Why is the Higgs boson SM-like ?

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Outline

1. LHC reveals a Standard Model (SM)-like Higgs boson

2. Non-minimal (extended) Higgs sectors

3. How to achieve a SM-like Higgs boson in the 2HDM

4. Classification of symmetries of the 2HDM bosonic sector

5. (Approximate) Higgs field alignment due to a (softly-broken) symmetry

6. Extending the symmetry to the Yukawa sector

7. Conclusions

This talk is primarily based on:

1. H.E. Haber and J.P. Silva, Phys. Rev. D **103**, 115012 (2021) [arXiv:2102.07136 [hep-ph]]

and

2. P. Draper, A. Ekstedt and H.E. Haber, JHEP **05**, 235 (2021) [arXiv:2011.13159 [hep-ph]].

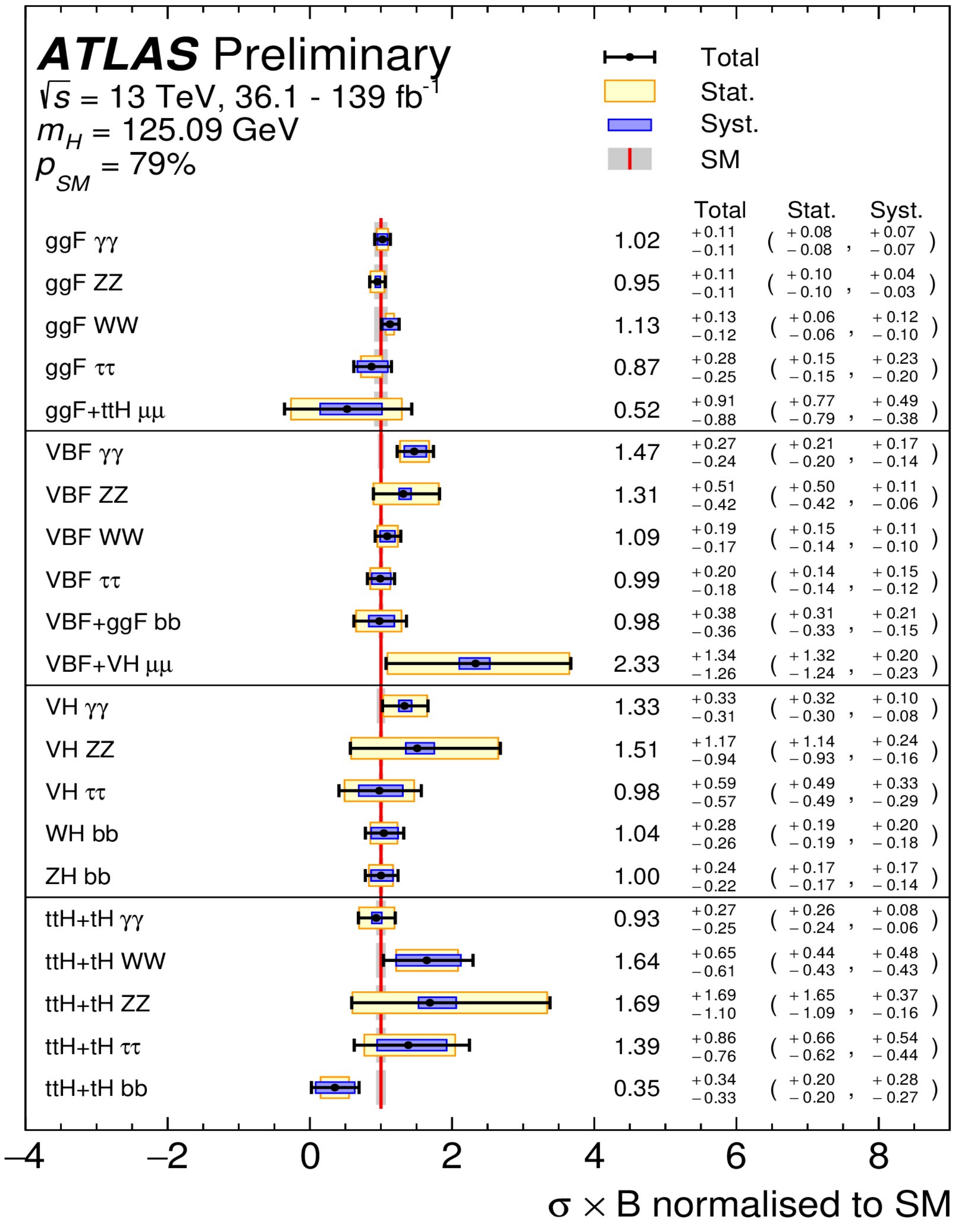
<u>Note:</u> Paper 2 extends results first obtained in:
P. Draper, H.E. Haber and J.T. Ruderman, JHEP **06**, 124 (2016) [arXiv:1605.03237 [hep-ph]].

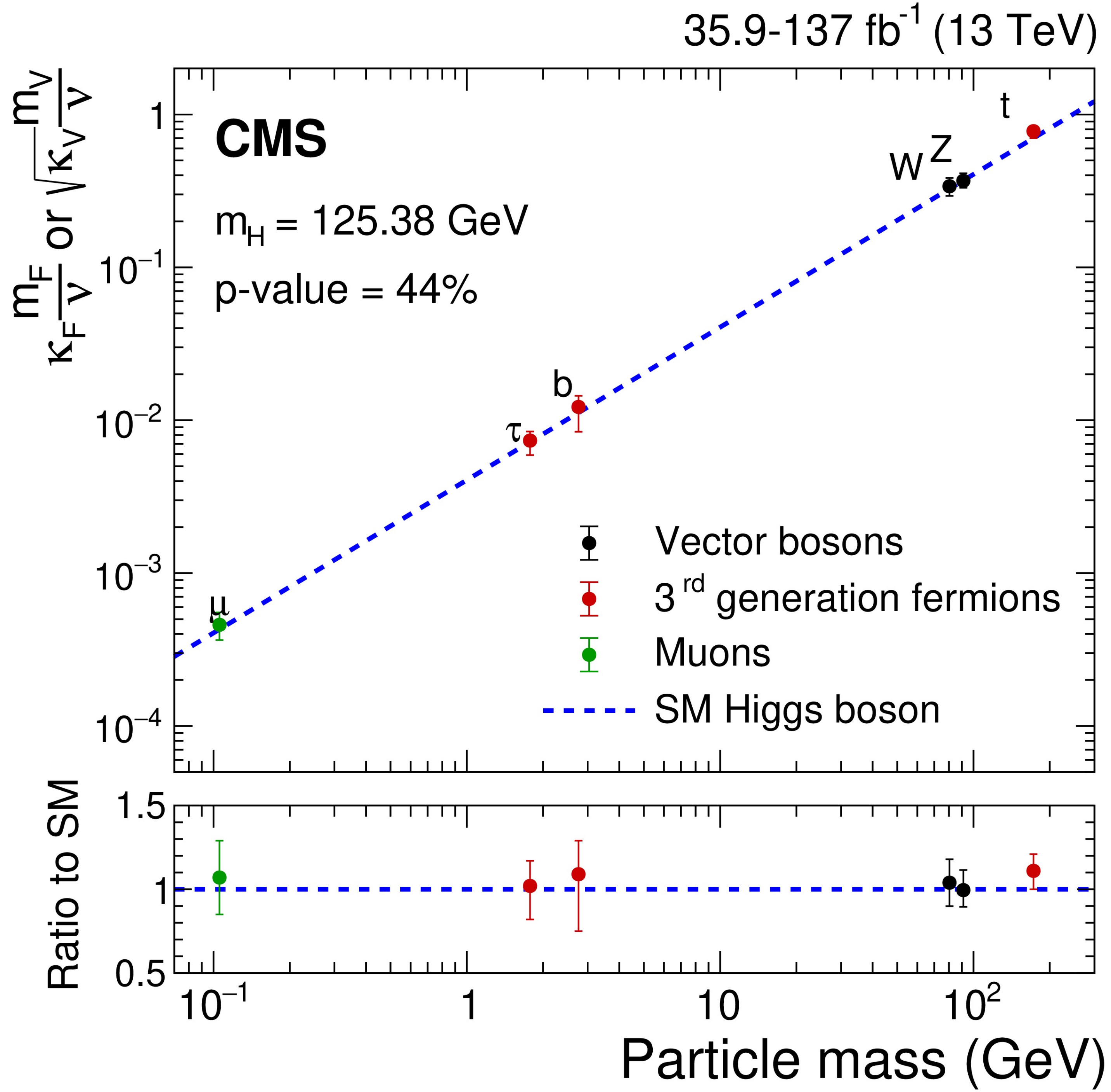
A SM-like Higgs boson

Nine years after the discovery of the Higgs boson, the data from Runs 1 and 2 of the LHC are well described by the Standard Model.

Taken from ATLAS collaboration, ATLAS-CONF-2021-053 (2 November 2021).

Cross sections times branching fraction for *ggF,* vector boson fusion (VBF), *VH* and *$t\bar{t}H$+tH* production in each relevant decay mode, normalized to their Standard Model (SM) predictions. The values are obtained from a simultaneous fit to all channels. The black error bars, blue boxes and yellow boxes show the total, systematic, and statistical uncertainties in the measurements, respectively. The gray bands show the theory uncertainties on the predictions. The level of compatibility between the measurement and the SM prediction corresponds to a *p*-value of p_{SM}=79%, computed using the procedure outlined in the text with 21 degrees of freedom.





Taken from CMS Collaboration, JHEP 01 (2021) 148.

The best fit estimates for the reduced coupling modifiers extracted for fermions and weak bosons from the resolved κ-framework compared to their corresponding prediction from the SM. The error bars represent 68% CL intervals for the measured parameters. In the lower panel, the ratios of the measured coupling modifiers values to their SM predictions are shown.

Nevertheless, given the current precision of the Higgs data, the possibility that the Higgs sector contains more that one physical scalar cannot be excluded.

Indeed, the structure of the Standard Model (SM) is far from being of minimal form. For example, there are three generations of quarks and leptons whereas one generation would have been sufficient ("Who ordered that?"). So why shouldn't the scalar sector be non-minimal as well?

