} \text{Re } H_d^0 - v_d) \cos \alpha + (\sqrt{2} \text{Re } H_u^0 - v_u) \sin \alpha ,$$

where we have now labeled the Higgs fields according to their electric charge. The angle α arises when the CP-even Higgs squared-mass matrix (in the H_d^0 — H_u^0 basis) is diagonalized to obtain the physical CP-even Higgs states.

All Higgs masses and couplings can be expressed in terms of two parameters usually chosen to be m_A and $\tan \beta$.

Tree-level MSSM Higgs masses

The charged Higgs mass is given by

$$m_{H^\pm}^2 = m_A^2 + m_W^2,$$

and the CP-even Higgs bosons h^0 and H^0 are eigenstates of the squared-mass matrix

$$\mathcal{M}_0^2 = \begin{pmatrix} m_A^2 \sin^2 \beta + m_Z^2 \cos^2 \beta & -(m_A^2 + m_Z^2) \sin \beta \cos \beta \\ -(m_A^2 + m_Z^2) \sin \beta \cos \beta & m_A^2 \cos^2 \beta + m_Z^2 \sin^2 \beta \end{pmatrix}.$$

The eigenvalues of \mathcal{M}_0^2 are the squared-masses of the two CP-even Higgs scalars

$$m_{H,h}^2 = \frac{1}{2} \left(m_A^2 + m_Z^2 \pm \sqrt{(m_A^2 + m_Z^2)^2 - 4m_Z^2 m_A^2 \cos^2 2\beta} \right),$$

and α is the angle that diagonalizes the CP-even Higgs squared-mass matrix. It follows that

$$m_h \leq m_Z |\cos 2\beta| \leq m_Z.$$

Note the contrast with the SM where the Higgs mass is a free parameter, $m_h^2 = \frac{1}{2}\lambda v^2$.

In the MSSM, all Higgs self-coupling parameters of the MSSM are related to the squares of the electroweak gauge couplings.

Tree-level MSSM Higgs couplings

1. Higgs couplings to gauge boson pairs ($V = W$ or Z)

$$g_{h^0 V V} = g_V m_V \sin(\beta - \alpha), \quad g_{H^0 V V} = g_V m_V \cos(\beta - \alpha),$$

where $g_V \equiv 2m_V/v$. There are no tree-level couplings of A^0 or H^\pm to VV .

2. Higgs couplings to a single gauge boson

The couplings of V to two neutral Higgs bosons (which must have opposite CP-quantum numbers) is denoted by $g_{\phi A^0 Z}(p_\phi - p_A^0)$, where $\phi = h^0$ or H^0 and the momenta p_ϕ and p_A^0 point into the vertex, and

$$g_{h^0 A^0 Z} = \frac{g \cos(\beta - \alpha)}{2 \cos \theta_W}, \quad g_{H^0 A^0 Z} = \frac{-g \sin(\beta - \alpha)}{2 \cos \theta_W}.$$

3. Summary of Higgs boson–vector boson couplings

The properties of the three-point and four-point Higgs boson-vector boson couplings are conveniently summarized by listing the couplings that are proportional to either $\sin(\beta - \alpha)$ or $\cos(\beta - \alpha)$ or are angle-independent. As a reminder, $\cos(\beta - \alpha) \rightarrow 0$ in the decoupling limit.

<u>$\cos(\beta - \alpha)$</u>	<u>$\sin(\beta - \alpha)$</u>	<u>angle-independent</u>
$H^0 W^+ W^-$	$h^0 W^+ W^-$	—
$H^0 Z Z$	$h^0 Z Z$	—
$Z A^0 h^0$	$Z A^0 H^0$	$Z H^+ H^-$, $\gamma H^+ H^-$
$W^\pm H^\mp h^0$	$W^\pm H^\mp H^0$	$W^\pm H^\mp A^0$
$Z W^\pm H^\mp h^0$	$Z W^\pm H^\mp H^0$	$Z W^\pm H^\mp A^0$
$\gamma W^\pm H^\mp h^0$	$\gamma W^\pm H^\mp H^0$	$\gamma W^\pm H^\mp A^0$
—	—	$V V \phi \phi$, $V V A^0 A^0$, $V V H^+ H^-$

where $\phi = h^0$ or H^0 and $V V = W^+ W^-$, $Z Z$, $Z \gamma$ or $\gamma \gamma$.

