SCIPP 91/06
March, 1991

LECTURES ON ELECTROWEAK SYMMETRY BREAKING⋆

Howard E. Haber
Santa Cruz Institute for Particle Physics
University of California, Santa Cruz, CA 95064

Abstract

In order to account for the existence of mass for the $W^{\pm}$ and Z gauge bosons and the quarks and charged leptons, the gauge symmetry of the electroweak interactions must be spontaneously broken. However, it is presently unknown how nature chooses to implement the mechanism of electroweak symmetry breaking. Although the elementary Higgs boson is the simplest manifestation of this mechanism, future experimental searches must be prepared for all eventualities. These lectures describe both theoretical and phenomenological aspects of electroweak symmetry breaking and the attempt to uncover its secrets in the next generation of particle physics experiments.

Invited Lectures presented at the Theoretical Advanced Study Institute
(TASI-90) Boulder, Colorado, June, 1990

⋆ Work supported, in part, by the Department of Energy.

Contents

Preface

After one year of running at LEP and roughly half a million Z^0 events accumulated, the Standard Model of particle physics continues to provide a remarkably detailed and complete description of all observed high energy physics phenomena. Yet, the final chapter of the story of the Standard Model is not yet written. The top quark remains to be discovered; present limits from the CDF Collaboration imply $m_t > 89$ GeV. The t-quark is probably just a mundane detail; its discovery simply requires higher luminosity or higher energy from our collider facilities. (Nevertheless, the existence of such a heavy t-quark is intriguing and will play an important role at various times in these lectures.) The final piece of the Standard Model puzzle which remains to be uncovered is the mechanism for electroweak symmetry breaking. These lectures will address what we know now and what we hope to learn about the origins of electroweak symmetry breaking. The central goal of particle physics in the 1990s and beyond is to uncover and elucidate the mechanism by which the W and Z (and fermions) get their mass. By conducting experiments that can probe the energy scale between 100 GeV and 1 TeV, there is an expectation that the secrets of electroweak symmetry breaking can be uncovered. Moreover, there are a number of theoretical arguments that strongly suggest that this endeavor will also lead to the first hints of deviations from the Standard Model. Thus, the exploration of the origins of electroweak symmetry breaking may reveal new phenomena with far reaching implications for future theories of particle physics.

These lectures are organized into six parts. In the first lecture, I will provide an introduction to electroweak symmetry breaking (ESB). The discussion here will be a little more general and formal than the introductory material given in most particle physics textbooks. However, the insight obtained in Lecture 1 will serve us well in future lectures. The simplest model incorporating ESB is the Standard Model with one weak doublet of elementary scalar (Higgs) fields. Lecture 2 will discuss in detail the theoretical properties of the (minimal) Higgs boson—its coupling to matter and gauge bosons, and expectations for its mass. Based on the theoretical properties elucidated in Lecture 2, a detailed description of the phenomenology of the Higgs boson is given in Lecture 3. There, I will summarize the present experimental limits on the Higgs boson mass, and discuss the prospects for discovering the Higgs boson at present and future colliders. Lecture 4 goes beyond the minimal Higgs model. The theory and phenomenology of a two-Higgs-doublet model is explored in detail, and some aspects of other non-minimal Higgs sectors are mentioned briefly. The most compelling argument for the two-Higgs-doublet model may be in the context of a supersymmetric extension of the Standard Model. The Higgs sector of the minimal supersymmetric model is examined in

Lecture 5; this provides one example of the possible connection between the origin of ESB and the necessity for physics beyond the Standard Model. Finally, in Lecture 6, I discuss an alternative approach to ESB in which the symmetry breaking is induced by some dynamical mechanism other than the generation of a nonzero vacuum expectation value for some elementary scalar field. The prospects for a successful theoretical approach of this kind are considered. The lectures end with some final thoughts on the implications of the search for the origins of electroweak symmetry breaking.

Much of the material presented in these lectures is treated in a recently published book, *The Higgs Hunter's Guide* (Addison-Wesley Publishing Company, Redwood City, CA, 1990), by John F. Gunion, Howard E. Haber, Gordon Kane and Sally Dawson. I am grateful to my co-authors for the wisdom that I gained from them during our collaboration, and I am pleased to be able to share some of their insights in these lectures. On occasion, I will refer the reader to additional information contained in this book (henceforth to be called the HHG). The HHG also contains a comprehensive list of references to the original literature. Therefore, I will not attempt to provide a complete bibliography here. Instead, at the end of each lecture, I will simply provide some suggestions for further reading, and list some of the key sources which treat the subject matter discussed in the lecture.

I would like to thank Paul Langacker for organizing such a stimulating and enjoyable summer school, and for the hospitality that he and the local TASI organizing committee provided during my stay in Boulder. I am grateful for helpful comments on Lecture 5 from Lance Dixon and on Lecture 6 from Tatsu Takeuchi. Finally, I greatly appreciated the interest and the perceptive questions of the TASI-90 students, which contributed greatly to my enjoyment in giving these lectures.

1. Electroweak Symmetry Breaking—An Introduction

1.1 Consistent Quantum Field Theories with Massive Vector Bosons

We begin by reviewing the components of the Standard Model. The elementary fields of this model (which have been observed to date in nature) are: spin 1/2 matter fields (quarks and leptons) and spin 1 gauge fields (gluons, γ, $W^\pm$ and Z). The theoretical structure which combines all these elements is renormalizable quantum field theory (QFT). At this point, I shall quote a theorem [due to Cornwall, Levin, and Tiktopoulos] which states that quantum field theories involving spin 1 gauge bosons are inconsistent (in perturbation theory) unless they belong to one of the following classes:

(*i*) massless U(1) gauge theories (*e.g.*, QED)

(*ii*) massive U(1) vector boson theories

(*iii*) non-abelian gauge theories.

or some combination of the above. Since the term $m^2 A^a_\mu A^{ua}$ violates gauge invariance, (iii) apparently describes a massless theory. Inserting such a mass term would result in a non-renormalizable, non-unitary theory.

How can I write down a consistent QFT containing the $W^\pm$ and Z? Indeed, the absence of $m^2 A^a_\mu A^{ua}$ means zero tree-level masses. Perhaps I can generate masses with quantum mechanical (loop) corrections. Unfortunately, I can (apparently) prove that vector boson mass generation is impossible. The "proof" goes as follows. Consider the generic Lagrangian for a non-abelian gauge field theory:

$$\mathcal{L} = -\tfrac{1}{4} F^a_{\mu v} F^{\mu v a} - \frac{1}{2\xi}(\partial_\mu A^\mu_a)^2 - \eta^{a*}\partial^\mu D^{ab}_\mu \eta^b \,. \tag{1.1}$$

Here, the η and η^* fields are Faddeev-Popov ghosts, and ξ is the gauge-fixing parameter. For simplicity, I have omitted matter (spin 0 and spin-1/2) fields, although they can be easily included. Although the gauge-fixing term violates the gauge invariance of the theory, the addition of the Faddeev-Popov term restores a more general gauge invariance called BRS-invariance. Under the BRS transformation, the fields of the model transform as follows

$$\begin{aligned} \delta_{\rm BRS} A^a_\mu(x) &= \theta D^{ab}_\mu \eta^b(x) \\ \delta_{\rm BRS} \eta^a(x) &= -\tfrac{1}{2}\theta g f^{abc}\eta^b\eta^c \\ \delta_{\rm BRS} \eta^{a*}(x) &= -\frac{\theta}{\xi}\,\partial^\mu A^a_\mu\,, \end{aligned} \tag{1.2}$$

where θ is an anti-commuting parameter ($\theta^2 = 0$), and D is the covariant derivative

$$D_\mu^{ab} = \delta^{ab}\partial_\mu + gf^{abc}A_\mu^c\,. \tag{1.3}$$

In particular, if we define the action to be

$$S = \int d^4x\,\mathcal{L}\,, \tag{1.4}$$

then the equation of motion for the η^* field is

$$\frac{\delta S}{\delta \eta_a^*} = -\partial^\mu D_\mu^{ab}\eta^b = 0\,, \tag{1.5}$$

which implies that

$$\delta_{\rm BRS}(\partial_\mu A_a^\mu) = -\theta\frac{\partial S}{\delta \eta_a^*}\,. \tag{1.6}$$

Consider the following Green's function

$$\langle 0|\,T\partial_\mu A_a^\mu(x)\eta_b^*(y)\,|0\rangle \equiv N^{-1}\int \mathcal{D}A_\mu\mathcal{D}\eta\mathcal{D}\eta^*\partial_\mu A_a^\mu(x)\eta_b^*(y)e^{iS}\,, \tag{1.7}$$

where

$$N \equiv \langle 0|0\rangle = \int \mathcal{D}A_\mu\mathcal{D}\eta\mathcal{D}\eta^* e^{iS} \tag{1.8}$$

is a field independent normalization factor. It is important to note that the right hand side of eq. (1.7) constitutes a definition of the T-product. This definition differs slightly from the conventional definition in that I am free to move the partial derivative ∂_μ outside the T-product without generating new terms. However, with this definition one cannot invoke the equations of motion on an operator which appears inside the T-product. The reason is clear: on the right hand side, we functionally integrate over all field configurations, and not just those that satisfy the equations of motion.

Since both the measure and the action S are BRS-invariant, I immediately get

$$\langle 0|\,T\delta_{\rm BRS}\left[\partial_\mu A_a^\mu(x)\right]\eta_b^*(y)\,|0\rangle + \langle 0|\,T\partial_\mu A_a^\mu(x)\delta_{\rm BRS}\;\eta_b^*(y)\,|0\rangle = 0\,. \tag{1.9}$$

Using eqs. (1.2) and (1.6), it follows that

$$\begin{aligned}\langle 0|\,T\partial_\mu A_a^\mu(x)\partial_\nu A_b^\nu(y)\,|0\rangle &= -\xi\,\langle 0|\,T\frac{\delta S}{\delta\eta_a^*(x)}\eta_b^*(y)\,|0\rangle \\ &= -i\xi\delta_{ab}\delta^4(x-y).\end{aligned} \tag{1.10}$$

The last line is a consequence of the following result that is obtained by a functional integration by parts

$$
\begin{aligned}
0 &= N^{-1} \int \mathcal{D}\Phi \frac{\delta}{\delta\Phi(x)} \left[\Phi(y) e^{iS[\Phi]}\right] \\
&= \delta^4(x-y) + i \int \mathcal{D}\Phi \frac{\delta S}{\delta\Phi(x)} \; \Phi(y) e^{iS[\Phi]} .
\end{aligned}
\tag{1.11}
$$

If we define the vector boson two-point function by

$$
\begin{aligned}
G^{ab}_{\mu\nu}(x-y) &\equiv \langle 0| \, A^a_\mu(x) A^b_\nu(y) \, |0\rangle \\
&= \int \frac{d^4p}{(2\pi)^4} \, e^{-ip(x-y)} G^{ab}_{\mu\nu}(p,-p) ,
\end{aligned}
\tag{1.12}
$$

then eq. (1.10) implies the following Ward Identity

$$
p^\mu p^\nu G^{ab}_{\mu\nu}(p,-p) = -i\xi\delta^{ab} . \tag{1.13}
$$

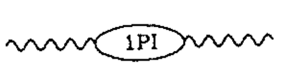
Let us consider the implications of this Ward identity. First, note that this result is satisfied by the exact two-point function of the theory. (Although we shall ignore the subtleties of renormalization, one can show that this identity must also apply to the renormalized Green's function as well.) Writing $G^{ab}_{\mu\nu}(p,-p) \equiv \delta^{ab} G_{\mu\nu}(p)$, and recalling the tree-level vector boson propagator

$$
G^0_{\mu\nu}(p) = \frac{-i}{p^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) - \frac{i\xi p_\mu p_\nu}{p^4} , \tag{1.14}
$$

it follows from eq. (1.13) that

$$
p^\mu p^\nu G_{\mu\nu}(p) = p^\mu p^\nu G^0_{\mu\nu}(p) = -i\xi . \tag{1.15}
$$

Thus, the longitudinal part of the vector-boson propagator is not renormalized. Moreover, as a consequence of the above analysis (*i.e.*, due to the BRS invariance and Lorentz invariance of the theory), one can express the one-particle-irreducible (1PI) vector-boson two-point function in the following form



$$
= i\Pi_{\mu\nu}(p) \equiv i(p_\mu p_\nu - p^2 g_{\mu\nu})\Pi(p^2) . \tag{1.16}
$$

Diagrammatically, $i\Pi_{\mu\nu}(p)$ is the sum of all 1PI Feynman diagrams with two external vector boson legs. The full two-point function can then be obtained by summing a geometric series, where the nth term of the series contains the 1PI bubble n times. The result is

$$G_{\mu\nu}(p) = \frac{-i\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)}{p^2[1+\Pi(p^2)]} - \frac{i\xi p_\mu p_\nu}{p^4}\,. \tag{1.17}$$

Observe that the pole at $p^2 = 0$ in the tree-level propagator [eq. (1.14)] is not shifted in the exact two-point function. Hence, the gauge boson remains massless even in the presence of interactions. This result is the origin of the (incorrect) statement which is sometimes made which states that the photon is massless due to gauge invariance.