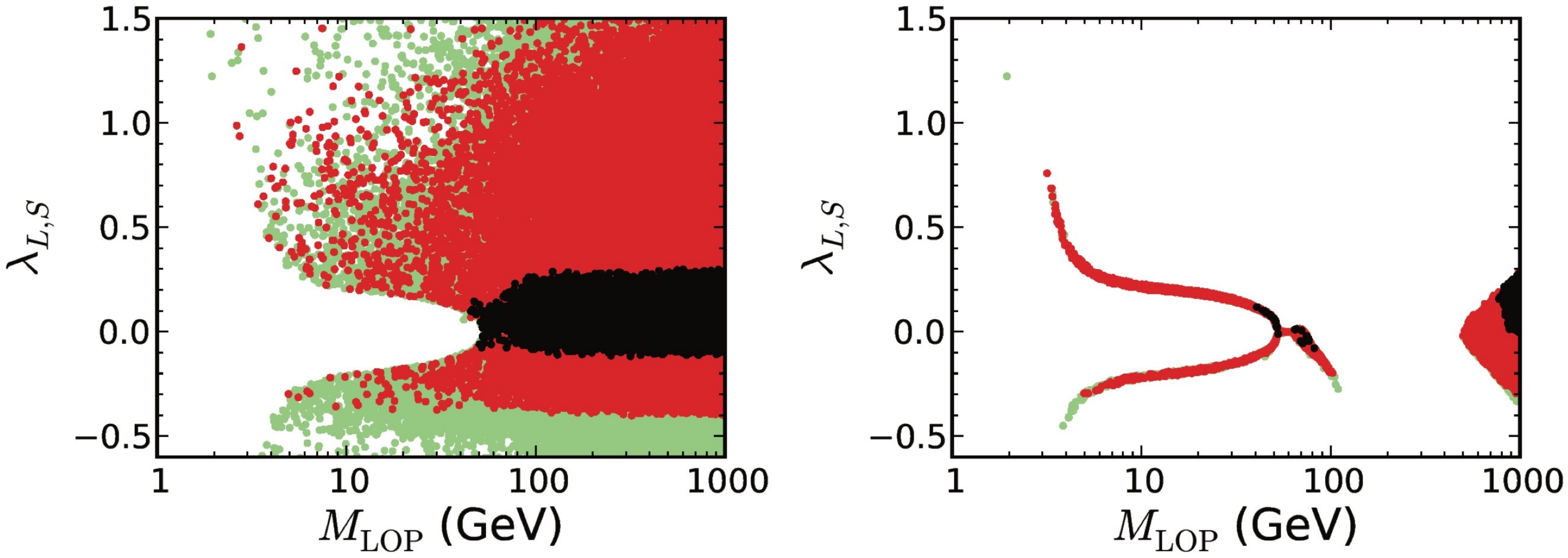
Non-minimal (extended) Higgs sectors

Motivations for Extended Higgs Sectors

- Extended Higgs sectors can modify the electroweak phase transition and facilitate baryogenesis.
- Extended Higgs sectors can enhance vacuum stability.
- Extended Higgs sectors can provide a dark matter candidate.
- Extended Higgs sectors can be employed to provide a solution to the strong CP problem ($\Longrightarrow$ axion)
- Models of new physics beyond the SM often require additional scalar Higgs states. E.g., two Higgs doublets are required in the minimal supersymmetric extension of the SM (MSSM).

A neutral scalar dark matter candidate—the inert doublet model (IDM)

The IDM is a 2HDM in which the scalar potential in a basis where $\langle H_1\rangle = v/\sqrt{2}$ and $\langle H_2\rangle = 0$ exhibits an exact $\mathbf{Z_2}$ discrete symmetry. All fields of the IDM—gauge bosons, fermions and the Higgs doublet field H_1 are even under $\mathbf{Z_2}$. Only the Higgs doublet field H_2 is $\mathbf{Z_2}$-odd. Hence, there is no mixing between H_1 and H_2. In particular, the SM Higgs boson h resides in H_1. The lightest $\mathbf{Z_2}$-odd particle (LOP) residing in H_2 is a candidate for the dark matter.



The viable IDM parameter space projected on the $(M_{\rm LOP}\,,\,\lambda_{L,S})$ plane imposing only the upper limit (left) and the upper and lower limits (right) of the WMAP range, $0.1018 \leq M_{\rm LOP}h^2 \leq 0.1234$. The green points correspond to all valid points in the scan, while the red and black regions show the points which remain valid when the model satisfies stability and perturbativity up to a scale $\Lambda = 10^4$ GeV and the GUT scale $\Lambda = 10^{16}$ GeV, respectively. Taken from A. Goudelis, B. Herrmann and O. Stål, JHEP **1309** (2013) 106.

Note: deviations of h from SM Higgs properties can arise at one-loop (e.g., $H^\pm$ loop corrections to $h \to \gamma\gamma$).

Extended Higgs Sectors are Highly Constrained

- The electroweak ρ parameter is very close to 1.
- One neutral Higgs scalar of the extended Higgs sector must be SM-like (and identified with the Higgs boson at mass 125 GeV).
- At present, only one Higgs scalar has been discovered.
- Higgs-mediated flavor-changing neutral currents (FCNCs) are suppressed.
- Higgs-mediated CP-violation has not yet been observed (with implications for electric dipole moments).
- Charged Higgs exchange at tree level (e.g. in $\bar{B} \to D^{(*)}\tau^-\overline{\nu}_\tau$) and at one-loop (e.g. in $b \to s\gamma$) can significantly constrain the charged Higgs mass and the Yukawa couplings.

The ρ-parameter constraint on extended Higgs sectors

Given that the electroweak ρ-parameter is very close to 1, it follows that a Higgs multiplet of weak-isospin T and hypercharge Y must satisfy,[1]

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1 \quad \Longleftrightarrow \quad (2T+1)^2 - 3Y^2 = 1\,,$$

independently of the Higgs vacuum expectation values (vevs). The simplest solutions are Higgs singlets $(T,Y) = (0,0)$ and hypercharge-one complex Higgs doublets $(T,Y) = (\frac{1}{2},1)$. For example, the latter is employed by the two Higgs doublet model (2HDM).

More generally, one can achieve $\rho = 1$ by fine-tuning if

$$\sum_{T,Y}\left[4T(T+1) - 3Y^2\right]|V_{T,Y}|^2 c_{T,Y} = 0\,,$$

where $V_{T,Y} \equiv \langle\Phi(T,Y)\rangle$ is the scalar vev, and $c_{T,Y} = 1$ for complex Higgs representations and $c_{T,Y} = \frac{1}{2}$ for real $Y = 0$ Higgs representations.

[1] Y is normalized such that the electric charge of the scalar field is $Q = T_3 + Y/2$.

Why is the observed Higgs boson SM-like?

- There is no extended Higgs sector.

- All other scalars (apart from the SM-like Higgs boson) are very heavy
  - This is the decoupling limit.

- A neutral scalar field with the tree-level properties of the SM Higgs boson is an approximate mass eigenstate (due to suppressed mixing with other neutral scalar fields of the extended Higgs sector).
  - This is the Higgs field alignment limit.
  - The other physical scalars of the model may or may not be significantly heavier than the SM Higgs boson. That is, the decoupling limit is a special case of the Higgs field alignment limit.

The Higgs field alignment limit: approaching the SM Higgs boson

Consider an extended Higgs sector with n hypercharge-one Higgs doublets Φ_i and m additional singlet Higgs fields ϕ_i.

After minimizing the scalar potential, we assume that only the neutral Higgs fields acquire vacuum expectation values (in order to preserve U(1)$_{\rm EM}$),

$$\langle\Phi_i^0\rangle = v_i/\sqrt{2}\,, \qquad\qquad \langle\phi_j^0\rangle = x_j\,.$$

Note that $v^2 \equiv \sum_i |v_i|^2 = 4m_W^2/g^2 = (246~\text{GeV})^2$.

The Higgs basis

Define new linear combinations of the hypercharge-one doublet Higgs fields (the so-called *Higgs basis*). In particular,

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} = \frac{1}{v}\sum_i v_i^* \Phi_i \,, \qquad\qquad \langle H_1^0\rangle = v/\sqrt{2}\,,$$

and $H_2, H_3, \ldots, H_n$ are the other linear combinations of doublet scalar fields such that $\langle H_i^0\rangle = 0$ (for $i = 2, 3, \ldots, n$).

That is H_1^0 is aligned in field space with the direction of the Higgs vacuum expectation value (vev). Thus, if $\sqrt{2}\,\mathrm{Re}(H_1^0) - v$ is a mass-eigenstate, then the tree-level couplings of this scalar to itself, to gauge bosons and to fermions are precisely those of the SM Higgs boson, h^0. This is the exact alignment limit.

A natural SM-like Higgs boson in an extended Higgs sector

- The naturalness of the SM-like Higgs boson mass will not be addressed here.

- The decoupling limit explanation of the SM-like Higgs boson is natural to the extent that the low-energy effective theory simply reduces to the SM.

- The central question of this talk is whether there is a natural mechanism that can produce approximate Higgs alignment without decoupling. The low energy effective theory will then contain the SM-like Higgs boson along with additional scalar states whose masses are not significantly larger than a few hundred GeV.

Naturalness à la 't Hooft: guaranteed by a symmetry that is either exact or is broken by soft symmetry breaking terms of positive mass dimension (i.e., by terms of dimension 3 or less in the Lagrangian).

How to achieve Higgs field alignment in the two Higgs doublet model (2HDM)

The Higgs alignment limit of the 2HDM

Define the scalar doublet fields of the Higgs basis,

$$\mathcal{H}_1 = \begin{pmatrix} \mathcal{H}_1^+ \\ \mathcal{H}_1^0 \end{pmatrix} \equiv c_\beta \Phi_1 + s_\beta e^{-i\xi} \Phi_2 \,, \qquad \mathcal{H}_2 = \begin{pmatrix} \mathcal{H}_2^+ \\ \mathcal{H}_2^0 \end{pmatrix} \equiv e^{i\eta} \big(-s_\beta e^{i\xi} \Phi_1 + c_\beta \Phi_2 \big),$$

such that $\langle \mathcal{H}_1^0 \rangle = v/\sqrt{2}$ and $\langle \mathcal{H}_2^0 \rangle = 0$. The Higgs basis is uniquely defined up to an overall rephasing that is parameterized by the phase angle η.[1]

The neutral scalar $\mathcal{H}_1^0$ is *aligned* in field space with the vacuum expectation value v. If $\sqrt{2}\,\mathrm{Re}\mathcal{H}_1^0 - v$ were a mass eigenstate, then its tree-level properties would coincide with those of the SM Higgs boson.