4. Higgs-fermion couplings

Supersymmetry imposes a Type-II structure for the Higgs-fermion Yukawa couplings. Since the neutral Higgs couplings to fermions are flavor-diagonal, we list only the Higgs coupling to 3rd generation fermions. The couplings of the neutral Higgs bosons to $f\bar{f}$ relative to the Standard Model value, $gm_f/2m_W$, are given by

$$\begin{aligned}
 h^0 b\bar{b} \quad (\text{or } h^0 \tau^+ \tau^-) : & \quad -\frac{\sin \alpha}{\cos \beta} = \sin(\beta - \alpha) - \tan \beta \cos(\beta - \alpha), \\
 h^0 t\bar{t} : & \quad \frac{\cos \alpha}{\sin \beta} = \sin(\beta - \alpha) + \cot \beta \cos(\beta - \alpha), \\
 H^0 b\bar{b} \quad (\text{or } H^0 \tau^+ \tau^-) : & \quad \frac{\cos \alpha}{\cos \beta} = \cos(\beta - \alpha) + \tan \beta \sin(\beta - \alpha), \\
 H^0 t\bar{t} : & \quad \frac{\sin \alpha}{\sin \beta} = \cos(\beta - \alpha) - \cot \beta \sin(\beta - \alpha), \\
 A^0 b\bar{b} \quad (\text{or } A^0 \tau^+ \tau^-) : & \quad \gamma_5 \tan \beta, \\
 A^0 t\bar{t} : & \quad \gamma_5 \cot \beta,
 \end{aligned}$$

where the γ_5 indicates a pseudoscalar coupling. Note that the $h^0 f\bar{f}$ couplings approach their SM values in the decoupling limit, where $\cos(\beta - \alpha) \rightarrow 0$.

Similarly, the charged Higgs boson couplings to fermion pairs, with all particles pointing into the vertex, are given by[‡]

$$g_{H^- t\bar{b}} = \frac{g}{\sqrt{2}m_W} \left[m_t \cot \beta P_R + m_b \tan \beta P_L \right],$$

$$g_{H^- \tau+\nu} = \frac{g}{\sqrt{2}m_W} \left[m_\tau \tan \beta P_L \right].$$

Epecially noteworthy is the possible $\tan \beta$ -enhancement of certain Higgs-fermion couplings. The general expectation in MSSM models is that $\tan \beta$ lies in a range:

$$1 \lesssim \tan \beta \lesssim \frac{m_t}{m_b}.$$

Near the upper limit of $\tan \beta$, we have roughly identical values for the top and bottom Yukawa couplings, $h_t \sim h_b$, since

$$h_b = \frac{\sqrt{2} m_b}{v_d} = \frac{\sqrt{2} m_b}{v \cos \beta}, \quad h_t = \frac{\sqrt{2} m_t}{v_u} = \frac{\sqrt{2} m_t}{v \sin \beta}.$$

[‡]Including the full flavor structure, the CKM matrix appears in the charged Higgs couplings in the standard way for a charged-current interaction.

Aside: the decoupling limit of the MSSM

The decoupling behavior of the MSSM Higgs sector is exhibited in the limit of $m_A \gg m_Z$, where the corresponding tree-level expressions are given by:

$$\begin{aligned} m_h^2 &\simeq m_Z^2 \cos^2 2\beta, & m_H^2 &\simeq m_A^2 + m_Z^2 \sin^2 2\beta, \\ m_{H^\pm}^2 &= m_A^2 + m_W^2, & \cos^2(\beta - \alpha) &\simeq \frac{m_Z^4 \sin^2 4\beta}{4m_A^4}. \end{aligned}$$