The loophole in the above argument was first discovered by Schwinger. Consider $\Pi(p^2)$ in the limit of $p^2 \to 0$. Suppose

$$\Pi(p^2) \underset{p^2\to 0}{\simeq} \frac{-g^2v^2}{p^2}\,. \tag{1.18}$$

Then $p^2[1+\Pi(p^2)] = p^2 - g^2v^2$, and we see that the pole of $G_{\mu\nu}(p)$ is no longer at $p^2 = 0$; it has been shifted to $p^2 = g^2v^2$, which is the mass of the gauge boson! Is the behavior of $\Pi(p^2)$ exhibited in eq. (1.18) possible? Yes! Since $i\Pi_{\mu\nu}(p)$ is the sum of all 1PI diagrams, the behavior $\Pi(p^2) \simeq 1/p^2$ means that we can "cut" the 1PI diagram and expose a massless scalar excitation in the sum over intermediate states. This is the massless Goldstone boson! The mass generation mechanism just exhibited is called the Higgs mechanism. I shall now illustrate it with two simple examples.

1.2 Vector Boson Mass Generation

Consider first an abelian gauge theory coupled to a complex elementary scalar field (sometimes called scalar QED). This model is well treated in the standard textbooks. The Lagrangian for the model is

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\Phi)^*(D^\mu\Phi) - \lambda\left(|\Phi|^2 - \tfrac{1}{2}v^2\right)^2 - \frac{1}{2\xi}(\partial_\mu A^\mu)^2\,, \tag{1.19}$$

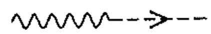
where $D_\mu \equiv \partial_\mu + ieA_\mu$ and $\Phi \equiv \sqrt{\tfrac{1}{2}}(\phi + i\chi)$. Note that the classical potential is minimized when $|\Phi| = v/\sqrt{2}$. For definiteness, assume that the potential minimum in field space points in the direction of ϕ. Then, we shift the fields: $\phi \to \phi + v$, $\chi \to$

χ so that $\langle\phi\rangle = \langle\chi\rangle = 0$. Expanding around the shifted vacuum, the Lagrangian can be rewritten as

$$\begin{aligned}\mathcal{L} = & -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \tfrac{1}{2}\left[(\partial_\mu\phi)^2 + (\partial_\mu\chi)^2\right] + \tfrac{1}{2}m_\phi^2\phi^2 \\ & + \tfrac{1}{2}e^2v^2 A_\mu A^\mu + evA_\mu\partial^\mu\chi + \text{ scalar interactions}\end{aligned} \tag{1.20}$$

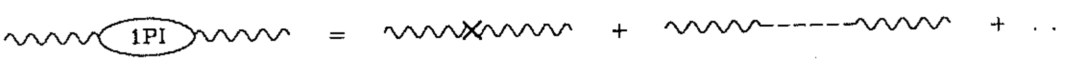
where $m_\phi^2 = 2\lambda v^2$. Note that no mass term for χ appears, *i.e.* $m_\chi = 0$. Consider the calculation of $\Pi_{\mu\nu}$ in this model. Normally, one must go to one-loop in order to obtain the first contribution to $\Pi_{\mu\nu}$. However, in this case, there is a tree-level contribution due to the appearance of two new "vertices" in the shifted Lagrangian

 $ie^2v^2g_{\mu\nu}$

 evp_μ .

p

Thus, we obtain $i\Pi_{\mu\nu}(p^2)$ to order e^2



$$i\Pi_{\mu\nu} = ie^2v^2\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) . \tag{1.21}$$

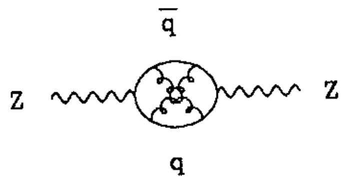
Note that the pole at $p^2 = 0$ arises explicitly due to the exchange of the massless χ field. This is the Goldstone boson of the model. It is well known that this Goldstone boson is not a physical degree of freedom. It can be eliminated by a (field-dependent) gauge transformation. Nevertheless, the remnant of the Goldstone boson remains as the longitudinal mode of the massive gauge boson. That the gauge boson is massive is clear from the shifted Lagrangian. Nevertheless, we can obtain the vector (photon) mass by noting that eq. (1.21) implies that

$$\Pi(p^2) = \frac{-e^2v^2}{p^2} . \tag{1.22}$$

It follows that $m_\gamma = ev$.

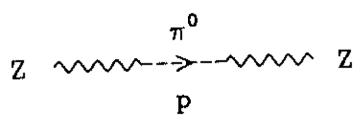
For our second example, we consider the Standard Model, but with no scalar Higgs field. This must be a very bad model of nature, since it predicts that the $W^\pm$ and Z (and the fermions as well) are all massless. But, these conclusions are based on a tree-level analysis of the model, so perhaps we are being too hasty.

To see what happens when loop effects are taken into account, I shall consider for simplicity a one generation model. To see whether loop effects can generate a vector boson mass, one must analyze corrections to the vector boson two-point functions. Naively, the theory possesses no massless scalar fields, so we should not expect a pole to develop in $\Pi(p^2)$ as in the previous example. However, due to the effects of the strong interactions, diagrams with gluon-exchange cannot be neglected.



If we could sum all such graphs to all orders, we would discover that pseudoscalar quark–antiquark bound states can be formed. Since our model possesses no quark mass terms, these pseudoscalars are in fact massless. In the one generation model, these are the pions: π^+, π^0, and π^-. In fact, these are Goldstone bosons which arise because a global $SU(2)_L \times SU(2)_R$ flavor symmetry is spontaneously broken down to a diagonal $SU(2)_{L+R}$ (called isospin). This is a dynamical breaking caused by the strong QCD forces which results in a non-zero vacuum expectation value for $\langle\overline{\psi}_L\psi_R\rangle = \langle\overline{\psi}_R\psi_L\rangle$.

As an exercise, let us compute the mass of the Z. To do this, we look for the leading contribution to $\Pi_{\mu\nu}(p)$ in which a pole at $p^2 = 0$ is generated. Since this model possesses massless pions, the leading contribution is the tree level process



The Z current can be read off from the Standard Model Lagrangian; it has the generic form

$$\mathcal{L} = g_Z Z^\mu j_\mu^Z \,. \tag{1.23}$$

The current j_μ^Z can create neutral pions from the vacuum

$$\langle 0 | j_\mu^Z(0) \, | \pi^0 \rangle = i f_\pi p_\mu \,, \tag{1.24}$$

where $f_\pi \simeq 93$ MeV is the pion decay constant. Thus, we immediately obtain

$$i\Pi_{\mu\nu}(p) = i g_Z^2 f_\pi^2 \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \,. \tag{1.25}$$

A few words of explanation are in order here. Clearly, the $p_\mu p_\nu/p^2$ term arises due to the massless pion exchange in the diagram shown above. It is rather remarkable that this term is so easy to obtain. In fact, this term depends only on the group-theoretical properties of the symmetry breaking pattern. Once we assume that the strong interactions spontaneously break chiral symmetry as indicated above, Goldstone's theorem tells us that massless scalars must exist in the spectrum with a matrix element given by eq. (1.24). The pole at $p^2 = 0$ in eq. (1.25) then immediately follows. In contrast, the $g_{\mu\nu}$ term is more difficult to obtain. Its origin is also a result of the strong interaction dynamics, but it cannot be computed directly without resorting to some nonperturbative analysis. Nevertheless, we are guaranteed by the gauge invariance of the model that this term must arise with precisely the coefficient shown in eq. (1.25). This is a consequence of the Ward identity obtained above [eq. (1.13)].

From eq. (1.25), we immediately obtain

$$\Pi(p^2) = \frac{-g_Z^2 f_\pi^2}{p^2}\,. \tag{1.26}$$

Following the steps outlined earlier, the vector boson mass is

$$\begin{aligned} m_Z = g_Z f_\pi &= \frac{M_Z f_\pi}{v} \\ &= 35\ \mathrm{MeV}\,, \end{aligned} \tag{1.27}$$

where, for convenience, I have compared the above result to the observed Z mass, $M_Z = 91$ GeV, and the vacuum expectation value of the Higgs field in the Standard Model, $v = 246$ GeV. By a similar computation (which involves the coupling of the $W^\pm$ current to the charged massless pions), we can obtain the W mass. Remarkably, we find

$$m_W = m_Z \cos\theta_W\,, \tag{1.28}$$

where θ_W is the usual weak mixing angle. This last result is very well respected by the experimental data, so perhaps there is a grain of truth in the above analysis. We will return to a more detailed examination of eq. (1.28) and its derivation in Lecture 2. For the moment, we must clearly admit to two phenomenologically disastrous predictions of this model:

1. The $\pi^\pm$ and π^0 are Goldstone bosons, and are therefore "eaten" by the $W^\pm$ and Z as a consequence of the Higgs mechanism. This results in a massive $W^\pm$ and Z, but the pions no longer exist as physical states in the theory. This is in contradiction to experiment which clearly observes pions as physical states.

2. The $W^{\pm}$ and Z masses computed above are too small (compared to experiment) by a factor of about 2600. Clearly, this is not a realistic model of nature. However, the above exercise has been particularly instructive, and we will have a chance to exploit some of its features later.

Two basic lessons can be drawn from the two models we have just examined. First, vector boson masses can be generated without destroying the consistency of quantum field theory. The gauge invariance of the theory (and the resulting Ward identities) have been successfully maintained. Second, we have seen that there are a number of possible mechanisms for vector boson mass generation. However, all mechanisms involve the existence of a Goldstone boson, which is required in order to generate the pole in $\Pi(p^2)$, which in turn produces the vector boson mass.

Therefore, we may conclude that with the discovery of the massive W and Z, we have in fact discovered the Goldstone boson and hence have "verified" the Higgs mechanism. Moreover, the search for the origin of electroweak symmetry breaking is equivalent to the search for the physics that generates the Goldstone bosons. On the other hand, the low energy dynamics of the Goldstone bosons (which should be interpreted as the longitudinal modes of the massive vector bosons—$W_L^{\pm}$ and Z_L) is independent of the dynamics of the physics which generates it, and depends only on the group-theoretical properties of the symmetry breaking. That is, the study of "low-energy" (near threshold) behavior of W's and Z's is not sufficient to uncover the electroweak symmetry breaking (ESB) mechanism. One must either detect the W_L and Z_L interactions at energies substantially above threshold, or detect directly the physics of the ESB sector.