[1] See R. Boto, T. V. Fernandes, H.E. Haber, J.C. Romão and J.P. Silva, Phys. Rev. D **101**, 055023 (2020).

In the Higgs basis, the scalar potential is given by:

$$\mathcal{V} = Y_1\mathcal{H}_1^\dagger\mathcal{H}_1 + Y_2\mathcal{H}_2^\dagger\mathcal{H}_2 + [Y_3 e^{-i\eta}\mathcal{H}_1^\dagger\mathcal{H}_2 + \text{h.c.}] + \tfrac{1}{2}Z_1(\mathcal{H}_1^\dagger\mathcal{H}_1)^2 \\ +\tfrac{1}{2}Z_2(\mathcal{H}_2^\dagger\mathcal{H}_2)^2 + Z_3(\mathcal{H}_1^\dagger\mathcal{H}_1)(\mathcal{H}_2^\dagger\mathcal{H}_2) + Z_4(\mathcal{H}_1^\dagger\mathcal{H}_2)(\mathcal{H}_2^\dagger\mathcal{H}_1) \\ + \left\{\tfrac{1}{2}Z_5 e^{-2i\eta}(\mathcal{H}_1^\dagger\mathcal{H}_2)^2 + \left[Z_6 e^{-i\eta}(\mathcal{H}_1^\dagger\mathcal{H}_1) + Z_7 e^{-i\eta}(\mathcal{H}_2^\dagger\mathcal{H}_2)\right]\mathcal{H}_1^\dagger\mathcal{H}_2 + \text{h.c.}\right\}.$$

Minimize the scalar potential: $Y_1 = -\frac{1}{2}Z_1 v^2$ and $Y_3 = -\frac{1}{2}Z_6 v^2$.

<u>Remark</u>:

Exact Higgs alignment $\Longleftrightarrow$ $Z_6 = 0$ (and $Y_3 = 0$ via the scalar potential minimum conditions), which implies no $\mathcal{H}_1^0$–$\mathcal{H}_2^0$ mixing.

Only the terms highlighted in red can yield an $\mathcal{H}_1^\dagger\mathcal{H}_2 + \text{h.c.}$ contribution to the quadratic terms of the scalar potential after imposing $\langle\mathcal{H}_1^0\rangle = v/\sqrt{2}$ and $\langle\mathcal{H}_2^0\rangle = 0$.

Approximate Higgs alignment in the CP-conserving 2HDM

With respect to Higgs basis states, $\{\sqrt{2}\,\mathrm{Re}\ H_1^0 - v\,,\sqrt{2}\,\mathrm{Re}\ H_2^0\}$,

$$\mathcal{M}_H^2 = \begin{pmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & m_A^2 + Z_5 v^2 \end{pmatrix}\,, \qquad \text{where } Z_5,\ Z_6 \in \mathbb{R}\,,$$

which yields two neutral CP-even scalars h and H with $m_h \leq m_H$, where m_A is the mass of the neutral CP-odd scalar A.

1. $m_A^2 \gg (Z_1 - Z_5)v^2$. This is the *decoupling limit*, where h is SM-like and $m_A^2 \sim m_H^2 \sim m_{H^\pm}^2 \gg m_h^2 \simeq Z_1 v^2$.

2. $|Z_6| \ll 1$. Then, h is SM-like if $m_A^2 + (Z_5 - Z_1)v^2 > 0$; otherwise, H is SM-like. $\Longrightarrow$ *Alignment without decoupling*.

In particular, the CP-even neutral scalar mass eigenstates are:

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_{\beta-\alpha} & -s_{\beta-\alpha} \\ s_{\beta-\alpha} & c_{\beta-\alpha} \end{pmatrix} \begin{pmatrix} \sqrt{2}\,\mathrm{Re}\ H_1^0 - v \\ \sqrt{2}\,\mathrm{Re}\ H_2^0 \end{pmatrix},$$

where $c_{\beta-\alpha} \equiv \cos(\beta-\alpha)$ and $s_{\beta-\alpha} \equiv \sin(\beta-\alpha)$ are defined in terms of the mixing angle α that diagonalizes the CP-even Higgs squared-mass matrix when expressed in the Φ_1–Φ_2 basis of scalar fields, $\{\sqrt{2}\,\mathrm{Re}\ \Phi_1^0 - v_1\,,\ \sqrt{2}\,\mathrm{Re}\ \Phi_2^0 - v_2\}$, and $\tan\beta \equiv v_2/v_1$.

Since the SM-like Higgs boson is approximately $\sqrt{2}\,\mathrm{Re}\ H_1^0 - v$,

- h is SM-like if $|c_{\beta-\alpha}| \ll 1$ (alignment with or without decoupling, depending on the value of m_A),
- H is SM-like if $|s_{\beta-\alpha}| \ll 1$ (alignment without decoupling).

If h is SM-like

Then, $m_h^2 \simeq Z_1 v^2 = 125~\text{GeV}$ (i.e., $Z_1 \simeq 0.26$) and

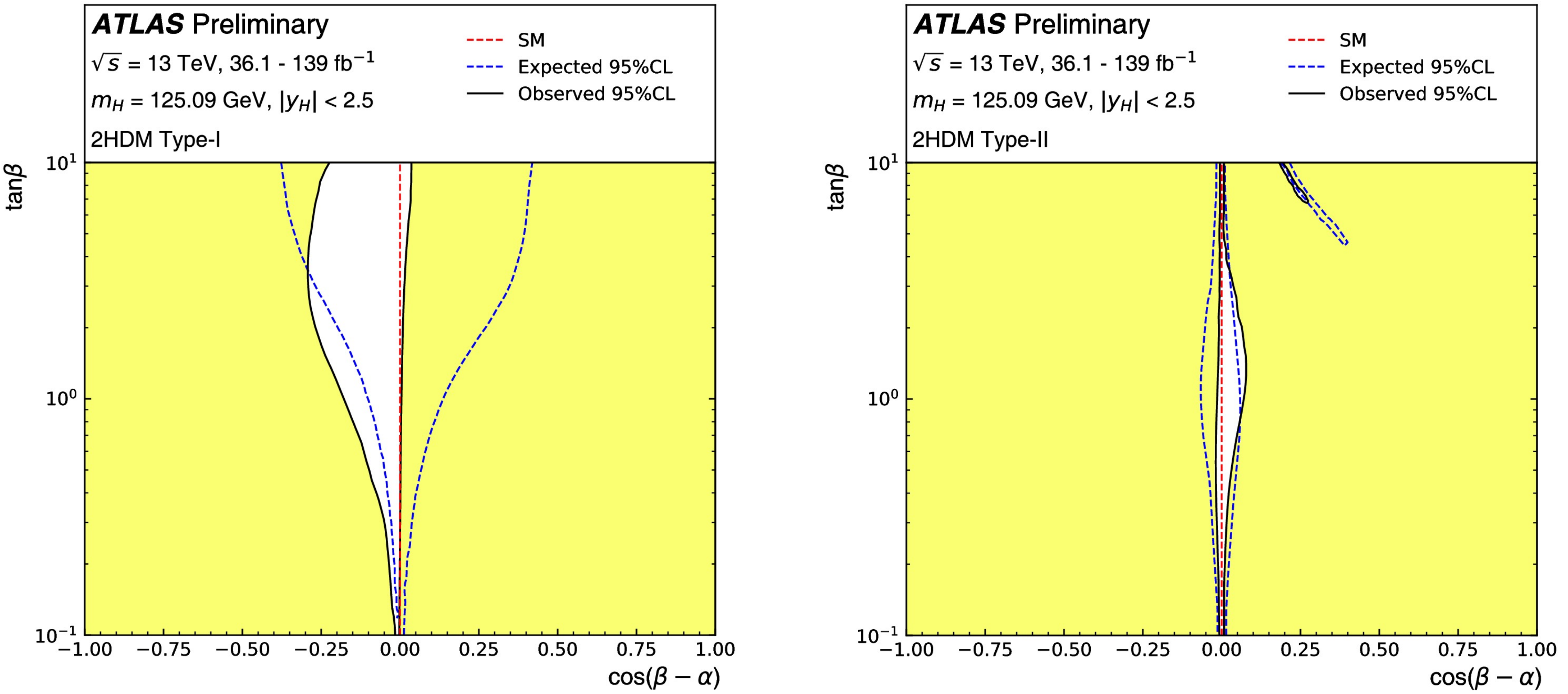
$$|c_{\beta-\alpha}| = \frac{|Z_6|v^2}{\sqrt{(m_H^2 - m_h^2)(m_H^2 - Z_1 v^2)}} \simeq \frac{|Z_6|v^2}{m_H^2 - m_h^2} \ll 1\,,$$

That is, h is SM-like, either

- in the decoupling limit where $m_H \gg m_h$,

or

- in the alignment limit without decoupling where $m_H \sim \mathcal{O}(v)$ and $|Z_6| \ll 1$.



Regions excluded by fits to the measured rates of the productions and decay of the Higgs boson (assumed to be h of the 2HDM). Contours at 95% CL. The observed best-fit values for cos(β - α) are -0.006 for the Type-I 2HDM and 0.002 for the Type-II 2HDM. Taken from ATLAS Collaboration, ATLAS-CONF-2021-053 (2 November 2021).

Achieving exact Higgs alignment in the 2HDM

The inert doublet model (IDM): There is a $\mathbb{Z}_2$ symmetry in the Higgs basis such that $\mathcal{H}_2 \to -\mathcal{H}_2$ is the only $\mathbb{Z}_2$-odd field. Then $Z_6 = 0$, and tree-level alignment is exact. Deviations from SM behavior can appear at loop level due to the virtual exchange of the scalar states that reside in $\mathcal{H}_2$.

Approximate Higgs alignment without decoupling

- Is this the result of an accidental choice of model parameters?
- Is this the result of a scalar potential that exhibits an exact symmetry at a very high energy scale that is not respected by the full Lagrangian, thereby generating deviations from exact alignment at the electroweak scale via renormalization group evolution?[2]
- Is this a consequence of an approximate (softly-broken) symmetry at the electroweak scale? Not possible in the IDM without additional symmetries.