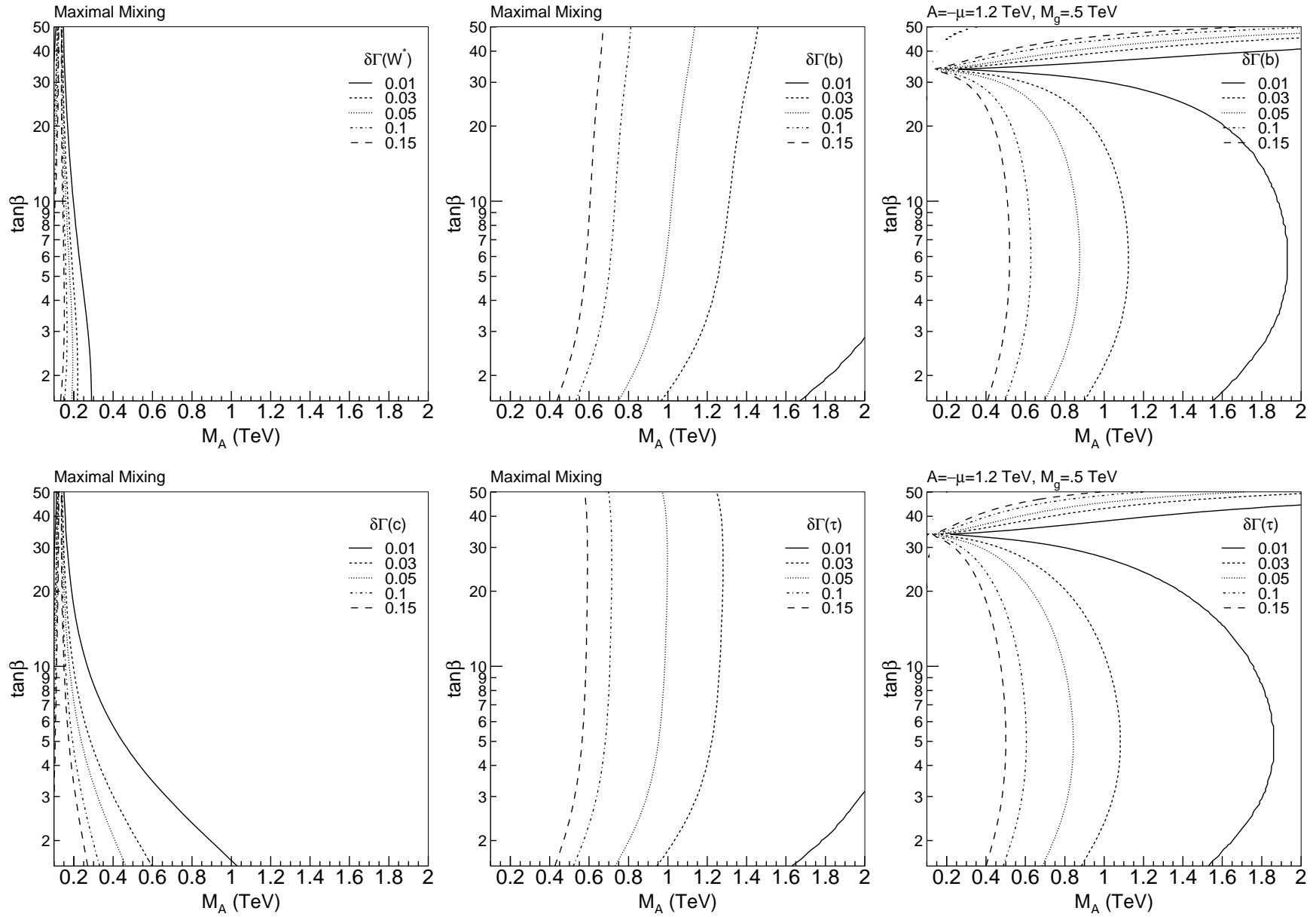
Indeed, $m_A \simeq m_H \simeq m_{H^\pm}$, up to corrections of $\mathcal{O}(m_Z^2/m_A)$, and $\cos(\beta - \alpha) = 0$ up to corrections of $\mathcal{O}(m_Z^2/m_A^2)$, as expected. In general, in the limit of $\cos(\beta - \alpha) \rightarrow 0$, all the h^0 couplings to SM particles approach their SM limits. In particular, if λ_V is a Higgs coupling to vector bosons and λ_f is a Higgs couplings to fermions, then

$$\frac{\lambda_V}{[\lambda_V]_{\text{SM}}} = \sin(\beta - \alpha) = 1 + \mathcal{O}\left(m_Z^4/m_A^4\right), \quad \frac{\lambda_f}{[\lambda_f]_{\text{SM}}} = 1 + \mathcal{O}\left(m_Z^2/m_A^2\right).$$

The behavior of the $h^0 f f$ coupling is seen from:

$$\begin{aligned} h^0 b\bar{b} \quad (\text{or } h^0 \tau^+ \tau^-) : & \quad -\frac{\sin \alpha}{\cos \beta} = \sin(\beta - \alpha) - \tan \beta \cos(\beta - \alpha), \\ h^0 t\bar{t} : & \quad \frac{\cos \alpha}{\sin \beta} = \sin(\beta - \alpha) + \cot \beta \cos(\beta - \alpha), \end{aligned}$$


Note the extra $\tan \beta$ enhancement in the deviation of λ_{hbb} from $[\lambda_{hbb}]_{\text{SM}}$.