Two basic approaches to electroweak symmetry breaking can be envisioned. The first approach would be of the type illustrated by our first example above. The Goldstone bosons arise from a weakly-coupled scalar (Higgs) sector made up of elementary scalar fields. Often, such models invoke supersymmetry, in order to provide a "natural" mechanism for generating a scalar vacuum expectation value many orders of magnitude smaller than the Planck scale (or other possible energy scales much larger than the electroweak scale). The *new* physics associated with the ESB scale would consist of the elementary Higgs scalars, and perhaps families of supersymmetric partners of the known particles. The second approach would be of the type illustrated by our second example above. The Goldstone bosons arise from a strongly-coupled sector. This sector could consist of Higgs scalars with large self-couplings or could involve new strong forces (such as technicolor) between hypothetical new (techni-) fermions. (More on such possibilities in Lecture 6.) In the latter case, scalars would arise as bound states of these new fermions. More general composite models (in which the known fermions and/or gauge bosons are composite as well) can also be envisioned. In this second approach, the physics associated with the ESB scale may involve W^+W^-, $W^\pm Z$ and ZZ resonances, techni-mesons and techni-baryons (*i.e.*, bound states of the techni-fermions), pseudo-Goldstone bosons, excited states of known Standard Model particles, *etc.* Unfortunately, one can also envision more pessimistic scenarios where the new physics associated with the ESB scale is far more subtle.

1.3 The Necessity of Higgs Bosons (or "Equivalent")—the Unitarity Argument

We have seen in the previous section that Goldstone bosons are a necessary ingredient in the generation of vector boson masses. Thus, it is tempting to ask the following question. Let us imagine that some theory generates the required Goldstone bosons. They are then "eaten" by the $W^\pm$ and Z. So, who needs the Higgs boson? More specifically, suppose that some complicated unknown dynamics, which perhaps operates at an energy scale much higher than those we can currently probe experimentally, is responsible for the generation of vector boson masses. Why must a Higgs boson exist in the effective low-energy theory (*i.e.*, at or near the energy scale which characterizes the electroweak interactions)?

I can rephrase this question as follows. Given the known mass spectrum of Standard Model particles (including the t-quark), but with the Higgs boson omitted, can I tell that the model is sick and must be fixed up not far from the scale of electroweak physics? In order to address this question, I shall make use of a fundamental principle of quantum mechanics—partial wave unitarity.

Consider the helicity amplitude $\mathcal{M}(\lambda_3\lambda_4;\lambda_1\lambda_2)$ for a $2\to 2$ scattering process, which is computed by evaluating the appropriate tree-level Feynman diagrams. The partial wave expansion is

$$\mathcal{M}(\lambda_3\lambda_4;\lambda_1\lambda_2)=\frac{8\pi\sqrt{s}}{(p_ip_f)^{1/2}}e^{i(\lambda_i-\lambda_f)\phi}\sum_{J=J_0}^{\infty}(2J+1)\mathcal{M}^J_\lambda(s)d^J_{\lambda_i\lambda_f}(\theta)\,, \qquad (1.29)$$

where p_i (p_f) is the incoming (outgoing) center-of-mass momentum, $\sqrt{s}$ is the center-of-mass energy of the scattering process, λ is shorthand for the helicities $\{\lambda_3,\lambda_4;\lambda_1,\lambda_2\}$, and

$$\begin{aligned} J_0 &\equiv \max\,\{\lambda_i,\lambda_f\}\,,\\ \lambda_i &\equiv \lambda_1-\lambda_2\,,\\ \lambda_f &\equiv \lambda_3-\lambda_4\,. \end{aligned} \qquad (1.30)$$

We can project out $\mathcal{M}^J$ from $\mathcal{M}$ in eq. (1.29) by using the orthogonality of the

$d^J(\theta)$. Partial wave unitarity requires

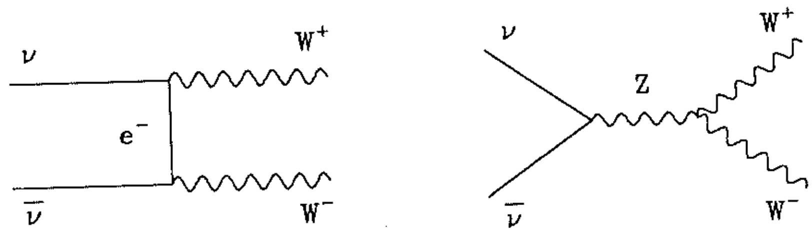
$$|\mathcal{M}^J_\lambda(s)| \leq 1\,, \tag{1.31}$$

so that the cross section for the J^{th} partial wave is bounded

$$\sigma_J \leq \frac{4\pi(2J+1)}{(2s_1+1)(2s_2+1)p_i^2}\,, \tag{1.32}$$

where s_1 and s_2 are the spins of the initial states. Note that for $\sqrt{s} \gg m_1, m_2$, we can use $p_i^2 = \frac{1}{4}s$.

Historically, unitarity arguments were used to prove that the four-fermi theory of weak interactions had to break down at some scale of $\mathcal{O}(G_F^{-1/2})$. Here, I shall simply focus on the Standard Model. First, consider the process $\nu\bar{\nu} \to W^+W^-$.



The corresponding matrix elements are proportional to $\varepsilon_{W^+} \cdot \varepsilon_{W^-}$. For a W moving in the z-direction with four momentum $(E_W; 0, 0, |\vec{k}|)$, the transverse and longitudinal polarization vectors are:

$$\begin{aligned} \varepsilon_T &= \left(0;\ \mp\frac{1}{\sqrt{2}}, \frac{-i}{\sqrt{2}}, 0\right)\,, \\ \varepsilon_L &= \frac{1}{m_W}\left(|\vec{k}|\ ; 0, 0, E_W\right)\,. \end{aligned} \tag{1.33}$$

Note that for $E_W \gg m_W$,

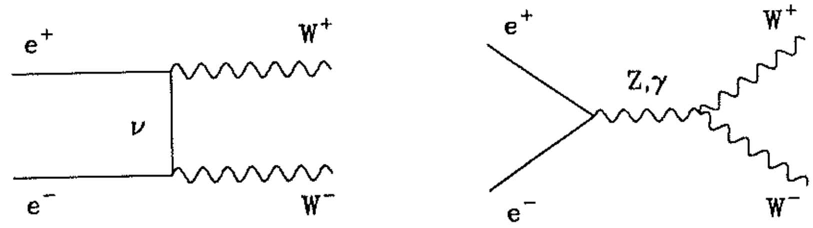
$$\varepsilon_L^\mu \simeq \frac{k^\mu}{m_W}\,. \tag{1.34}$$

It follows that

$$\varepsilon^L_{W^+} \cdot \varepsilon^L_{W^-} \simeq \frac{k_{W^+} \cdot k_{W^-}}{m_W^2} \simeq \frac{s}{2m_W^2}\,, \tag{1.35}$$

which can lead to bad high energy behavior, that is, a violation of the unitarity bound at large s due to a violation of $|\mathcal{M}^J(s)| \leq 1$. However, in this example, the magic of gauge symmetry results the cancellation of the (potentially) bad high energy behavior that each Feynman diagram separately exhibits! This magic is implemented by the precise form of the triple vector boson (W^+W^-Z) coupling.

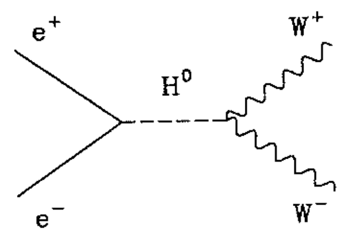
Let us next look at $e^+e^- \to W^+W^-$.



Gauge symmetry magic is in operation here as well, and the leading bad high energy behavior [of $\mathcal{O}(s)$] cancels as in the previous example. However, in this example, the magic falls a little short. The leading behavior of the scattering amplitude for two longitudinally polarized gauge bosons is

$$\mathcal{M}(L, L; \lambda', \lambda) \sim G_F m_e \bar{v}_{\lambda'} u_\lambda \, . \tag{1.36}$$

For $\lambda = \lambda'$, $\bar{v}_\lambda u_\lambda \sim \mathcal{O}(\sqrt{s})$. Actually, because m_e is so small, $(G_F m_e)^{-1} \simeq 2 \times 10^5$ TeV is very large. Thus, it is very unlikely that experiment will be able to probe directly this energy scale in our lifetimes. Nevertheless, it is an interesting to consider how the gauge theory repairs this unitarity violation.* In the Standard Model, it is the Higgs boson which comes to the rescue and removes the unitarity violation.



Due to the form of the $H^0 e^+ e^-$ coupling

$$g_{H^0 e^+ e^-} = \frac{g m_e}{2 m_W} \, , \tag{1.37}$$

the leading behavior of $\mathcal{M}$ is

$$\mathcal{M}(L, L; \lambda', \lambda) \sim G_F m_e \bar{v}_{\lambda'} u_\lambda \left[1 + \frac{s}{m_H^2 - s} \right] \, . \tag{1.38}$$

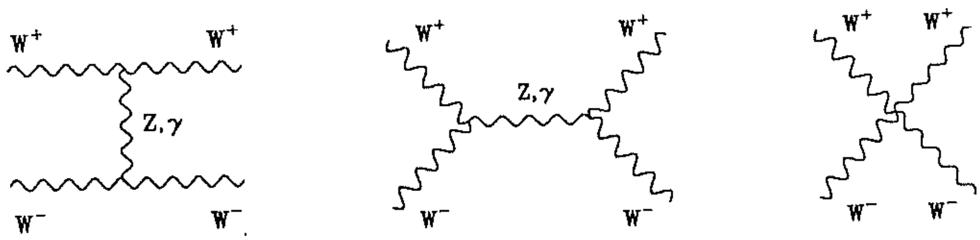
For $m_H \gg s$, we simply recover our previous result [eq. (1.36)]. However, for large

---

$\star$ I could consider the process $t\bar{t} \to W^+W^-$. By the same arguments as those presented above, I would find that unitarity is violated at $\sqrt{s} \sim \mathcal{O}[(G_F m_t)^{-1}] \sim 1$ TeV, which is much nearer the energy scale which can be probed at colliders in the near future.

energy ($s \gg m_H$), the term in brackets behaves as m_H^2/s and the bad high energy behavior is removed!

As a final example, let us examine the process $W^+W^- \to W^+W^-$.



As before, the largest potential violation of unitarity occurs in the scattering of longitudinal gauge bosons. Thus, we examine $W_L^+W_L^- \to W_L^+W_L^-$. Separately, each Feynman diagram is proportional to

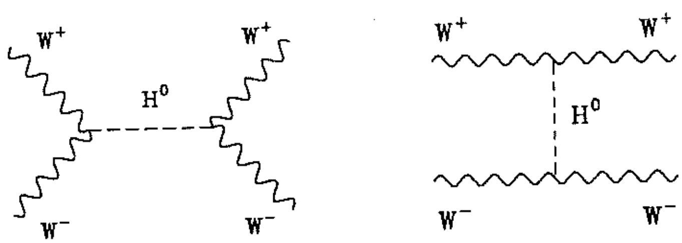
$$\varepsilon_{W^+} \cdot \varepsilon_{W^-} \, \varepsilon_{W^+} \cdot \varepsilon_{W^-} \sim \frac{s^2}{m_W^4} \,. \tag{1.39}$$

After adding up the diagrams above, we find

$$\mathcal{M}(L, L; L, L) \sim \sqrt{2} G_F (s + t) \,, \tag{1.40}$$

where t is the four-momentum transfer of the scattering process. Indeed, some cancellation has occurred (again, due to the magic of gauge symmetry), but not enough to avoid bad high energy behavior.