[2] See, e.g., N. Darvishi and A. Pilaftsis, PoS **CORFU2019**, 064 (2020) [arXiv:2004.04505 [hep-ph]].

Classification of global symmetries of the 2HDM bosonic sector

Symmetries of the 2HDM bosonic sector

The gauge covariant kinetic energy terms of the scalar fields are invariant under global U(2) transformations, $\Phi \to U\Phi$.

The 2HDM scalar potential in the Φ-basis is given by,

$$\begin{aligned}\mathcal{V}(\Phi) &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}] + \tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 \\ &\quad +\tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &\quad + \left\{\tfrac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \big[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\big]\Phi_1^\dagger\Phi_2 + \text{h.c.}\right\}\,.\end{aligned}$$

After minimizing the scalar potential, $\langle\Phi_1^0\rangle = vc_\beta/\sqrt{2}$ and $\langle\Phi_2^0\rangle = vs_\beta e^{i\xi}/\sqrt{2}$, where $v \equiv 2m_W/g = 246$ GeV, $s_\beta \equiv \sin\beta$ and $c_\beta \equiv \cos\beta$, with $0 \leq \beta \leq \frac{1}{2}\pi$.

The Φ-basis is meaningful once a particular (global) symmetry [e.g., a subgroup of U(2)] is imposed.

Family and GCP symmetries of the 2HDM bosonic sector

Higgs family symmetries[3]

$\mathbb{Z}_2$: $\qquad \Phi_1 \to \Phi_1, \qquad \Phi_2 \to -\Phi_2$

Π_2 : $\qquad \Phi_1 \longleftrightarrow \Phi_2$

U(1)$_{\rm PQ}$ [Peccei-Quinn]: $\qquad \Phi_1 \to e^{-i\theta}\Phi_1, \qquad \Phi_2 \to e^{i\theta}\Phi_2$

SO(3): $\qquad \Phi_a \to U_{ab}\Phi_b\,, \qquad U \in {\rm U}(2)/{\rm U}(1)_{\rm Y}$

Generalized CP (GCP) transformations

GCP1 : $\qquad \Phi_1 \to \Phi_1^*, \qquad \Phi_2 \to \Phi_2^*$

GCP2 : $\qquad \Phi_1 \to \Phi_2^*, \qquad \Phi_2 \to -\Phi_1^*$

GCP3 : $\qquad \Phi_1 \to \Phi_1^* c_\theta + \Phi_2^* s_\theta, \qquad \Phi_2 \to -\Phi_1^* s_\theta + \Phi_2^* c_\theta,$ for any $0 < \theta < \frac{1}{2}\pi$

where $c_\theta \equiv \cos\theta$ and $s_\theta \equiv \sin\theta$.

[3]Note that custodial symmetries are not symmetries of the gauge covariant scalar kinetic energy terms.

Possible symmetries of the 2HDM bosonic sector

A complete classification of possible Higgs family and generalized CP symmetries of the scalar potential (in the Φ-basis) has been obtained.[4]

symmetry	m_{22}^2	m_{12}^2	λ_2	λ_4	$\mathrm{Re}\lambda_5$	$\mathrm{Im}\lambda_5$	λ_6	λ_7
$\mathbb{Z}_2$		0					0	0
Π_2	m_{11}^2	real	λ_1			0		λ_6^*
$\mathbb{Z}_2 \otimes \Pi_2$	m_{11}^2	0	λ_1			0	0	0
U(1)		0			0	0	0	0
U(1)$\otimes\Pi_2$	m_{11}^2	0	λ_1		0	0	0	0
SO(3)	m_{11}^2	0	λ_1	$\lambda_1 - \lambda_3$	0	0	0	0
GCP1		real				0	real	real
GCP2	m_{11}^2	0	λ_1					$-\lambda_6$
GCP3	m_{11}^2	0	λ_1		$\lambda_1 - \lambda_3 - \lambda_4$	0	0	0

Note that $\Pi_2 \Longleftrightarrow \mathbb{Z}_2$ symmetry in a different Φ'-basis; $\mathbb{Z}_2 \otimes \Pi_2 \Longleftrightarrow$ GCP2 in a different basis; U(1)$\otimes\Pi_2 \Longleftrightarrow$ GCP3 in a different basis, where $\Phi' = V\Phi$ for a suitably chosen V.

[4] I.P. Ivanov, Phys. Rev. D **77**, 015017 (2008) [arXiv:0710.3490]; P.M. Ferreira, H.E. Haber and J.P. Silva, Phys. Rev. D **79**, 116004 (2009) [arXiv:0902.1537].

GCP2 symmetry yields an Exceptional Region of the 2HDM Parameter Space

ERPS: $m_{11}^2 = m_{22}^2$, $m_{12}^2 = 0$, $\lambda_1 = \lambda_2$, and $\lambda_7 = -\lambda_6$ (5 dofs)

ERPS4: $\lambda_1 = \lambda_2$ and $\lambda_7 = -\lambda_6$ (8 dofs)

The ERPS4 corresponds to a softly-broken GCP2-symmetric scalar potential.

Theorem: If $\lambda_1 = \lambda_2$ and $\lambda_7 = -\lambda_6$, then a basis of scalar fields exists (which is not unique) such that $\lambda_6 = \lambda_7 = 0$ and $\lambda_5 \in \mathbb{R}$, corresponding to a softly-broken $\mathbb{Z}_2 \otimes \Pi_2$-symmetric scalar potential.

Exceptional features of the ERPS4

- If $\lambda_1 = \lambda_2$ and $\lambda_7 = -\lambda_6$ is satisfied in one scalar field basis, then it is satisfied in any choice of scalar field basis.

- Even when $\lambda_6 = \lambda_7 = 0$, the condition $\mathrm{Im}\big(\lambda_5^*[m_{12}^2]^2\big) \neq 0$ does not necessarily imply a CP-violating scalar potential.[3]

- Exact Higgs alignment holds *naturally* in the ERPS and can also be achieved in a significant region of the ERPS4.

- The custodial symmetric subregion of the ERPS4 can allow for a CP-even scalar that is degenerate in mass with $H^\pm$.

[3] In the ERPS where $m_{12}^2 = 0$, CP is automatically conserved by the scalar potential and vacuum.

The ERPS4 with $\mathrm{Im}\big(\lambda_5^*[m_{12}^2]^2\big) \neq 0$

Outside of the ERPS4, $\mathrm{Im}\big(\lambda_5^*[m_{12}^2]^2\big) \neq 0$ necessarily implies a CP-violating scalar potential. However, consider the following two cases:

- softly-broken $Z_2 \otimes \Pi_2$ with complex m_{12}^2 and $\beta = \frac{1}{4}\pi$.

  Softly-broken $Z_2 \otimes \Pi_2$ implies that λ_5 is real and nonzero. Hence, $\mathrm{Im}\big(\lambda_5^*[m_{12}^2]^2\big) \neq 0$. Nevertheless the scalar potential and vacuum are CP-conserving. This model possesses an unbroken GCP1′ symmetry,

  $$\mathrm{GCP1}': \qquad \Phi_1 \to \Phi_2^*, \qquad\qquad \Phi_2 \to \Phi_1^*$$

  which imposes the conditions $m_{11}^2 = m_{22}^2$, $\lambda_1 = \lambda_2$ and $\lambda_6 = \lambda_7$, consistent with the $Z_2 \otimes \Pi_2$ symmetry constraints (since $\beta = \frac{1}{4}\pi$ requires $m_{11}^2 = m_{22}^2$). However, no reality condition is imposed on m_{12}^2 or λ_5. Moreover, since $\langle\Phi_1^0\rangle = \langle\Phi_2^0\rangle$, the vacuum also respects the $\mathrm{GCP1}'$ symmetry!

- softly-broken GCP3 with complex m_{12}^2, for arbitrary $\tan\beta$.

  Softly broken GCP3 implies that $\lambda_5 = \lambda_1 - \lambda_3 - \lambda_4$ is real and nonzero. Hence, $\mathrm{Im}\big(\lambda_5^*[m_{12}^2]^2\big) \neq 0$. Nevertheless the scalar potential and vacuum are CP-conserving. One can construct the relevant GCP transformation that is preserved (its ugly!).

  However, it is easier to transform to the scalar field basis where the U(1)$\otimes\Pi_2$ symmetry is manifestly realized. In this basis, m_{12}^2 is still complex but $\lambda_5 = 0$. That is, in this basis $\mathrm{Im}\big(\lambda_5^*[m_{12}^2]^2\big) = 0$ and there is no possibility of spontaneous CP violation.

Remark:

In the ERPS4, an explicit CP-violating scalar potential arises if $s_{2\beta} \neq 0$, $\sin 2\xi \neq 0$, $m_{11}^2 \neq m_{22}^2$, and $\mathrm{Im}[m_{12}^2]^2 \neq 0$. If latter condition is replaced by $\mathrm{Im}[m_{12}^2]^2 = 0$, then spontaneous CP violation arises if $0 < |m_{12}^2| < \frac{1}{2}\lambda_5 v^2 s_{2\beta}$.

Higgs alignment without decoupling due to a (softly-broken) symmetry

Symmetry origin for exact Higgs alignment

In the Φ-basis, $\langle\Phi_1^0\rangle = vc_\beta/\sqrt{2}$ and $\langle\Phi_2^0\rangle = vs_\beta e^{i\xi}/\sqrt{2}$. The scalar potential parameters in this basis are related to the corresponding Higgs basis parameters; e.g.,

$$Y_3 = \left[\tfrac{1}{2}(m_{22}^2 - m_{11}^2)s_{2\beta} - \mathrm{Re}(m_{12}^2 e^{i\xi})c_{2\beta} - i\mathrm{Im}(m_{12}^2 e^{i\xi})\right]e^{-i\xi}\,.$$

If $m_{11}^2 = m_{22}^2$ and $m_{12}^2 = 0$, then $Y_3 = 0$, independently of the choice of β and ξ. The scalar potential minimum condition ($Y_3 = -\frac{1}{2}Z_6 v^2$) then yields $Z_6 = 0$, i.e. exact Higgs alignment.[5]

Note: if $Y_3 = 0$ by virtue of a particular choice of β and ξ, then the resulting model is simply the IDM.