Deviations of Higgs partial widths from their SM values in two different MSSM scenarios (Carena, Haber, Logan and Mrenna).

Saving the MSSM Higgs sector—the impact of radiative corrections

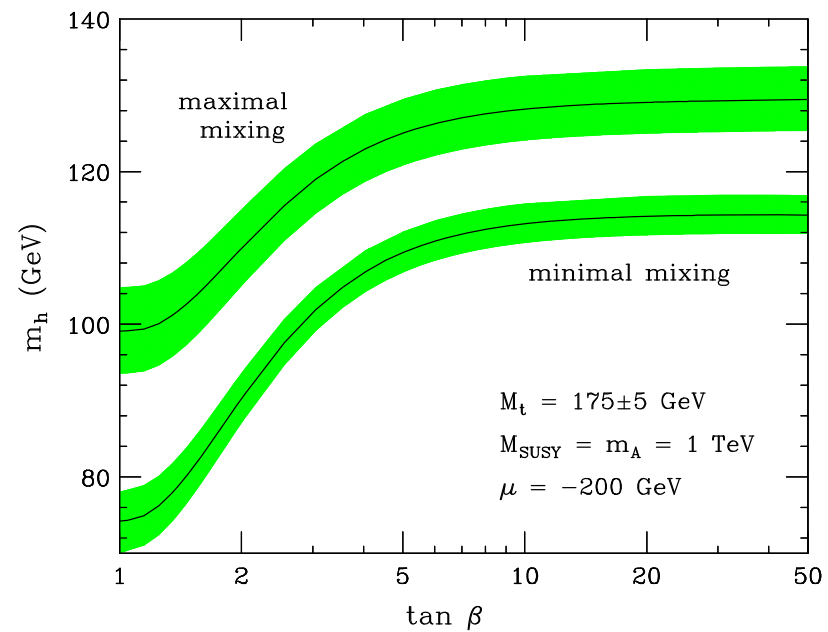
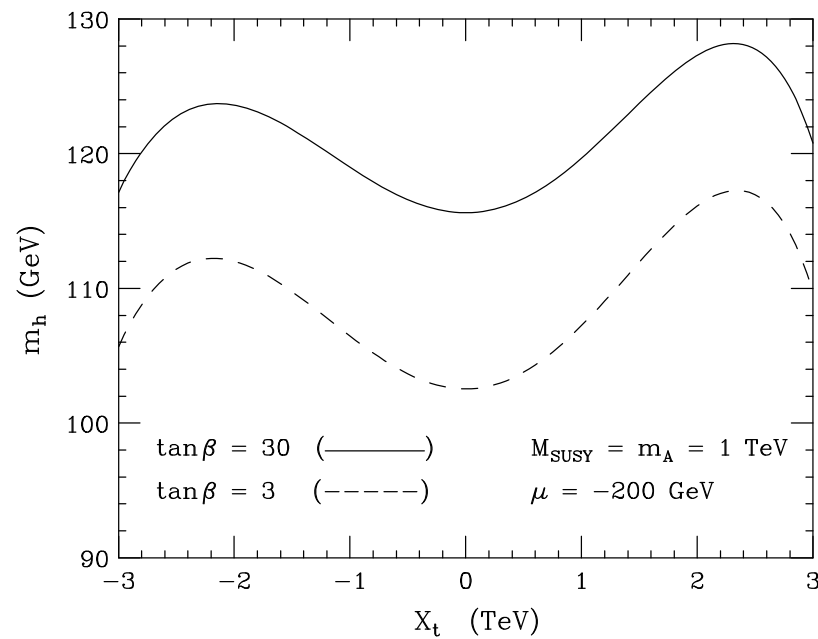
We have already noted the tree-level relation $m_h \leq m_Z$, which is already ruled out by LEP data. But, this inequality receives quantum corrections. The Higgs mass can be shifted due to loops of particles and their superpartners (an incomplete cancellation, which would have been exact if supersymmetry were unbroken):



$$m_h^2 \lesssim m_Z^2 + \frac{3g^2 m_t^4}{8\pi^2 m_W^2} \left[\ln \left(\frac{M_S^2}{m_t^2} \right) + \frac{X_t^2}{M_S^2} \left(1 - \frac{X_t^2}{12M_S^2} \right) \right],$$

where $X_t \equiv A_t - \mu \cot \beta$ governs stop mixing and M_S^2 is the average squared-mass of the top-squarks \tilde{t}_1 and \tilde{t}_2 (which are the mass-eigenstate combinations of the interaction eigenstates, \tilde{t}_L and \tilde{t}_R).