Once again, the Higgs boson comes to the rescue and removes the unitarity violation.



This occurs due to the form of the $H^0W^+W^-$ coupling

$$g_{H^0W^+W^-} = g m_W \,. \tag{1.41}$$

Adding the Higgs contributions to the previous result,

$$\mathcal{M}(L, L; L, L) \sim -\sqrt{2} G_F m_H^2 \left(\frac{s}{s - m_H^2} + \frac{t}{t - m_H^2} \right) \,, \tag{1.42}$$

and for $\sqrt{s} \gg m_H$, the bad high energy behavior is cancelled.

Note that m_H cannot be arbitrarily large, since otherwise there would be a range of $\sqrt{s}$ where unitarity is violated. Thus, we roughly expect $G_F m_H^2 \lesssim \mathcal{O}(1)$. That is, the Standard Model as it is currently observed experimentally (*i.e.*, with the Higgs boson at present undiscovered) must be "repaired" at an energy scale of order $G_F^{-1/2} \simeq 300$ GeV. This estimate very rough, since I have not paid careful attention to all the numerical factors which appear in the above equations. A more careful analysis will be presented near the end of Lecture 2.

I have demonstrated above how a single Higgs scalar of the Standard Model succeeds in repairing the violation of unitarity in certain $2 \to 2$ tree-level scattering processes. Of course, the single physical Higgs boson is not the unique solution for recovering tree-level unitarity of scattering amplitudes. Nevertheless, the above analysis implies that a new sector of physics, associated with electroweak symmetry breaking, must exist at an energy scale of $\mathcal{O}(G_F^{-1/2})$. The condition of tree-level unitarity is a very strong condition. Consider an arbitrary QFT involving scalars, fermions and vector bosons, all of which may be massive, and let us impose the requirement of tree level unitarity on the amplitudes of all $2 \to n$ (tree-level) scattering processes. Such a theory can only be of the following type: a spontaneously broken non-abelian gauge theory, plus an unbroken gauge theory of massless gauge bosons, plus a sector which can contain massive U(1) vector bosons.

Suggestions for Further Reading and a Brief Guide to the Literature

A discussion of the formalism introduced in this lecture can be found in

1. Ta-Pei Cheng and Ling-Fong Li, *Gauge Theory of Elementary Particle Physics*, (Oxford University Press, Oxford, 1984).
2. John Collins, *Renormalization* (Cambridge University Press, Cambridge, 1984).
3. Stefan Pokorski, *Gauge Field Theories* (Cambridge University Press, Cambridge, 1987).

The calculation of vector boson masses in a model without Higgs bosons is given in

4. E. Farhi and L. Susskind, *Phys. Rep.* **74**, 277 (1981).

Tree-unitarity of weak interactions and its implications are discussed in

5. C.H. Llewellyn Smith, in *Proceedings of the Fifth Hawaii Topical Conference in Particle Physics*, edited by P.N. Dobson, Jr., V.Z. Peterson and S.F. Tuan (University of Hawaii Press, Honolulu, 1973); *Phys. Lett.* **46B**, 233 (1973).
6. J.M. Cornwall, D.N. Levin and G. Tiktopoulos, *Phys. Rev. Lett.* **30**, 1268 (1973); *Phys. Rev.* **D10**, 1145 (1974).
7. B.W. Lee, C. Quigg and G.B. Thacker, *Phys. Rev. Lett.* **38**, 883 (1977); *Phys. Rev.* **D16**, 1519 (1977).
8. *The Higgs Hunter's Guide*, §2.6.

The relevance of the Higgs boson mass as a new energy threshold for weak interactions is described in

9. M. Veltman, *Acta Phys. Polon.* **B8**, 475 (1977).

2. Theoretical Properties of the Standard Model Higgs Boson

The Standard Model with the minimal Higgs structure consisting of one complex doublet of scalars provides the simplest realization of the Higgs mechanism which generates mass for the $W^{\pm}$ and Z gauge bosons. In this approach, the Goldstone bosons will be generated by the dynamics of elementary scalar fields, and precisely one neutral Higgs scalar remains in the physical spectrum. In this lecture, I will discuss the theoretical properties of this minimal Higgs boson. Although this is indeed the simplest model of electroweak symmetry breaking, strong theoretical doubts have been raised as to its viability as a consistent theory. These profound theoretical misgivings are associated with the "naturalness" and gauge hierarchy problems. In general, scalar field masses in a QFT are driven to the largest mass scale in the theory. Thus, in the presence of the Planck scale associated with gravity ($M_P \simeq 10^{19}$ GeV), it is difficult to understand how the Standard Model can generate a Higgs boson mass on the scale of electroweak physics, without a very unnatural fine-tuning of parameters in the fundamental Planck scale theory. In the next three lectures, I will ignore these problems, but I will return to these issues in Lectures 5 and 6. Nevertheless, I shall argue that the phenomenology of the elementary Higgs boson is not simply an academic exercise, but may be quite relevant to our search for the origin of electroweak symmetry breaking.

2.1 Goldstone Bosons from the Dynamics of Elementary Scalars

Goldstone bosons can arise easily in a scalar field theory. First, consider the following tree-level analysis. Let ϕ_i be a multiplet of real scalar fields: (There is no loss of generality here, since I can always replace a complex field with two real fields.) The Lagrangian of the scalar fields is given by

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\phi_i\partial^\mu\phi^i - V(\phi_i)\,. \tag{2.1}$$

This Lagrangian is assumed to possess a global symmetry. That is, $\mathcal{L}$ is invariant under $\phi \longrightarrow \phi + \delta\phi$, where

$$\delta\phi_i = -i\theta^a T^a_{ij}\phi_j\,, \tag{2.2}$$

where the generators iT^a are real antisymmetric matrices and the θ^a are real parameters. The invariance of the Lagrangian under eq. (2.2) restricts the form for the potential V. That is, $\delta\mathcal{L} = 0$ implies

$$\delta V = \frac{\partial V}{\partial\phi_i}\,\delta\phi_i = \frac{\partial V}{\partial\phi_i}\,T^a_{ij}\phi_j = 0\,, \tag{2.3}$$

which must be true for all a. Suppose that the minimum of the potential $V(\phi)$

occurs at $\phi_i = v_i$, *i.e.*,

$$\left.\frac{\partial V}{\partial \phi_i}\right|_{\phi_i=v_i} = 0\,, \tag{2.4}$$

such that $e^{-i\theta^a T^a} v \neq v$. (An equivalent condition, obtained by taking θ infinitesimal, is $T^a v \neq 0$.) In this case, the vacuum does not respect the symmetry. Choose a definite vacuum state [*i.e.*, choose a definite direction for ϕ_i satisfying eq. (2.4)] and express the Lagrangian in terms of the shifted field

$$\tilde{\phi} \equiv \phi - v\,. \tag{2.5}$$

Then,

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu \tilde{\phi}_i \partial^\mu \tilde{\phi}^i - \tfrac{1}{2} M^2_{ij} \tilde{\phi}_i \tilde{\phi}_j + \text{interactions}\,, \tag{2.6}$$

where M^2 is a non-negative symmetric matrix

$$M^2_{ij} \equiv \left.\frac{\partial V}{\partial \phi_i \partial \phi_j}\right|_{\phi_i=v_i}\,. \tag{2.7}$$

Differentiating eq. (2.3) with respect to ϕ_j, and setting $\phi_i = v_i$ gives

$$M^2_{ik} T^a_{ij} v_j = 0\,. \tag{2.8}$$

That is, $\phi_i T^a_{ij} v_j$ is an eigenvector of M^2 with zero eigenvalue. There is one Goldstone boson for each broken generator $T^a v \neq 0$. In general, if we break the global symmetry group G to a subgroup H, then there are $\dim G - \dim H$ Goldstone bosons. The remaining scalars are massive; these are the Higgs bosons.

Suppose we now embed this structure in a non-abelian gauge theory with gauge group G. That is, the Lagrangian is now

$$\mathcal{L} = \mathcal{L}_{\rm YM} + \tfrac{1}{2}(D_\mu \phi)^T (D^\mu \phi) - V(\phi)\,, \tag{2.9}$$

where $\mathcal{L}_{\rm YM}$ is the standard Yang-Mills Lagrangian, and D is the covariant derivative

$$D_\mu \equiv \partial_\mu + igT^a A^a_\mu\,. \tag{2.10}$$

As before, we use a convention where all scalar fields are real so that the iT^a are real antisymmetric matrices. In addition, we assume that the scalar potential is minimized at $\phi_i = v_i$. When we shift $\phi_i \to \phi_i + v_i$, we generate

$$(D_\mu \phi)^T (D^\mu \phi) = M^2_{ab} A^a_\mu A^{\mu b} + \ldots \tag{2.11}$$

with

$$M^2_{ab} = g^2 v^T T^a T^b v \,. \tag{2.12}$$

For each unbroken generator (*i.e.*, $T^a v = 0$), the corresponding vector boson remains massless. The remaining vector bosons acquire mass and the corresponding Goldstone bosons are eaten.

The analysis we have just presented is a tree-level analysis and therefore applies only at the classical level. To incorporate quantum effects, we must go beyond the tree-level. For example, it is possible that the conclusions as to which symmetries are spontaneously broken would change when radiative corrections are taken into account. Thus, we must develop a more general formalism to address the question of symmetry breaking. To do this, we shall introduce the concept of the effective potential. Here is a lightning review of the necessary formalism. We start with the path integral representation of the generating functional for the Green's functions of the QFT

$$W[J] = e^{iZ[J]} = N^{-1} \int \mathcal{D}\phi \exp i \left\{ S[\phi] + \int d^4x J(x)\phi(x) \right\} , \tag{2.13}$$

where $Z[J]$ generates the connected Green's functions. $S[\phi]$ is the classical action [see eq. (1.4)]. Next, we define the classical field $\phi_c(x)$ by

$$\phi_c(x) \equiv \frac{\delta Z[J]}{\delta J(x)} = \frac{\langle 0|\, \phi(x)\, |0\rangle_J}{\langle 0|0\rangle_J} \,. \tag{2.14}$$

We perform a functional Legendre transformation by defining the effective action $\Gamma[\phi_c]$

$$\Gamma[\phi_c] = Z[J] - \int d^4x J(x)\phi_c(x) \,. \tag{2.15}$$

where $J(x)$, which appears on the right-hand side of eq. (2.15) should be re-expressed in terms of $\phi_c(x)$ by inverting eq. (2.14). The origin of the name *effective action* derives from the result

$$\Gamma[\phi_c] = S[\phi_c] + \mathcal{O}(\hbar) \,. \tag{2.16}$$

One can show that $\Gamma[\phi_c]$ generates the 1PI Green's functions of the theory. In a scalar field theory, the effective action takes the following form

$$\Gamma[\phi_c] = \int d^4x \left[\tfrac{1}{2}Z(\phi_c)\partial_\mu\phi_c\partial^\mu\phi_c - V_{\rm eff}(\phi_c) + \text{higher derivative terms}\right] , \qquad (2.17)$$

where

$$\begin{aligned} Z(\phi_c) &= 1 + \mathcal{O}(\hbar) , \\ V_{\rm eff}(\phi_c) &= V(\phi_c) + \mathcal{O}(\hbar) . \end{aligned} \qquad (2.18)$$

$V_{\rm eff}$ is called the effective potential and it is the quantum generalization of the classical potential $V(\phi)$. To demonstrate this claim, suppose that the scalar field acquires a vacuum expectation value. That is,

$$v = \langle 0|\,\phi(x)\,|0\rangle = \lim_{J\to 0} \phi_c(x) , \qquad (2.19)$$

where the last equality follows from eq. (2.14). Using

$$J(x) = -\frac{\delta\Gamma[\phi_c]}{\delta\phi_c(x)} , \qquad (2.20)$$

which follows from eq. (2.15), we can conclude that

$$\left.\frac{\delta\Gamma[\phi_c]}{\delta\phi_c(x)}\right|_{\phi_c=v} = 0 . \qquad (2.21)$$

Since v is a constant, it follows from eq. (2.17) that this condition is equivalent to

$$\left.\frac{\partial V_{\rm eff}}{\partial\phi_c}\right|_{\phi_c=v} = 0 . \qquad (2.22)$$

To summarize, in order to determine whether spontaneous symmetry breaking occurs, one minimizes the effective potential

$$V_{\rm eff}(\phi) = V^{(0)}(\phi) + \hbar V^{(1)}(\phi) + \cdots \qquad (2.23)$$

and searches for the existence of a symmetry-breaking global minimum by solving the equation

$$\left.\frac{dV_{\rm eff}(\phi)}{d\phi}\right|_{\phi=v} = 0 . \qquad (2.24)$$

Suppose we adopt this procedure for constructing a realistic model of electroweak symmetry breaking. There are a number of possible outcomes.