[5]See, e.g., P.S. Bhupal Dev and A. Pilaftsis, JHEP **1412**, 024 (2014) [Erratum: JHEP **1511**, 147 (2015)].

Beyond the IDM, exact Higgs alignment arises in the ERPS, which can exhibit the following symmetries:

symmetry	m_{22}^2	m_{12}^2	λ_2	λ_4	λ_5	λ_6	λ_7
$\mathbb{Z}_2 \otimes \Pi_2$	m_{11}^2	0	λ_1		real	0	0
GCP2	m_{11}^2	0	λ_1				$-\lambda_6$
U(1)$\otimes\Pi_2$	m_{11}^2	0	λ_1		0	0	0
GCP3	m_{11}^2	0	λ_1		$\lambda_1 - \lambda_3 - \lambda_4$ (real)	0	0
SO(3)	m_{11}^2	0	λ_1	$\lambda_1 - \lambda_3$	0	0	0

As previously noted, $\mathbb{Z}_2 \otimes \Pi_2$ and U(1)$\otimes\Pi_2$ are not independent symmetries, since a change of basis can be performed in each case to a new basis in which the GCP2 and GCP3 symmetries, respectively, are manifestly realized.

However, it is remarkable that in many cases, exact alignment is preserved even if the above symmetries are softly broken. In all such cases, exact Higgs alignment is achieved in the *inert limit* where $Y_3 = Z_6 = Z_7 = 0$. A complete classification of 2HDM scalar potentials with Higgs alignment due to a symmetry has been obtained.

Approximate Higgs alignment due to soft symmetry breaking

In the models below, $Z_6 \neq 0$ but can be "naturally" small.

Scalar potentials with a softly-broken $\mathbb{Z}_2 \otimes \Pi_2$ symmetry

β	$\sin 2\xi$	$m_{11}^2,\, m_{22}^2$	m_{12}^2	CP-violation?	comment
$s_{2\beta} \neq 0$	$\neq 0$	$m_{11}^2 \neq m_{22}^2$	complex	explicit	$\mathrm{Im}\,[m_{12}^2]^2 \neq 0$
$s_{2\beta} \neq 0$	$\neq 0$	$m_{11}^2 \neq m_{22}^2$	$\mathrm{Im}\,[m_{12}^2]^2 = 0$	spontaneous	$0 < \lvert m_{12}^2 \rvert < \frac{1}{2}\lambda_5 v^2 s_{2\beta}$
$s_{2\beta} \neq 0$	$\neq 0$	$m_{11}^2 \neq m_{22}^2$	$\mathrm{Im}\,[m_{12}^2]^2 = 0$	no	$\lvert m_{12}^2 \rvert > \frac{1}{2}\lambda_5 v^2 s_{2\beta}$
$c_{2\beta} = 0$	$\neq 0$	$m_{11}^2 = m_{22}^2$	complex	no	$m_{12}^2 \neq 0$
$s_{2\beta} c_{2\beta} \neq 0$	0	$m_{11}^2 \neq m_{22}^2$	$\mathrm{Im}\,[m_{12}^2]^2 = 0$	no	

Scalar potentials with a softly-broken U(1)$\otimes \Pi_2$ symmetry

β	$m_{11}^2,\, m_{22}^2$	$m_{12}^2 e^{i\xi}$	R	comment
$s_{2\beta} c_{2\beta} \neq 0$	$m_{11}^2 \neq m_{22}^2$	> 0	$R \neq 1$	
$s_{2\beta} c_{2\beta} \neq 0$	$m_{11}^2 \neq m_{22}^2$	0	$\lvert R \rvert < 1$	$m_A^2 = 0$

$R \equiv (\lambda_3 + \lambda_4)/\lambda_1$ and $\lambda_5 = 0$.

Scalar potentials with a softly-broken GCP3 symmetry (primed parameters refer to the GCP3 basis)

β'	ξ'	$m_{11}'^2,\, m_{22}'^2$	$m_{12}'^2$	comment
$s_{2\beta'} c_{2\beta'} \neq 0$	$\sin 2\xi' \neq 0$	$m_{11}'^2 \neq m_{22}'^2$	complex	
$s_{2\beta'} c_{2\beta'} \neq 0$	$\cos \xi' = 0$	$m_{11}'^2 \neq m_{22}'^2$	pure imaginary	
$s_{2\beta'} c_{2\beta'} \neq 0$	$\sin 2\xi' \neq 0$	$m_{11}'^2 = m_{22}'^2$	pure imaginary ($\neq 0$)	$m_A^2 = 0$
$c_{2\beta'} = 0$	$\sin 2\xi' \neq 0$	$m_{11}'^2 = m_{22}'^2$	complex	$m_A^2 > 0$

Has natural approximate Higgs alignment been successfully achieved?

Only if the softly-broken symmetry can be extended to the entire model Lagrangian.

- The scalar gauge covariant kinetic energy term conserves CP and is U(2)-invariant, which contains all the family symmetries previously considered.

- The Yukawa interactions do not respect the GCP2 and GCP3 symmetries.
  - An attempt to extend softly-broken GCP2 and GCP3 symmetries to the Yukawa interactions (with three quark generations) in P.M. Ferreira and J.P. Silva, Eur. Phys. J. C **69**, 45 (2010) yielded phenomenologically unacceptable results.

- To extend the GCP2 and GCP3 symmetries to the Yukawa sector, we shall add vector-like top (and bottom) quark partners to the Standard Model. These symmetries will be broken softly by vector-like top mass parameters, which also provides a mechanism for generating $m_{11}^2 \neq m_{22}^2$ and $m_{12}^2 \neq 0$.

Extending the ERPS (and ERPS4) to the Yukawa sector

U(1)$\otimes\Pi_2$-symmetric 2HDM with vector-like fermions

The 2HDM with a GCP3-symmetric scalar potential can be realized in another basis as a $\mathrm{U}(1)\otimes\Pi_2$ symmetry, where

$$m_{11}^2 = m_{22}^2\,, \quad \lambda_1 = \lambda_2\,, \quad m_{12}^2 = \lambda_5 = \lambda_6 = \lambda_7 = 0\,.$$

To extend this symmetry to the Yukawa sector, we introduce vector-like fermions U and $\overline{U}$. SM two-component fermions are denoted by lower case letters (e.g. doublet fields $q = (u, d)$ with $Y = 1/3$ and singlet fields $\bar{u}$ with $Y = -4/3$); vector-like singlet two-component fermions by upper case letters. Note that $Y_{\bar{u}} = Y_{\bar{U}} = -Y_U$. Under the symmetries,[6]

symmetry	Φ_1	Φ_2	q	$\bar{u}$	$\overline{U}$	U
Π_2	Φ_2	Φ_1	q	$\overline{U}$	$\bar{u}$	U
U(1)	$e^{-i\theta}\Phi_1$	$e^{i\theta}\Phi_2$	q	$e^{-i\theta}\bar{u}$	$e^{-i\theta}\overline{U}$	$e^{\pm i\theta}U$

[6] The down-type quarks and leptons can also be included by introducing the appropriate vector-like fermions, in which case the Type-I, II, X or Y Higgs-fermion Yukawa couplings can be realized.

The Yukawa couplings consistent with the $\mathrm{U}(1) \otimes \Pi_2$ symmetry and the SU(2)×U(1)$_{\mathrm{Y}}$ gauge symmetry are

$$\mathscr{L}_{\mathrm{Yuk}} \supset y_t \left(q\Phi_2 \bar{u} + q\Phi_1 \overline{U} \right) + \mathrm{h.c.}$$

The model is not phenomenologically viable due to

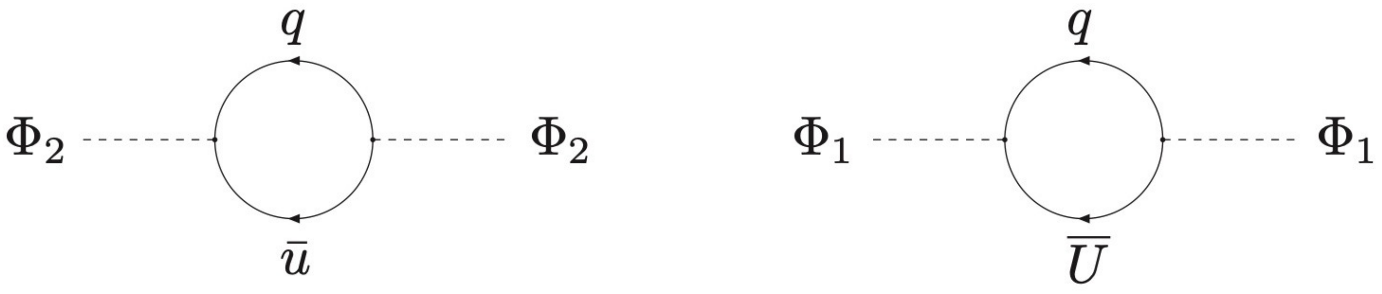
- experimental limits on vector-like fermion masses
- existence of a massless scalar if the global U(1) is spontaneously broken

Thus, we introduce SU(2)×U(1)$_{\mathrm{Y}}$ preserving mass terms,

$$\mathscr{L}_{\mathrm{mass}} \supset M_U \overline{U} U + M_u \bar{u} U + \mathrm{h.c.}$$

The U(1) symmetry is explicitly broken if $M_U M_u \neq 0$. The Π_2 discrete symmetry is also explicitly broken if $M_U \neq M_u$. The symmetry breaking is soft, so that corrections to the scalar potential squared-mass parameters are protected from quadratic sensitivity to the cutoff scale Λ of the theory.