The state-of-the-art computation includes the full one-loop result, all the significant two-loop contributions, some of the leading three-loop terms, and renormalization-group improvements. The final conclusion is that $m_h \lesssim 130$ GeV [assuming that the top-squark mass is no heavier than about 2 TeV].



Maximal mixing corresponds to choosing the MSSM Higgs parameters in such a way that m_h is maximized (for a fixed $\tan\beta$). This occurs for $X_t/M_S \sim 2$. As $\tan\beta$ varies, m_h reaches its maximal value, $(m_h)_{\text{max}} \simeq 130$ GeV, for $\tan\beta \gg 1$ and $m_A \gg m_Z$.

Beyond the MSSM Higgs sector

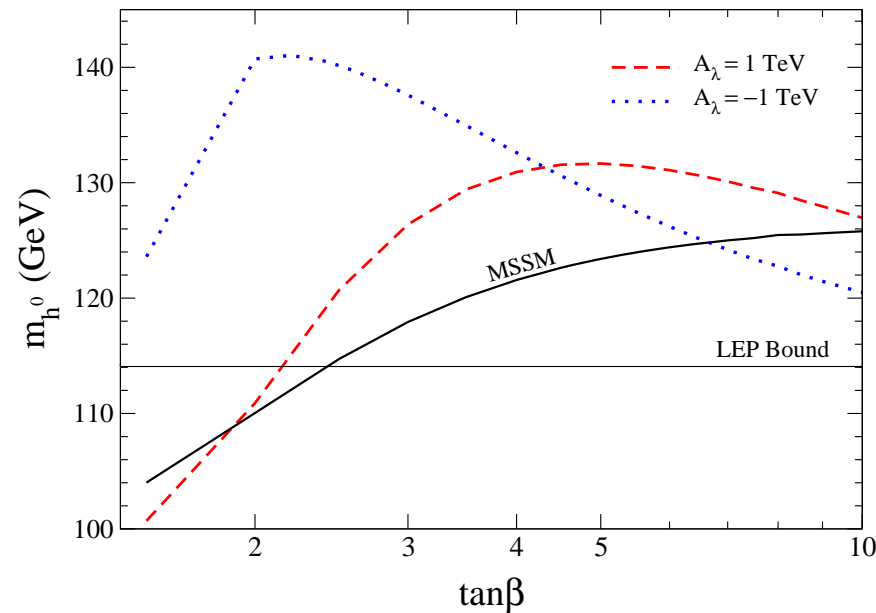
Why go beyond the MSSM? The observed Higgs mass of 125 GeV is somewhat uncomfortable for the MSSM, as the mass of h^0 is somewhat close to its maximally allowed value, which requires heavy stop masses and significant stop mixing. The absence of observed SUSY particles emphasizes this *little hierarchy problem* that seems to require at least 1% fine-tuning of MSSM parameters to explain the magnitude of the EWSB scale.

In the NMSSM, a Higgs singlet superfield \hat{S} is added to the MSSM. The corresponding superpotential terms,

$$(\mu + \lambda\hat{S})\hat{H}_u\hat{H}_d + \frac{1}{2}\mu_S\hat{S}^2 + \frac{1}{3}\kappa\hat{S}^3,$$

and soft-SUSY-breaking terms $B_s S^2 + \lambda A_\lambda S H_u H_d$ add additional parameters to the model, which can modify the bounds on the lightest Higgs mass.

For example, Delgado, Kolda, Olson and de la Puente obtain:



Other authors (e.g. Dermisek and Gunion) have advocated NMSSM models as a way to partially alleviate the little hierarchy problem. More generally, there is a large literature (beginning with Haber and Sher in 1987) suggesting the possibility of relaxing the Higgs mass upper bound in extensions of the MSSM.