1. The symmetry is unbroken. *Try another model.*
2. The symmetry is broken at tree-level. *If this is true, we may have a good candidate for a model of electroweak symmetry breaking. Although one should check the effect of radiative corrections, it is usually a good assumption that the inclusion of higher order effects will not alter the conclusion that the symmetry is spontaneously broken.*
3. The symmetry is not broken at tree-level, but is found to be broken when radiative corrections [e.g., $V^{(1)}(\phi)$] are included. *This is a very interesting mechanism which we will examine in section 2.3.*

The most common model of electroweak symmetry breaking by elementary scalars is one in which the symmetry is broken at tree-level. Thus, we first turn to a detailed discussion of the minimal model of this type.

2.2 The Standard Model with Minimal Higgs Structure

The Standard Model is an SU(2) $\times$ U(1) gauge theory with massive $W^{\pm}, Z^0$ gauge bosons and a massless photon. This requires the following symmetry breaking pattern

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \rightarrow \mathrm{U}(1)_{\mathrm{EM}}\,, \tag{2.25}$$

where the generator of the electric charge is a linear combination of the two diagonal generators of SU(2) $\times$ U(1)

$$Q = T_3 + \tfrac{1}{2}Y\,. \tag{2.26}$$

The physical Z and photon are linear combinations of the $\mathrm{SU}(2)_Y$ neutral gauge field W^3 and the $\mathrm{U}(1)_Y$ gauge field B

$$\begin{aligned} Z_\mu &\equiv \cos\theta_W W^3_\mu - \sin\theta_W B_\mu \\ A_\mu &\equiv \sin\theta_W W^3_\mu + \cos\theta_W B_\mu\,. \end{aligned} \tag{2.27}$$

Three Goldstone bosons are required to generate mass for the $W^{\pm}$ and Z. This can be accomplished in the most economical way by introducing a minimal Higgs structure consisting of one complex $Y = 1$ doublet

$$\Phi \equiv \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} \qquad \text{with } \langle \Phi^0 \rangle = \frac{v}{\sqrt{2}}\,. \tag{2.28}$$

The vacuum expectation value (VEV), $v = (\sqrt{2}G_F)^{-1/2} = 246$ GeV sets the mass scale for the electroweak interactions. This minimal Higgs structure automatically guarantees that

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1 \tag{2.29}$$

at tree level. In models with more complicated Higgs structures, the parameter ρ is an independent parameter of the model. In the minimal model, the special value of $\rho = 1$ arises due to an extra symmetry in the Higgs potential which is given by

$$\begin{aligned} V(\Phi) &= \lambda\left(\Phi^+\Phi - \tfrac{1}{2}v^2\right)^2 \\ &= -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2\,, \end{aligned} \tag{2.30}$$

where $\mu^2 = \lambda v^2$ and a constant field independent term has been dropped. To expose the full symmetry of this potential, let us write

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} w_1 + iw_2 \\ H + iw_3 \end{pmatrix}, \tag{2.31}$$

where $\langle H\rangle = v$. If $\vec{\mathrm{w}} = (w_1, w_2, w_3)$, then $\Phi^\dagger\Phi = \frac{1}{2}(\vec{\mathrm{w}}^{\,2} + H^2)$ so that $V(\Phi)$ clearly possesses an $\mathrm{SO}(4) \simeq \mathrm{SU}(2)_L\times\ \mathrm{SU}(2)_R$ global symmetry, which spontaneously breaks to $\mathrm{SO}(3) \simeq \mathrm{SU}(2)_{L+R}$. The latter SU(2) is the diagonal subgroup of $\mathrm{SU}(2)_L\times\ \mathrm{SU}(2)_R$. To verify this explicitly, we can rewrite Φ in matrix form

$$\Phi = \begin{pmatrix} \Phi^0 & -\Phi^+ \\ \Phi^- & \Phi^{0*} \end{pmatrix}. \tag{2.32}$$

Then, Φ transforms under an $\mathrm{SU}(2)_L\times\ \mathrm{SU}(2)_R$ transformation as

$$\Phi \to U_L \Phi U_R^\dagger\,, \tag{2.33}$$

where U_L and U_R are $SU(2)$ matrices. It is simple to write out the corresponding transformations of $\vec{\mathrm{w}}$ and H. We can identify the diagonal subgroup $\mathrm{SU}(2)_{L+R}$ by taking $U_L = U_R = \exp(-i\vec{\theta}\cdot\vec{\sigma}/2)$. The infinitesimal form of the transformation under $\mathrm{SU}(2)_{L+R}$ is then

$$\begin{aligned} \delta\vec{\mathrm{w}} &= \delta\vec{\theta}\times\vec{w} \\ \delta H &= 0\,, \end{aligned} \tag{2.34}$$

which is clearly preserved when H acquires a vacuum expectation value. After symmetry breaking, the $\vec{\mathrm{w}}$ is the triplet of Goldstone bosons which will get eaten by W^+, W^-, Z, and $H^0 \equiv H - v$ is the physical Higgs particle. It is straightforward to compute the gauge boson masses by standard textbook techniques. Here, I shall do the computation making use of the technology introduced in Lecture 1. First, one identifies the currents which couple to the gauge bosons. We can rewrite the relevant part of the gauge boson contribution to the Standard Model Lagrangian as follows

$$\mathcal{L} = gW_a^\mu T_{\mu L}^a + \tfrac{1}{2}g' B^\mu Y_\mu \,, \tag{2.35}$$

where T_μ is the left-handed isospin current, and Y_μ is the hypercharge current. Using $Q = T_3 + Y/2$, and noting that Q is a vector current, we may decompose $\mathcal{L}$ into vector and axial vector pieces

$$\mathcal{L} = -\tfrac{1}{2}gW_a^\mu j_{\mu 5}^a + \tfrac{1}{2}g' B^\mu j_{\mu 5}^3 + \text{ vector current couplings}\,. \tag{2.36}$$

In analogy with eq. (1.24),

$$\langle 0 \mid j_{\mu 5}^a(0) \mid w^b \rangle = ivp_\mu \delta^{ab}\,. \tag{2.37}$$

The δ^{ab} is a consequence of the $\mathrm{SU}(2)_{L+R}$ symmetry (which is called the *custodial* SU(2) symmetry) that remains even after H acquires a vacuum expectation value. Using eq. (2.37), it is a simple matter to follow the steps outlined in Lecture 1 and compute the 4×4 vacuum polarization matrix $\Pi(p)_{ij}$. The resulting vector boson squared mass matrix is given by

$$\frac{v^2}{4}\begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg' \\ 0 & 0 & -gg' & g'^2 \end{pmatrix}. \tag{2.38}$$

Diagonalizing this matrix yields

$$\begin{aligned} m_W^2 &= \tfrac{1}{4}g^2v^2 \\ m_Z^2 &= m_W^2/\cos^2\theta_W\,, \end{aligned} \tag{2.39}$$

which implies that $\rho = 1$.

The symmetry arguments given above were based on the global symmetries of the scalar sector. In fact, the custodial SU(2) is not an exact symmetry of the full theory, since when the gauge interactions are turned on, only $\mathrm{U}(1)_{\mathrm{EM}}$ survives as an exact symmetry. In particular, the hypercharge gauge interactions break the custodial SU(2) symmetry. As a result, the relation $\rho = 1$ acquires a (finite) radiative correction of $O(g'^2)$. Other violations of the custodial SU(2) symmetry can be introduced when new sectors are coupled to the gauge–Higgs theory. Perhaps the most dramatic violation of the custodial SU(2) symmetry can arise due to isospin violation in the fermion sector. Specifically, because $m_t \neq m_b$, the relation $\rho - 1 = 0$ suffers a radiative correction given by

$$\delta\rho = \frac{g^2 N_c}{64\pi^2 m_W^2}\left[m_t^2 + m_b^2 - \frac{2m_t^2 m_b^2}{m_t^2 - m_b^2}\ln\left(\frac{m_t^2}{m_b^2}\right)\right], \tag{2.40}$$

where $N_c = 3$ for the contribution of a quark doublet. Note that $\delta\rho = 0$ if $m_t = m_b$ and $\delta\rho > 0$ otherwise. For $m_t \gg m_b$, $\delta\rho$ grows quadratically with m_t. Thus, accurate experimental measurements of the W and Z masses can impose nontrivial constraints on the maximum allowed value for the undiscovered top-quark mass as well as on a possible fourth generation of quarks and leptons.

Fermion Masses via the Higgs mechanism

Until now, we have concentrated our efforts on generating masses for the W and Z gauge bosons. But in the Standard Model, f_L and f_R have different SU(2) $\times$ U(1) quantum numbers, so that a bare mass term

$$m_f[\bar{f}_L f_R + \text{h.c.}] \tag{2.41}$$

would violate gauge invariance. This implies that all quarks and leptons are massless, which contradicts experimental observations. Remarkably, one can make use of the elementary Higgs field to generate fermion masses in a simple way. Consider the gauge-invariant interaction of fermions and scalars

$$g_f[\bar{f}_L \phi f_R + \text{h.c.}]\,. \tag{2.42}$$

When ϕ acquires a vacuum expectation value, $\phi \to \phi + v/\sqrt{2}$, a fermion mass is generated:

$$m_f = \frac{g_f v}{\sqrt{2}}\,. \tag{2.43}$$

This mechanism is easily applied to the Standard Model. For simplicity, we consider one generation of quarks: $Q_L \equiv \binom{t_L}{b_L}$, t_R, b_R. The most general Yukawa interaction between quarks and the scalar Higgs doublet Φ which is consistent with the gauge symmetry is

$$-\mathcal{L}_{\text{Yukawa}} = g_b\overline{Q}_L\Phi b_R + g_t\overline{Q}_L\widetilde{\Phi}t_R + \text{h.c.} \tag{2.44}$$

where $\widetilde{\Phi} \equiv i\sigma_2\Phi^*$. Note that weak isospin breaking has been put in by hand; *i.e.*, $g_b \neq g_t$ in order that $m_b \neq m_t$. Nevertheless, this is very elegant: the fermions acquire mass simultaneously with the vector bosons when the gauge symmetry is spontaneously broken. (Keep this in mind when we explore technicolor models in Lecture 6.)

The extension to the full three generation model is straightforward. One finds that the quark mass eigenstates need not be identical to the electroweak interaction eigenstates defined above. After diagonalizing the quark mass matrices, one finds the well known Cabibbo-Kobayashi-Maskawa structure for the charged weak currents; however, no tree-level flavor changing neutral currents mediated by the Z or Higgs boson are generated.

How Does the Charged Pion Decay?