Effects of the softly-broken symmetries



$$\Delta m^2 \equiv m_{22}^2 - m_{11}^2 \sim \kappa(M_U^2 - M_u^2) - \frac{3y_t^2(M_U^2 - M_u^2)}{4\pi^2} \ln(\Lambda/M) \,,$$

where $M \equiv (M_U^2 + M_u^2)^{1/2}$. The above result includes a finite threshold corrections proportional to κ. Note that when $M_U = M_u$, the Π_2 symmetry is unbroken and hence the relation $m_{11}^2 = m_{22}^2$ is protected. Likewise,

$$m_{12}^2 \sim \kappa_{12} M_U M_u + \frac{3y_t^2 M_U M_u}{4\pi^2} \ln(\Lambda/M) \,,$$

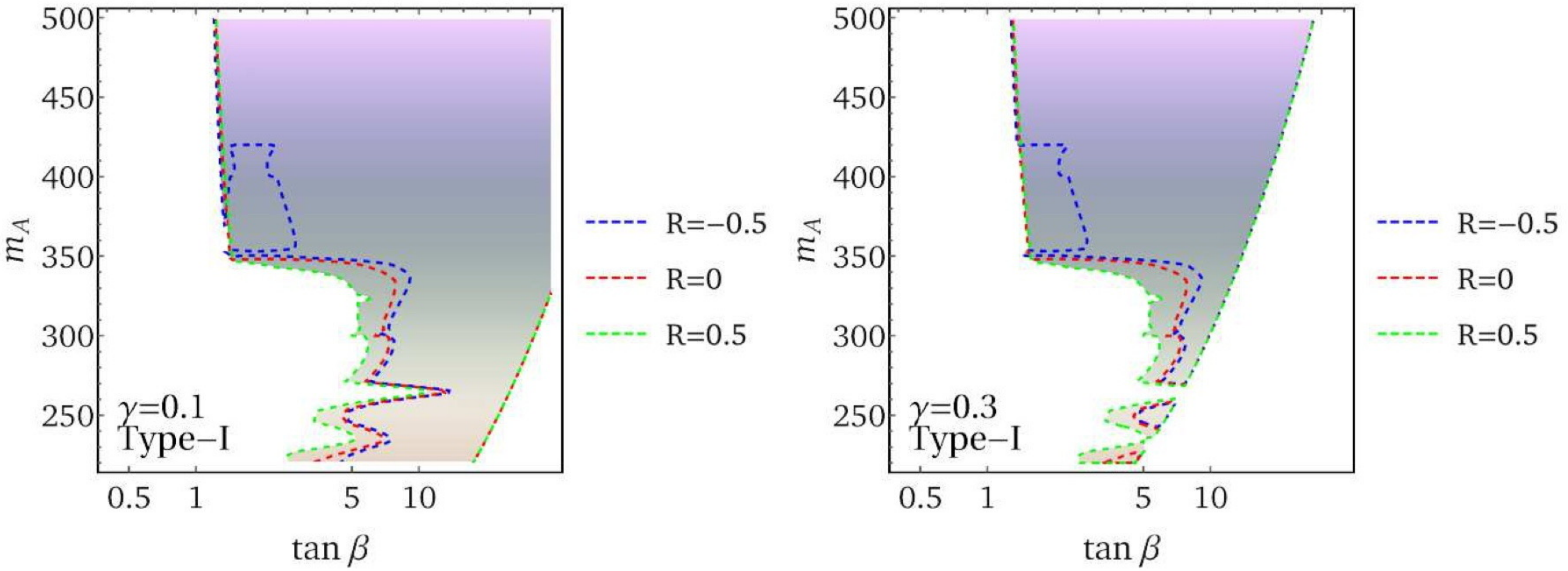
which includes a finite threshold corrections proportional to κ_{12}. In our numerical scans we chose $\ln(\Lambda/M) = 3$ and examined two benchmark points, $\gamma = 0.1$ and $\gamma = 0.3$, where $\tan\gamma \equiv M_u/M_U$.

Regions of approximate alignment without decoupling

In addition to a SM-like Higgs boson (consistent with LHC data), we have also imposed:

- Non-SM Higgs bosons in the parameter regime of Higgs alignment without decoupling should have so far evaded LHC detection.
- Constraints on the charged Higgs mass from flavor constraints in the Type-I 2HDM.
- Vectorlike top quark mass bounds [we chose $M_T \gtrsim 1.5$ TeV].
- Constraints on mixing between the top quark and its vectorlike fermion partner (the mixing is governed by the parameters γ, β, m_t and M_T).[7]
- Avoid excessive fine-tuning while keeping small the size of the effects due to the soft breaking of the U(1)$\otimes\Pi_2$ symmetry.

[7] See, e.g., A. Arhrib et al., Phys. Rev. D 97, 095015 (2018).



Regions allowed by experimental bounds and tuning constraints for different values of $R \equiv (\lambda_3 + \lambda_4)/\lambda$, with an m_{12}^2 and Δm^2 tuning of at most 5% [assuming that $\ln(\Lambda/M) = 3$]. Each panel shows three different R curves; the white regions of the parameter space are ruled out. The ruled out areas expand somewhat as R decreases, with the borders of the allowed shaded regions indicated by the corresponding contours. For $R = -0.5$, the area enclosed by the closed dashed blue contour in panel (a) is also ruled out. Type-I Yukawa couplings are employed and, two choices for γ are shown. Taken from P. Draper, A. Ekstedt and H.E. Haber, JHEP **05**, 235 (2021) [arXiv:2011.13159 [hep-ph]].

Note: The shrinking of the allowed parameter space as γ increases is due primarily to the behavior of the measure of fine-tuning of the parameter m_{12}^2.

Conclusions

Take home messages

- If an extended Higgs sector with additional "light" scalars exits, then one needs to understand why the observed Higgs boson is SM-like.

- Evidence for the additional Higgs scalars will first emerge either through their direct discovery or via the detection of deviations of the *h* couplings from their SM predictions.

- In the case of the IDM, the deviations of the *h* couplings from their SM predictions are radiatively induced and thus will be quite small.

- If additional Higgs scalars are found and deviations of *h* from its SM behavior are confirmed (which are too large to be compatible with the IDM), then a symmetry-based explanation for why the Higgs boson is SM-like would suggest the presence of new physics in the Yukawa sector that involves top quark fermionic partners (and perhaps partners to other quarks and leptons).

Backup Slides

More family and GCP symmetries of the 2HDM

Higgs family symmetries

Π_2' : $\quad \Phi_1 \to \Phi_2, \qquad \Phi_2 \to -\Phi_1$

U(1)$'$: $\quad \Phi_1 \to \Phi_1 c_\theta + \Phi_2 s_\theta \qquad \Phi_2 \to -\Phi_1 s_\theta + \Phi_2 c_\theta$

U(1)$''$: $\quad \Phi_1 \to \Phi_1 c_\theta + i\Phi_2 s_\theta \qquad \Phi_2 \to i\Phi_1 s_\theta + \Phi_2 c_\theta$

GCP transformations

$\mathrm{GCP1}'$: $\quad \Phi_1 \to \Phi_2^*, \qquad \Phi_2 \to \Phi_1^*$

GCP3$'$: $\quad \Phi_1 \to \Phi_1^* c_\theta - i\Phi_2^* s_\theta \qquad \Phi_2 \to i\Phi_1^* s_\theta - \Phi_2^* c_\theta\,,$ for any $0 < \theta < \frac{1}{2}\pi$

symmetry	m_{22}^2	m_{12}^2	λ_2	$\mathrm{Re}\lambda_5$	$\mathrm{Im}\lambda_5$	λ_6	λ_7
Π_2'	m_{11}^2	pure imaginary	λ_1		0		$-\lambda_6^*$
$\Pi_2 \otimes \Pi_2'$	m_{11}^2	0	λ_1		0	0	0
U(1)$'$	m_{11}^2	pure imaginary	λ_1	$\lambda_1 - \lambda_3 - \lambda_4$	0	0	0
U(1)$''$	m_{11}^2	real	λ_1	$\lambda_3 + \lambda_4 - \lambda_1$	0	0	0
U(1)$' \otimes \mathbb{Z}_2$	m_{11}^2	0	λ_1	$\lambda_1 - \lambda_3 - \lambda_4$	0	0	0
U(1)$'' \otimes \mathbb{Z}_2$	m_{11}^2	0	λ_1	$\lambda_3 + \lambda_4 - \lambda_1$	0	0	0
GCP1$'$	m_{11}^2		λ_1				λ_6
GCP3$'$	m_{11}^2	0	λ_1	$\lambda_3 + \lambda_4 - \lambda_1$	0	0	0

Note that $\Pi_2' \Longleftrightarrow \mathbb{Z}_2$ symmetry in a different basis; GCP1$' \Longleftrightarrow$ GCP1 in a different basis; GCP3$' \Longleftrightarrow$ GCP3 in a different basis. Moreover, the constraints on the scalar potential parameters due to the $\mathbb{Z}_2 \otimes \Pi_2$, GCP3 and GCP3$'$ symmetries coincide with those of the $\Pi_2 \otimes \Pi_2'$, U(1)$' \otimes \mathbb{Z}_2$ and U(1)$'' \otimes \mathbb{Z}_2$ symmetries, respectively.

Symmetry	soft-breaking	parameter constraints	residual unbroken symmetry of scalar potential	vacuum
$\mathbb{Z}_2$	none	$s_{2\beta} = 0$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
U(1)	none	$s_{2\beta} = 0$	U(1)	U(1)
$\mathbb{Z}_2 \otimes \Pi_2$	$m_{11}^2 \neq m_{22}^2$	$s_{2\beta} = 0$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
$\mathbb{Z}_2 \otimes \Pi_2$	$\mathrm{Re} m_{12}^2 \neq 0$	$c_{2\beta} = \sin\xi = 0$	Π_2	Π_2
$\mathbb{Z}_2 \otimes \Pi_2$	$\mathrm{Im} m_{12}^2 \neq 0$	$c_{2\beta} = \cos\xi = 0$	Π'_2	Π'_2
$\mathbb{Z}_2 \otimes \Pi_2$	none	$s_{2\beta} = 0$	$\mathbb{Z}_2 \otimes \Pi_2$	$\mathbb{Z}_2$
$\mathbb{Z}_2 \otimes \Pi_2$	none	$c_{2\beta} = \sin 2\xi = 0$	$\mathbb{Z}_2 \otimes \Pi_2$	Π_2
U(1)$\otimes\Pi_2$	$m_{11}^2 \neq m_{22}^2$	$s_{2\beta} = 0$	U(1)	U(1)
U(1)$\otimes\Pi_2$	$\mathrm{Re}(m_{12}^2 e^{i\xi}) \neq 0$	$c_{2\beta} = 0$	$\Pi_2^{(\xi)}$	$\Pi_2^{(\xi)}$
U(1)$\otimes\Pi_2$	none	$s_{2\beta} = 0$	U(1)$\otimes\Pi_2$	U(1)
U(1)$\otimes\Pi_2$	none	$c_{2\beta} = 0$	U(1)$\otimes\Pi_2$	Π_2

Part I of the classification of symmetries of the 2HDM scalar potential that yield exact Higgs alignment. Note that $m_{11}^2 = m_{22}^2$ and $\mathrm{Re}(m_{12}^2 e^{i\xi}) = \mathrm{Im}(m_{12}^2 e^{i\xi}) = 0$ unless otherwise indicated. In cases where the vacuum preserves a U(1) symmetry, $m_H = m_A \neq 0$. Since GCP3 is equivalent to $\mathrm{U}(1) \otimes \Pi_2$ when expressed in a different scalar field basis, there is a one-to-one mapping between the corresponding entries in this Table and the one that follows. Taken from H.E. Haber and J.P. Silva, Phys. Rev. D **103**, 115012 (2021).