In Lecture 1, we discussed the generation of masses for the $W^\pm$ and Z in a model with no Higgs bosons. We noted that because $\mathrm{SU}(2)_L\times\ \mathrm{SU}(2)_R$ flavor symmetry was spontaneously broken by QCD, the pions would be Goldstone bosons which were "eaten" by the $W^\pm$ and Z. Later, when we introduced the Higgs sector (or equivalent), we conveniently neglected the pions. If we now consider the fate of the pions, we seem to run into a small mystery.

We now have two triplets of Goldstone bosons: π^a and w^a. Then, the total axial vector current is

$$j_5^{\mu a} = j_{5,QCD}^{\mu a} + v\partial^\mu w^a \,. \tag{2.45}$$

so that in addition to eq. (2.37), we have

$$\langle 0|\, j_5^{\mu a} |\pi^b\rangle = i f_\pi p^\mu \delta^{ab} \,. \tag{2.46}$$

In particular, the true Goldstone bosons which are eaten by the vector bosons are $f_\pi \,|\pi\rangle + v\,|w\rangle$. Let $|G\rangle$ be the true (normalized) Goldstone boson state

$$|G\rangle = \frac{1}{\sqrt{f_\pi^2+v^2}}\left[f_\pi\,|\pi\rangle + v\,|w\rangle\right] . \tag{2.47}$$

The physical pion is then identified as the state orthogonal to $|G\rangle$

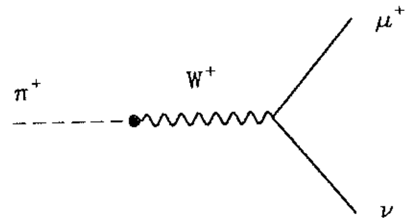
$$|\pi\rangle_{\rm phys} = \frac{1}{\sqrt{f_\pi^2+v^2}}\left[v\,|\pi\rangle - f_\pi\,|w\rangle\right] . \tag{2.48}$$

It is then a simple matter to check that

$$\langle 0|\, j_{\mu 5}^a |G^b\rangle = i(f_\pi^2+v^2)^{1/2} p_\mu \delta^{ab} \,, \tag{2.49}$$

$$\langle 0|\, j_{\mu 5}^a |\pi^b\rangle_{\rm phys} = 0 \,. \tag{2.50}$$

The following mystery now arises. In particle physics textbooks, one often sees that the charged pion decays through the following diagram



where the π–W vertex is given by $\langle 0|\, j_5^{\mu a}|\pi^b\rangle$. But eq. (2.50) implies that this matrix element between the vacuum and the physical pion vanishes! So how does the charged pion decay?

The solution to the π-decay mystery is as follows. The Yukawa coupling of the electroweak theory couples leptons to the full Higgs doublet

$$\mathcal{L}_{\rm Yukawa} = \frac{\sqrt{2}m_\ell}{v}\,\overline{E}_L\phi\ell_R + \text{h.c.} \tag{2.51}$$

where

$$E = \begin{pmatrix} \nu \\ \ell_L \end{pmatrix}, \qquad \phi = \begin{pmatrix} w^+ \\ \frac{1}{\sqrt{2}}(v + H^0 + iw_3) \end{pmatrix}. \tag{2.52}$$

This leads to a Yukawa interaction between the leptons and the w field

$$-\mathcal{L}_{w^-\ell^+\nu} = G_F m_\ell \bar{\ell}(1-\gamma_5)\nu w^- + \text{h.c.} \tag{2.53}$$

But, we can express w in terms of the true Goldstone boson, G and the physical pion field $\pi_{\rm phys}$

$$|w\rangle = \frac{1}{\sqrt{f_\pi^2 + v^2}}\,\left[v\,|G\rangle - f_\pi\,|\pi\rangle_{\rm phys}\right]. \tag{2.54}$$

In the unitary gauge, $|G^a\rangle$ is eaten and disappears. Thus, since $v \gg f_\pi$, we obtain

$$\mathcal{L}_{\pi_{\rm phys}\ell^+\nu} = f_\pi G_F m_\ell\, \bar{\ell}(1-\gamma_5)\nu\pi^- + \text{h.c.} \tag{2.55}$$

which leads to the "standard" matrix element for $\pi^+_{\rm phys} \to \ell^+\nu$ given in the textbooks.

Summary of the Minimal Higgs Sector of the Standard Model

The Standard Model with the minimal Higgs sector can be summarized by the Feynman rules of fig. 1. The features of this model are briefly summarized as follows:

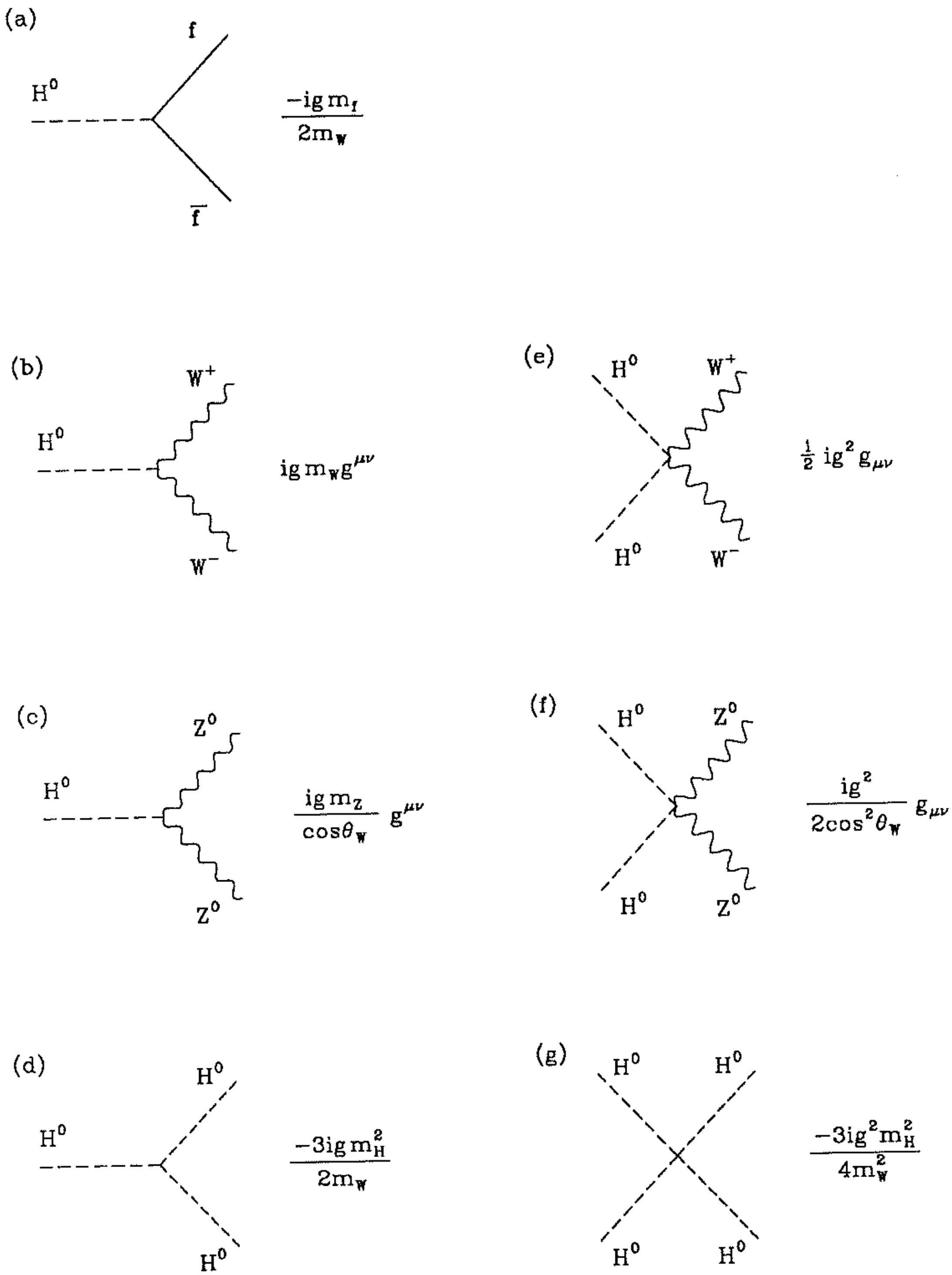


Figure 1 Feynman rules for the Standard Model Higgs boson.

1. The electroweak gauge symmetry is broken because the neutral Higgs field possesses a nonzero vacuum expectation value. As a result, the $W^\pm$ and Z gauge bosons acquire mass. In the minimal Higgs model, there is a relation

$$m_W = m_Z \cos\theta_W \left[1 + \mathcal{O}(g'^2) + \mathcal{O}\left(\frac{g^2 m_t^2}{m_W^2}\right)\right] , \tag{2.56}$$

due to a custodial SU(2) symmetry of the Higgs potential which is only violated by hyperchange gauge interactions and unequal fermion doublet masses.

2. Arbitrary Higgs-fermion Yukawa couplings are in one-to-one correspondence with corresponding fermion mass matrices. In the three-generation model, this naturally leads to a CKM structure for the charged weak current and an absence of tree-level flavor changing neutral currents.

3. The coupling strengths of the Higgs interactions with all particles are fixed and are proportional to the corresponding particle masses. This is summarized by the basic Higgs vertices of the Standard Model shown in fig. 1.

4. The physical Higgs mass is proportional to its self-coupling and hence m_H is undetermined. This is evident by rewriting the Higgs potential in terms of the physical Higgs field $H \to H + v$. Using eqs. (2.30) and (2.31),

$$\begin{aligned} V(\vec{\mathrm{w}}, H) &= \tfrac{1}{4}\lambda\left(\vec{\mathrm{w}}^{\,2} + H^2 + 2vH\right)^2 \\ &= \tfrac{1}{4}\lambda\left(\vec{\mathrm{w}}^{\,2} + H^2\right)^2 + \lambda v H\left(\vec{\mathrm{w}}^{\,2} + H^2\right) + \tfrac{1}{2}m_H^2 H^2 , \end{aligned} \tag{2.57}$$

with $m_H^2 = 2\lambda v^2$. The Higgs mass depends on the unknown Higgs self-coupling parameter λ.

2.3 Symmetry Breaking Through Radiative Corrections

In the Standard Model described in the previous section, electroweak symmetry breaking is generated at tree-level. In this section, we examine the intriguing possibility that electroweak symmetry breaking can be generated by radiative corrections. In section 2.1 we learned that one must compute the effective potential in order to determine whether symmetry breaking occurs. Thus, we must now discuss how to compute $V_{\text{eff}}(\phi)$ in perturbation theory.

Effective Potential Formalism

The first step is to functionally expand the effective action $\Gamma[\phi_c]$ about an arbitrary constant field Φ,

$$\Gamma[\phi_c] = \sum_{n=1}^{\infty} \frac{1}{n!} \int d^4x_1 \ldots d^4x_n \, \Gamma_\Phi^{(n)}(x_1, \ldots, x_n)[\phi_c(x_1) - \Phi] \cdots [\phi_c(x_n) - \Phi]. \quad (2.58)$$

We have used the notation $\Gamma_\Phi^{(n)}$ to indicate that these are the Green's functions of the *shifted* theory (*i.e.*, the theory obtained by shifting the scalar field by a constant: $\phi_c(x) \to \phi_c(x) + \Phi$). If we take $\phi_c(x) \equiv \phi_c$ independent of x, we can read off V_{eff} by comparing eqs. (2.17) and (2.58). The result takes on a particularly simple form if we make use of the momentum space Green's functions

$$\begin{aligned} &\widetilde{\Gamma}_\Phi^{(n)}(p_1, \ldots, p_n)(2\pi)^4 \delta^4(p_1 + \ldots + p_n) \\ &\qquad = \int d^4x_1 \ldots d^4x_n \, e^{i(p_1 \cdot x_1 + \ldots + p_n \cdot x_n)} \, \Gamma_\Phi^{(n)}(x_1, \ldots, x_n). \end{aligned} \quad (2.59)$$

If we identify $(2\pi)^4 \delta^4(0)$ with $\int d^4x$, we obtain

$$V_{\text{eff}}(\phi_c) = -\sum_{n=1}^{\infty} \frac{1}{n!} \widetilde{\Gamma}_\Phi^{(n)}(0)(\phi_c - \Phi)^n, \quad (2.60)$$

where $\widetilde{\Gamma}_\Phi^{(n)}(0)$ indicates that all external momenta are set to zero. Now, if we take the derivative with respect to ϕ_c and then set $\phi_c = \Phi$, only one term survives,

$$\left. \frac{dV_{\text{eff}}(\phi_c)}{d\phi_c} \right|_{\phi_c = \Phi} = -\widetilde{\Gamma}_\Phi^{(1)}(0) = i$$ 

. (2.61)

Thus, in order to compute the effective potential, all we need to do is to compute the tadpole (one-point function) $i\widetilde{\Gamma}_\Phi^{(1)}(0)$ of the shifted theory and integrate once to obtain $V_{\text{eff}}(\phi_c)$. Clearly, a constant of integration is of no concern since only the field dependent terms of V_{eff} are relevant. Note that if we take $\Phi = v$ where v is the true vacuum expectation value of the theory, then the sum of all tadpoles must vanish. We then regain the minimum condition for V_{eff}, as expected.

By similar calculation, we find

$$\left. \frac{d^2V_{\text{eff}}(\phi_c)}{d\phi_c^2} \right|_{\phi_c = \Phi} = -\Gamma_\Phi^{(2)}(0). \quad (2.62)$$

If we take $\Phi = v$ where v is the true vacuum expectation value, then $\Gamma_\Phi^{(2)}(0)$ is the two-point 1PI Green's function of the shifted theory. Traditionally, one designates the sum of all Feynman diagrams (one-loop or higher) contributing to the mass shift by $-i\Sigma(k^2)$. That is,

$$\begin{aligned}\Gamma^{(2)}_{\Phi}(k) &= k^2 - m^2 - \Sigma(k^2) \\ &= k^2\left[1 - \Sigma'(k^2)\right] - m^2 - \Sigma(0)\,,\end{aligned} \tag{2.63}$$

where $\Sigma'(k)$ is defined by

$$\Sigma(k) \equiv \Sigma(0) + k^2\Sigma'(k^2)\,. \tag{2.64}$$

Thus, we identify

$$m^2 + \Sigma(0) = \left.\frac{d^2V_{\text{eff}}}{d\phi^2}\right|_{\langle\phi\rangle}, \tag{2.65}$$

where the notation $\langle\phi\rangle$ indicates that the second derivative should be evaluated at the minimum of V_{eff}. However, the physical mass (m_H^2) should be identified with the pole in the propagator (or the zero of the inverse propagator) which requires a solution of

$$k^2 = \left.\frac{d^2V_{\text{eff}}}{d\phi^2}\right|_{\langle\phi\rangle}\left[1 - \Sigma'(k^2)\right]^{-1}. \tag{2.66}$$

For example, in a one-loop computation, we can expand the right hand side and drop higher order terms. The result is

$$m_H^2 = \left.\frac{d^2V_{\text{eff}}}{d\phi^2}\right|_{\langle\phi\rangle} + \Sigma(m_0^2) - \Sigma(0)\,, \tag{2.67}$$

where m_0 is the tree-level scalar mass. The corresponding formula at two loops is more complicated, but can easily be worked out.

Let us now consider the practical question: how does one compute the effective potential? There are a number of techniques which are well described in the textbooks. Here I shall focus on an alternative method based on eq. (2.61). [See Marc Sher's review for further details.] Here, only the one-loop computation will be discussed, so I shall write

$$V_{\text{eff}}(\phi) = V^{(0)}(\phi) + \hbar V^{(1)}(\phi)\,. \tag{2.68}$$

Eq. (2.61) instructs us to consider the shifted theory. That is, take the tree-level action which depends on ϕ_c and let $\phi_c \to \phi_c + \phi$. (Here, ϕ is any field configuration, and need not be constant.) The tree-level Feynman rules corresponding to this new action are easily obtained. For example, noting that

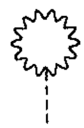
$$V(\phi_c + \phi) = V(\phi) + \phi_c \frac{dV}{d\phi_c}\bigg|_{\phi_c=\phi} + \cdots \tag{2.69}$$

we see that there is a tree-level contribution to the tadpole due to the Feynman rule

$$-i\,\frac{dV}{d\phi}$$

where the dashed line represents ϕ_c and $V(\phi)$ is the classical potential of the unshifted theory expressed as a function of ϕ. Comparing with eq. (2.61), we obtain the expected result $V^{(0)}(\phi) = V(\phi)$.

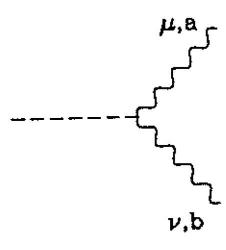
Next, we consider the one-loop contributions to the tadpoles. Here, we shall exhibit the contribution of the (vector) gauge fields.



At this point, we run into a minor complication since V_{eff} is not gauge invariant. Although there are a number of subtleties in this regard, I will simply choose a convenient gauge and argue that any physical quantity must be gauge invariant. Traditionally, the Landau gauge ($\xi = 0$ in the R_ξ-gauge) is the gauge of choice for reasons of simplicity. For example, the coupling of the scalars to the Faddeev-Popov ghost fields vanish for $\xi = 0$. To evaluate the diagram above, we need to determine the $\phi_c A^a_\mu A^{\mu a}$ coupling. Starting with the kinetic energy term $\frac{1}{2}(D_\mu\phi_c)^T(D^\mu\phi_c)$ and inserting $D_\mu = \partial_\mu + igT^a A^a_\mu$ yields a term $\frac{1}{2}M^2_{ab}(\phi_c)A^a_\mu A^{\mu b}$, where $M^2_{ab}(\phi_c) \equiv g^2\phi_c^T T^a T^b \phi_c$. In the shifted Lagrangian ($\phi_c \to \phi_c + \phi$), the vector bosons acquire a (field-dependent) squared mass matrix $M^2_{ab}(\phi)$, and the following trilinear coupling is generated

$$\frac{1}{2}\frac{dM^2_{ab}}{d\phi_c}\bigg|_{\phi_c=\phi} \phi_c A^a_\mu A^{\mu b}\,, \tag{2.70}$$

which results in the Feynman rule



$$i g^{\mu\nu}\,\frac{dM^2_{ab}}{d\phi}\,.$$

The simplest way to proceed is to diagonalize the vector boson squared mass matrix. Then,

$$i\widetilde{\Gamma}_{\Phi}^{(1)}(0) = \tfrac{3}{2}\int \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2 - M^2(\phi)}\,\frac{dM^2(\phi)}{d\phi}\,, \tag{2.71}$$

where a factor of $\frac{1}{2}$ has been inserted for the case of a neutral gauge boson (this is a symmetry factor of the tadpole graph). Note that the factor of 3 above arises because we have used the Landau gauge vector boson propagator. In particular, $g^{\mu\nu}(g_{\mu\nu}-p_\mu p_\nu/p^2) = 3$ simply counts the number of degrees of freedom of a massive vector boson. Integrating to get $V_{\text{eff}}^{(1)}$,

$$V^{(1)}(\phi) = -\tfrac{3}{2}\,i\int \frac{d^4k}{(2\pi)^4}\ln\left[k^2 - M^2(\phi)\right]. \tag{2.72}$$

I am free to normalize the one-loop effective potential such that $V^{(1)}(\phi = 0) = 0$. Then, I claim that the complete result for $V^{(1)}(\phi)$, where arbitrary particles of spin J are allowed to run around inside the loop, is

$$V^{(1)}(\phi) = \frac{-i}{2}\,\text{Str}\int \frac{d^4k}{(2\pi)^4}\ln\left[\frac{k^2 - M^2(\phi)}{k^2 - M^2(0)}\right]\,, \tag{2.73}$$

where

$$\text{Str}\,\{\cdots\} = \sum_i (-1)^{2J_i}(2J_i+1)C_i\,\{\cdots\}\,, \tag{2.74}$$

and C_i counts the electric charge and color degrees of freedom of particle i (*e.g.*, $C = 2$ for the $W^\pm$ gauge boson and $C = 6$ for a colored quark, since we count both particle and antiparticle). We can regulate this integral with a momentum cutoff Λ. Rotating to Euclidean space and noting that $d^4k = k^3dkd\Omega_3$, with $\int d\Omega_3 = 2\pi^2$, the integrals are straightforward. The final result (ignoring constant terms) is

$$V^{(1)}(\phi) = \frac{\Lambda^2}{32\pi^2}\,\text{Str}\,M_i^2(\phi) + \frac{1}{64\pi^2}\,\text{Str}\left\{M_i^4(\phi)\left[\ln\frac{M_i^2(\phi)}{\Lambda^2} - \tfrac{1}{2}\right]\right\}. \tag{2.75}$$

To complete the calculation, one must perform the standard renormalization procedure and absorb the ultraviolet infinities into coupling constant and mass parameter redefinitions. One way to do this is to write the renormalized effective potential as

$$V_{\text{eff}}(\phi) = V^{(0)}(\phi) + V^{(1)}(\phi) + V_{\text{c.t.}} \tag{2.76}$$

where $V_{\text{c.t.}}$ has the same form as $V^{(0)}(\phi)$. By choosing suitable renormalization conditions (thereby defining the physical masses and couplings), one ends up with a finite and renormalized $V_{\text{eff}}(\phi)$.

In order to illustrate the above formalism, let us examine the famous calculation of Coleman and E. Weinberg. Consider the tree-level potential in a theory with a multiplet of scalar fields ϕ_i. The tree-level potential is assumed to be of the following form

$$V^{(0)}(\phi) = \tfrac{1}{2}a\phi^4 , \qquad \phi^2 \equiv \sum_i \phi_i^2 , \tag{2.77}$$

i.e., we have arbitrarily set all tree-level dimensionful parameters such as scalar masses to zero. Classically, this model does not exhibit spontaneous symmetry breaking.

To perform the one-loop computation of $V_{\rm eff}$, we must first compute the tree-level ϕ-dependent particle masses in the shifted theory as indicated above. For example, $m_W^2(\phi) = \frac{1}{4}g^2\phi^2$. [The physical W mass is obtained by taking $|\phi| = v$, with $v = (\sqrt{2}G_F)^{-1/2} = 246$ GeV.] Clearly, all gauge bosons and fermion ϕ-dependent masses are proportional to ϕ in the Standard Model. For the scalars, this would be true as well if there were no tree-level scalar masses as in the Coleman-Weinberg scenario. However, it is useful to perform a more general computation which will be of use to us at a later point in these lectures. Consider the scalar potential for an $O(N)$ symmetric scalar sector which contains tree-level quadratic terms

$$V^{(0)}(\phi) = \tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4}\lambda\phi^4 . \tag{2.78}$$

Then, the elements of the (symmetric) squared mass matrix are

$$\begin{aligned} M_{ii}^2(\phi) &= \mu^2 + \lambda\phi^2 + 2\lambda\phi_i^2 , \\ M_{ij}^2(\phi) &= 2\lambda\phi_i\phi_j \qquad (i \neq j) . \end{aligned} \tag{2.79}$$

The eigenvalues of this squared mass matrix are easily computed: $\mu^2+3\lambda\phi^2$ appears once and $\mu^2 + \lambda\phi^2$ appears $N-1$ times. This suggests that it would be useful to compute the contribution to the effective potential of a particle with a ϕ-dependent mass of the form*

$$M^2(\phi) = \mu^2 + \lambda\phi^2 . \tag{2.80}$$

Adding the appropriate counterterms, I find

$$V^{(1)}(\phi) = A + B\phi^2 + C\phi^4 + \frac{\Lambda^2}{32\pi^2}(\mu^2 + \lambda\phi^2) + \frac{1}{64\pi^2}(\mu^2+\lambda\phi^2)^2 \ln\frac{\mu^2+\lambda\phi^2}{\Lambda^2} . \tag{2.81}$$

---

* This is clearly not the most general form for $M_i^2(\phi)$. For example, more complicated forms arise in supersymmetric models for the ϕ-dependent masses of the neutralino and charginos.