Symmetry	soft-breaking	parameter constraints	residual unbroken symmetry of scalar potential	residual unbroken symmetry of vacuum
GCP3	$m_{11}'^2 \neq m_{22}'^2$, $\mathrm{Re}\, m_{12}'^2 \neq 0$	$s_{2\beta'}c_{2\beta'} \neq 0$, $\sin\xi' = 0$	$\overline{\Pi}_2^{(\alpha)}$	$\overline{\Pi}_2^{(\alpha)}$
GCP3	$m_{11}'^2 \neq m_{22}'^2$	$s_{2\beta'} = 0$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
GCP3	$\mathrm{Re}\, m_{12}'^2 \neq 0$	$c_{2\beta'} = 0$, $\sin\xi' = 0$	Π_2	Π_2
GCP3	$\mathrm{Im}\, m_{12}'^2 \neq 0$	$c_{2\beta'} = 0$, $\cos\xi' = 0$	U(1)$'$	U(1)$'$
GCP3	none	$s_{2\beta'} = 0$	U(1)$' \otimes \mathbb{Z}_2$	$\mathbb{Z}_2$
GCP3	none	$s_{2\beta'} \neq 0$, $\sin\xi' = 0$	U(1)$' \otimes \mathbb{Z}_2$	$\overline{\Pi}_2^{(\alpha)}$
GCP3	none	$c_{2\beta'} = 0$, $\cos\xi' = 0$	U(1)$' \otimes \mathbb{Z}_2$	U(1)$'$
SO(3)	$m_{11}'^2 \neq m_{22}'^2$, $\mathrm{Re}(m_{12}'^2 e^{i\xi'}) \neq 0$	$s_{2\beta'}c_{2\beta'} \neq 0$	U(1)$_{\rm H}$	U(1)$_{\rm H}$
SO(3)	$\mathrm{Re}(m_{12}'^2 e^{i\xi'}) \neq 0$	$c_{2\beta'} = 0$	U(1)$_{\rm H}$	U(1)$_{\rm H}$
SO(3)	$m_{11}'^2 \neq m_{22}'^2$	$s_{2\beta'} = 0$	U(1)	U(1)
SO(3)	none	none	SO(3)	U(1)$_{\rm H}$

Part II of the classification of symmetries of the 2HDM scalar potential that yield exact Higgs alignment. Note that $m_{11}'^2 = m_{22}'^2$ and $\mathrm{Re}(m_{12}'^2 e^{i\xi'}) = \mathrm{Im}(m_{12}'^2 e^{i\xi'}) = 0$ unless otherwise indicated, where the primed parameters correspond to the GCP3 scalar field basis. The symmetry group U(1)$_{\rm H}$ refers to a Peccei-Quinn U(1) symmetry that is manifestly realized in the Higgs basis. In cases where the vacuum preserves a U(1) symmetry, $m_H = m_A \neq 0$ (with the exception of the unbroken SO(3)-symmetric scalar potential where both H and A are massless). Taken from H.E. Haber and J.P. Silva, Phys. Rev. D **103**, 115012 (2021).

What is natural Higgs alignment?

In P.S. Bhupal Dev and A. Pilaftsis, JHEP **1412**, 024 (2014), "natural" alignment is defined as Higgs alignment due to a symmetry that is independent of $\tan\beta$. That is, the solution to $Y_3 = Z_6 = 0$ should not depend on $\tan\beta$,

$$Z_6 = \Big\{-\tfrac{1}{2}s_{2\beta}\left[\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345}c_{2\beta} - i\mathrm{Im}(\lambda_5 e^{2i\xi})\right] + c_\beta c_{3\beta}\mathrm{Re}(\lambda_6 e^{i\xi})$$

$$+ s_\beta s_{3\beta}\mathrm{Re}(\lambda_7 e^{i\xi}) + ic_\beta^2\mathrm{Im}(\lambda_6 e^{i\xi}) + is_\beta^2\mathrm{Im}(\lambda_7 e^{i\xi})\Big\}e^{-i\xi}\,,$$

where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \mathrm{Re}(\lambda_5 e^{2i\xi})$.

Application: GCP3-conserving scalar potential with $m_{11}^2 = m_{22}^2$, $m_{12}^2 = 0$, $\lambda_1 = \lambda_2 = \lambda_3 + \lambda_4 + \mathrm{Re}\lambda_5$, and $\mathrm{Im}\lambda_5 = \lambda_6 = \lambda_7 = 0$, produces

$$Z_6 = i\lambda_5 s_{2\beta}\sin\xi e^{-i\xi}\big(\cos\xi + ic_{2\beta}\sin\xi\big)\,.$$

The potential minimum conditions yield $s_{2\beta}\sin 2\xi = c_{2\beta}\sin^2\xi = 0$, which implies that $\sin\xi = 0$ for arbitrary β or $\cos\xi = 0$ for $\beta = \frac{1}{4}\pi$. Bhupal Dev and Pilaftsis assumed that $\sin\xi = 0$, implying $Z_6 = 0$ independently of $\tan\beta$.

Transforming to the U(1)$\otimes\Pi_2$ basis, the corresponding minimum conditions of the scalar potential yield $s_{2\beta}c_{2\beta}=0 \iff \beta=0, \frac{1}{4}\pi$ or $\frac{1}{2}\pi$. For these values, $Y_3=Z_6=0$. Is this still an example of "natural" alignment?

Likewise, $Z_6=0$ after applying the minimum conditions for the GCP2 and $\mathbb{Z}_2\otimes\Pi_2$-conserving scalar potentials, which are not viewed by Bhupal Dev and Pilaftsis as examples of "natural" alignment.

I believe that what Bhupal Dev and Pilaftsis really meant by "natural" alignment is that $Y_3=Z_6=0$ *independently of the scalar potential minimum conditions*. With this definition, neither the GCP2 nor GCP3-conserving scalar potentials exhibit natural alignment. Only the SO(3)-conserving scalar potential (i.e., GCP3 with $\lambda_5=0$) would qualify.

I prefer the term "natural alignment" to imply Higgs alignment as a consequence of a symmetry (which may be softly broken). That is, naturalness in the sense of 't Hooft, where a symmetry is enlarged when a parameter is set to zero.

Equivalence of $\mathbb{Z}_2 \otimes \Pi_2$ and GCP2 symmetries of the 2HDM

The 2HDM scalar potential in the Φ-basis can be written as,

$$\mathcal{V}(\Phi) = Y_{ab}(\Phi_a^\dagger \Phi_b) + \tfrac{1}{2} Z_{ac,bd}(\Phi_a^\dagger \Phi_b)(\Phi_c^\dagger \Phi_d).$$

Define a three-vector whose components P_B (for $B = 1, 2, 3$) are given by

$$P_B = \tfrac{1}{4}(Z_{ab,cd} + \overline{Z}_{ab,cd})\delta_{ca}\sigma^B_{db} = \begin{pmatrix} \mathrm{Re}(\lambda_6+\lambda_7) & -\mathrm{Im}(\lambda_6+\lambda_7) & \tfrac{1}{2}(\lambda_1-\lambda_2) \end{pmatrix},$$

and a 3×3 real symmetric matrix whose elements D_{AB} are given by

$$D_{AB} = \tfrac{1}{4}(Z_{ab,cd} + \overline{Z}_{ab,cd})\sigma^A_{ca}\sigma^B_{db} - \tfrac{1}{12}(Z_{ab,ab} + \overline{Z}_{ab,ab})\delta^{AB}$$

$$= \begin{pmatrix} -\tfrac{1}{3}\Delta + \mathrm{Re}\,\lambda_5 & -\mathrm{Im}\,\lambda_5 & \mathrm{Re}\,(\lambda_6-\lambda_7) \\ -\mathrm{Im}\,\lambda_5 & -\tfrac{1}{3}\Delta - \mathrm{Re}\,\lambda_5 & -\mathrm{Im}\,(\lambda_6-\lambda_7) \\ \mathrm{Re}\,(\lambda_6-\lambda_7) & -\mathrm{Im}\,(\lambda_6-\lambda_7) & \tfrac{2}{3}\Delta \end{pmatrix},$$

where $\Delta \equiv \tfrac{1}{2}(\lambda_1+\lambda_2) - \lambda_3 - \lambda_4$ and $\overline{Z}_{ab,cd} \equiv Z_{ba,cd} = Z_{ab,dc}$.

Under a change of scalar field basis, $\Phi \to \Phi' = V\Phi$ (where V is unitary),

$$P_B \to P'_B = \mathcal{R}_{BD} P_D \,, \qquad D_{AB} \to D'_{AB} = \mathcal{R}_{AC}\mathcal{R}_{BD} D_{CD} = (\mathcal{R} D \mathcal{R}^{\mathsf{T}})_{AB} \,,$$

after employing the identity $V^\dagger \sigma^A V = \mathcal{R}_{AB}\sigma^B$, where $\mathcal{R}$ is a real orthogonal matrix that is explicitly given by $\mathcal{R}_{AB} = \frac{1}{2}\mathrm{Tr}(V^\dagger \sigma^A V \sigma^B)$.

Theorem: If $\lambda_1 = \lambda_2$ and $\lambda_7 = -\lambda_6$ in the Φ-basis, then there exists a Φ'-basis, defined by $\Phi' = U\Phi$, in which $\lambda'_1 = \lambda'_2$ and $\mathrm{Im}\lambda'_5 = \lambda'_6 = \lambda'_7 = 0$.

If $\lambda_1 = \lambda_2$ and $\lambda_7 = -\lambda_6$ in the Φ-basis [GCP2 symmetry] then it follows that $P = 0$. Moreover, D is a real traceless symmetric matrix, which can always be transformed into a real diagonal matrix via an orthogonal similarity transformation. Thus, there exists a real orthogonal matrix $\mathcal{R}$ such that

$$P' = \mathcal{R}P = 0 \quad \text{and} \quad D' = \mathcal{R} D \mathcal{R}^{\mathsf{T}} \text{ is diagonal.}$$

Noting the explicit forms of P and D previously given, it follows that $\lambda'_1 = \lambda'_2$ and $\mathrm{Im}\lambda'_5 = \lambda'_6 = \lambda'_7 = 0$ in the Φ'-basis [$\mathbb{Z}_2 \otimes \Pi_2$ symmetry].

Translation between the U(1)$\otimes\Pi_2$ basis and the GCP3 basis

Consider the following unitary basis transformation, $\Phi \to \Phi' = V\Phi$, where

$$V = \frac{e^{i\phi}}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}, \qquad \text{where } e^{i\phi} = \frac{c_\beta + i s_\beta e^{-i\xi}}{(1 + s_{2\beta}\sin\xi)^{1/2}}.$$

Starting from the U(1)$\otimes\Pi_2$-basis,

$$\begin{aligned}
\lambda' = \lambda'_1 = \lambda'_2 &= \tfrac{1}{2}\lambda(1+R)\,, \\
\lambda'_3 &= \lambda_3 + \tfrac{1}{2}\lambda(1-R)\,, \\
\lambda'_4 &= \lambda_4 + \tfrac{1}{2}\lambda(1-R)\,, \\
\lambda'_5 &= -\tfrac{1}{2}\lambda(1-R)\,, \\
\lambda'_6 &= -\lambda'_7 = 0\,,
\end{aligned}$$

where $R \equiv (\lambda_3 + \lambda_4)/\lambda$. In particular, $\lambda'_5 = \lambda' - \lambda'_3 - \lambda'_4$ is real and $\lambda'_6 = \lambda'_7 = 0$, corresponding to the GCP3 basis.

The corresponding soft-breaking squared mass parameters are,

$$m'^{\,2}_{11} = \tfrac{1}{2}(m^2_{11} + m^2_{22}) + {\rm Im}m^2_{12}\,,$$

$$m'^{\,2}_{22} = \tfrac{1}{2}(m^2_{11} + m^2_{22}) - {\rm Im}m^2_{12}\,,$$

$$m'^{\,2}_{12} = {\rm Re}m^2_{12} + \tfrac{1}{2}i(m^2_{22} - m^2_{11})\,.$$

The vevs, $v'_1 \equiv vc_{\beta'}$ and $v'_2 \equiv vs_{\beta'}$ are real and positive,

$$c_{\beta'} = \frac{1}{\sqrt{2}}\bigl(1 + s_{2\beta}\sin\xi\bigr)^{1/2}\,, \qquad s_{\beta'} = \frac{1}{\sqrt{2}}\bigl(1 - s_{2\beta}\sin\xi\bigr)^{1/2}\,,$$

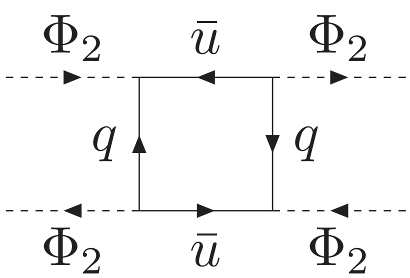
which yields, $s^2_{2\beta'} = 1 - s^2_{2\beta}\sin^2\xi$. Likewise, the relative phase angle, ξ' is given by

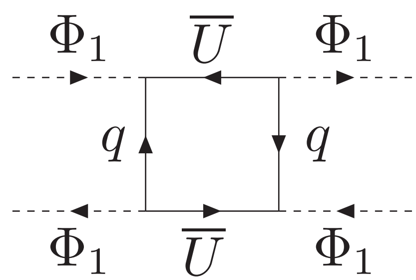
$$\sin\xi' = \frac{-c_{2\beta}}{(1 - s^2_{2\beta}\sin^2\xi)^{1/2}}\,, \qquad \cos\xi' = \frac{s_{2\beta}\cos\xi}{(1 - s^2_{2\beta}\sin^2\xi)^{1/2}}\,.$$

Finally, if $\beta = \frac{1}{4}\pi$ and $\sin\xi = \pm 1$, then one of the vevs vanishes. It then follows that $s_{2\beta'} = 0$, in which case ξ' is indeterminate if $s_{\beta'} = 0$ and $\xi' = 0$ if $c_{\beta'} = 0$.

Small corrections to the ERPS4 conditions

Integrating out the vector-like fermions below the scale M, one generates a small splitting between λ_1 and λ_2 and nonzero values of $\lambda_{5,6,7}$. For example, above the scale M, the diagrams





contribute equally to $\lambda_2(\Phi_2^\dagger\Phi_2)^2$ and $\lambda_1(\Phi_1^\dagger\Phi_1)^2$, respectively. Below the scale M, the diagrams with internal U lines decouple, which then yields

$$\Delta\lambda \equiv |\lambda_1 - \lambda_2| \sim \frac{3y_t^4}{4\pi^2}\left(\frac{M_U^2 - M_u^2}{M_U^2 + M_u^2}\right)\log(M/m_t) \sim \mathcal{O}(0.1)\,,$$

for $M \sim \mathcal{O}(1\ \mathrm{TeV})$. This is a small correction, which in first approximation can be neglected in our analysis. Likewise, explicit breaking of the U(1) symmetry will generate small nonzero values of λ_5, λ_6 and λ_7.

Top quark–vectorlike quark mixing

After electroweak symmetry breaking, the fermion mass eigenstates are obtain by Takagi-diagonalization of the following 4×4 mass matrix.

$$-\mathscr{L}_{\rm mass} = \tfrac{1}{2}(u \quad U \quad \bar{u} \quad \overline{U}) \begin{pmatrix} 0 & 0 & Y s_\beta & Y c_\beta \\ 0 & 0 & M_u & M_U \\ Y s_\beta & M_u & 0 & 0 \\ Y c_\beta & M_U & 0 & 0 \end{pmatrix} \begin{pmatrix} u \\ U \\ \bar{u} \\ \overline{U} \end{pmatrix} + \text{h.c.}\,,$$

where $Y \equiv y_t v/\sqrt{2}$. States with the same electric charge, i.e. $\{u, U\}$ and $\{\bar{u}, \overline{U}\}$, can separately mix (with mixing angles θ_L and θ_R, respectively). This yields two Dirac fermions—the top quark t and its vector-like top partner T, with corresponding masses and mixing angles (assuming $m_t \ll M_T$),

$$m_t \simeq Y|s_{\beta-\gamma}| \left(1 - \frac{Y}{M} c_{\beta-\gamma}\right)\,, \qquad M_T \simeq M \left[1 + \frac{m_t^2}{2M^2} \cot^2(\beta-\gamma)\right]\,,$$

$$\theta_L \simeq \frac{m_t}{M_T} |\cot(\beta-\gamma)|\,, \qquad \theta_R \simeq \gamma + \frac{m_t^2}{M_T^2} \cot(\beta-\gamma)\,.$$

The Higgs sector of the softly-broken U(1)$\otimes\Pi_2$-symmetric 2HDM

The important parameters of the scalar potential are:

$$m^2 \equiv \tfrac{1}{2}(m_{11}^2 + m_{22}^2)\,, \qquad \Delta m^2 \equiv m_{22}^2 - m_{11}^2\,, \qquad R \equiv \frac{\lambda_3+\lambda_4}{\lambda}\,, \qquad m_{12}^2\,,$$

with $\lambda \equiv \lambda_1 = \lambda_2$ and $\lambda_5 = \lambda_6 = \lambda_7 = 0$. We impose $\lambda > 0$ and $R > -1$ to ensure that the vacuum is bounded from below. Solving for the potential minimum yields,

$$2m^2 = \bar{m}^2 - \tfrac{1}{2}\lambda v^2(1+R)\,, \qquad \Delta m^2 = \epsilon\left(\bar{m}^2 + \tfrac{1}{2}\lambda v^2(1-R)\right)\,,$$

where $\bar{m}^2 \equiv 2m_{12}^2/\sin 2\beta$ and

$$\tan\beta \equiv \frac{v_2}{v_1} = \sqrt{\frac{1-\epsilon}{1+\epsilon}}\,, \qquad \text{where} \quad \epsilon \equiv \cos 2\beta\,.$$

The positivity of v_1^2 and v_2^2 requires $|\epsilon| < 1$.

Approximate alignment without decoupling

The relevant Higgs basis parameters are given by,

$$Z_1 = \tfrac{1}{2}\lambda\left[1 + R + \epsilon^2(1 - R)\right] ,$$

$$m_A^2 + Z_5 v^2 = 2m^2 + \lambda v^2\left[1 - \tfrac{1}{2}\epsilon^2(1 - R)\right] ,$$

$$Z_6 = \tfrac{1}{2}\lambda(R - 1)\epsilon\sqrt{1 - \epsilon^2} ,$$

Approximate alignment without decoupling requires that $|Z_6| \ll 1$ and $m^2 \sim \mathcal{O}(v^2)$. To avoid $\tan\beta$ very large or very small, we consider two limiting cases: $|\epsilon| \ll 1$ and $|R - 1| \ll 1$.

In the limit of $|\epsilon| \ll 1$,

$$m_h^2 = \tfrac{1}{2}\lambda v^2(1 + R) , \qquad m_H^2 = 2m^2 + \lambda v^2 , \qquad c_{\beta-\alpha} = \frac{\lambda v^2(1 - R)\epsilon}{4m^2 + \lambda v^2(1 - R)} .$$

In the limit of $|R - 1| \ll 1$,

$$m_h^2 = \lambda v^2 , \qquad m_H^2 = 2m^2 + \lambda v^2 , \qquad c_{\beta-\alpha} = \frac{\lambda v^2(1 - R)\epsilon\sqrt{1 - \epsilon^2}}{4m^2} .$$