The counterterms can be fixed by the following renormalization conditions:

$(i)\ V^{(1)}(0) = 0$ [field independent terms are irrelevant]

$(ii)\ \frac{dV^{(1)}}{d\phi^2}\Big|_{\phi=0} = 0$ [fixes the mass parameter appearing in $V^{(0)}$]

$(iii)\ \frac{d}{d\phi^2}\frac{dV^{(1)}}{d\phi^2}\Big|_{\phi=0} = 0$ [fixes the coupling parameter appearing in $V^{(0)}$].

Imposing these conditions allows us to solve for A, B, and C. If we insert the results obtained back into eq. (2.81), the ultraviolet cutoff disappears and we get

$$V^{(1)}(\phi) = \frac{1}{64\pi^2}\left[(\mu^2+\lambda\phi^2)^2 \ln\left(\frac{\mu^2+\lambda\phi^2}{\mu^2}\right) - \lambda\phi^2(\mu^2+\tfrac{3}{2}\lambda\phi^2)\right]. \qquad (2.82)$$

This form is not very convenient for the Coleman-Weinberg computation, where we would like to set $\mu^2 = 0$. The logarithmic divergence at $\mu = 0$ is an infrared problem which arises in the massless theory due to the on-shell renormalization condition chosen above. In fact, we can circumvent this problem by imposing the minimum condition on the full potential

$$\frac{d}{d\phi^2}[V^{(0)} + V^{(1)}]\Big|_{\phi=v} = 0. \qquad (2.83)$$

If $V^{(0)}(\phi) = \frac{1}{2}a\phi^4$, then I can solve for a in terms of v

$$a = \frac{-\lambda}{32\pi^2 v^2}\left[(\mu^2+\lambda v^2)\ln\left(\frac{\mu^2+\lambda v^2}{\mu^2}\right) - \lambda v^2\right]. \qquad (2.84)$$

The full effective potential then takes on the following form

$$V(\phi) = \frac{1}{64\pi^2}\left[M^4(\phi)\ln\left(\frac{M^2(\phi)}{\mu^2}\right) - M^2\frac{\lambda\phi^4}{v^2}\ln\left(\frac{M^2}{\mu^2}\right) - \lambda\mu^2\phi^2 - \frac{\lambda^2\phi^4}{2}\right], \qquad (2.85)$$

where $M^2(\phi)$ is defined in eq. (2.80) and $M^2 \equiv M^2(v) = \mu^2 + \lambda v^2$. In the simplest model containing one Higgs doublet, $\mu = 0$ for all particles contributing to the one-loop effective potential. Setting $\mu = 0$ in eq. (2.85), we obtain

$$V(\phi) = \frac{M^4\phi^4}{64\pi^2 v^4}\left[\ln\left(\frac{\phi^2}{v^2}\right) - \tfrac{1}{2}\right], \qquad (2.86)$$

which is the famous result of Coleman and Weinberg. Thus, whereas the tree-level potential of the massless theory exhibits no symmetry breaking, the one-loop effective potential possesses a global minimum at $\phi = v$. Thus, we have demonstrated the possibility of radiatively induced symmetry breaking. The same conclusion can be reached in the more general case of $\mu \neq 0$. One can easily check that eq. (2.85) satisfies $V(0) = 0$ and $V(v) < 0$ for all choices of λ and μ. (We will discuss shortly whether the conclusion of radiatively induced symmetry breaking is valid given the nature of the approximations made.)

The Higgs Boson Mass in the Coleman-Weinberg Model

In the Coleman-Weinberg model, the Higgs boson mass is zero at tree-level. Then, the scalar mass matrix takes the form

$$M_{ij}^2 = \frac{d^2 V}{d\phi_i d\phi_j}\bigg|_{\phi=v} . \tag{2.87}$$

As above, we take ϕ to be an N-component scalar. For an $O(N)$ symmetric potential, V depends only on ϕ^2, so we can write

$$M_{ij}^2 = 4v_i v_j \frac{d}{d\phi^2}\frac{dV}{d\phi^2}\bigg|_{\phi=v} \tag{2.88}$$

This mass matrix possesses $N-1$ zero eigenvalues (corresponding to the Goldstone bosons) and one nonzero eigenvalue which corresponds to a physical Higgs boson with squared mass

$$m_H^2 = 4v^2 \frac{d}{d\phi^2}\frac{dV}{d\phi^2}\bigg|_{\phi=v} . \tag{2.89}$$

Inserting the general expression for V obtained in eq. (2.85) yields

$$m_H^2 = \frac{\lambda^2 v^2}{8\pi^2}\left[1 - \frac{\mu^2}{\lambda v^2}\ln\left(\frac{M^2}{\mu^2}\right)\right] . \tag{2.90}$$

This expression is more general than our present needs (although we will make use of this more general result in Lecture 4). In the Coleman-Weinberg model with a single Higgs multiplet, $\mu = 0$ so that $M^2 = \lambda v^2$ and

$$m_H^2 = \frac{M^4}{8\pi^2 v^2} . \tag{2.91}$$

The generation of a nonzero scalar mass by radiative corrections indicates that the scale invariance of the classical scalar Lagrangian is violated by quantum mechanical effects. Thus, setting the tree-level scalar masses to zero in the Coleman-Weinberg model is "unnatural" and is just as arbitrary as any other choice for the tree-level scalar masses.

The analysis above suggests that it is possible to take a gauge theory coupled to a multiplet of scalar fields in which the gauge symmetry is unbroken at tree-level, but is broken when radiative corrections are taken into account. Since this conclusion is based on an approximate (*i.e.*, one-loop) computation, one must check whether the conclusion is reliable. In order to address this question, we must examine the validity of the perturbation expansion. Consider massless scalar electrodynamics, in which a massless complex scalar field is coupled to an abelian $U(1)$ gauge theory. The effective potential can be computed as outlined above, although as remarked above it is somewhat inconvenient to choose on-shell renormalization conditions due to potential infrared problems. Coleman and Weinberg replaced the third condition below eq. (2.81) with $(d^4V^{(1)}/d\phi^4)_{\phi=\mathcal{M}}=0$; the other two conditions were left unchanged. The resulting effective potential at one-loop in scalar electrodynamics is

$$V(\phi)=\tfrac{1}{4}\lambda\phi^2+\left(\frac{5\lambda^2}{32\pi^2}+\frac{3e^4}{64\pi^2}\right)\phi^4\left[\ln\left(\frac{\phi^2}{\mathcal{M}^2}\right)-\frac{25}{6}\right]. \tag{2.92}$$

Note that through the renormalization condition, λ depends implicitly on $\mathcal{M}$. This dependence can be deduced from the requirement that V be independent of $\mathcal{M}$.

The potential in eq. (2.92) exhibits spontaneous symmetry breaking of the $U(1)$ symmetry. In order to discuss the validity of this conclusion, first suppose that $e=0$. Then one must reject the conclusion that the symmetry is spontaneously broken, since the radiatively induced minimum occurs at values of ϕ such that $\lambda\ln(\phi^2/\mathcal{M}^2)/\pi^2\sim\mathcal{O}(1)$, which is outside the region of validity of perturbation theory. Basically, the $\mathcal{O}(\lambda^2)$ term in eq. (2.92) must be more important than the $\mathcal{O}(\lambda)$ term in order to change the potential minimum from the symmetric ($\phi=0$) to the asymmetric ($\phi\neq0$) point. The above analysis can be improved using renormalization group (RG) techniques. The RG-improved analysis implies that $\phi=0$ is in fact the true minimum. Thus the one-loop analysis is misleading—the symmetry is unbroken.

Now consider what happens If $e\neq0$. If we assert that $e^4\sim\mathcal{O}(\lambda)$, then we must drop the $\mathcal{O}(\lambda^2)$ term in eq. (2.92) for consistency. Nevertheless, the conclusion that the symmetry is spontaneously broken is trustworthy since higher order corrections to the effective potential are indeed small! These conclusions are not altered by the RG analysis.

Thus, we can now write down Coleman and Weinberg's "prediction" for the Higgs boson mass in the Standard Model. According to the last remark, we drop $\mathcal{O}(\lambda^2)$ terms. This is equivalent to deleting the effects of the scalar ϕ in $V^{(1)}(\phi)$.

We can immediately conclude from eqs. (2.74) and (2.91) that

$$m_H^2 = \frac{1}{8\pi^2 v^2} \sum_i (-1)^{2J_i}(2J_i+1)C_i M_i^4 \tag{2.93}$$

where we sum over vector bosons and fermions of mass M_i and spin J_i (and C_i counts color and electric charge degrees of freedom). Explicitly,

$$m_H^2 \equiv m_{CW}^2 \simeq \frac{6m_W^4 + 3m_Z^4 - 12m_t^4}{8\pi^2 v^2} \simeq (10 \text{ GeV})^2 \left(1 - 1.09\frac{m_t^4}{m_W^4}\right) . \tag{2.94}$$

This was an interesting prediction in the days when the top quark was thought to be light (compared to the gauge bosons). If we accept the CDF experimental limit of $m_t \geq 89$ GeV, this would yield $m_H^2 < 0$ implying an instability. The origin of this instability is most easily seen by writing the effective potential in the form given in eq. (2.86)

$$V(\phi) = \frac{\phi^4}{64\pi^2 v^4} \text{Str } M_i^4 \left[\ln\left(\frac{\phi^2}{v^2}\right) - \tfrac{1}{2}\right] . \tag{2.95}$$

For Str $M_i^4 < 0$, the potential is sick since $V(\phi) \to -\infty$ for large ϕ, which means that there is no stable minimum. To repair this instability, one could add new particles to the model (*e.g.*, scalars) such that Str $M_i^4 > 0$. Otherwise, one must conclude that the vanishing quadratic term in the tree-level scalar potential is untenable.

2.4 Lower and Upper Higgs Mass Bounds

A Lower Higgs Mass Bound: The Linde-Weinberg Bound

Let us suspend our belief in the heavy top quark for a moment and assume that Str $M_i^4 > 0$ so that the Coleman-Weinberg mechanism can work. Consider what happens if we perturb the picture by adding a small positive mass-squared term at tree-level

$$V(\phi) = \tfrac{1}{2}m^2\phi^2 + \tfrac{1}{2}a\phi^4 + V_{CW}^{(1)}(\phi) . \tag{2.96}$$

If the effect of the $m^2\phi^2$ term on the Higgs mass is of the same order as the Higgs mass computed from $V_{CW}^{(1)}(\phi)$ then it is consistent to treat $m^2\phi^2$ as a perturbation without including its effects on the calculation of $V_{CW}^{(1)}(\phi)$. Thus, we can simply repeat the calculations above with minor modifications. When we reach the step of trading a for v, we obtain

$$a = \frac{-m^2}{2v^2} + a_0 \,. \tag{2.97}$$

where $a_0 \equiv a(m = 0)$ is the expression for a given in eq. (2.84). The final result for the potential is

$$V(\phi) = \tfrac{1}{2}m^2\phi^2\left(1 - \frac{\phi^2}{2v^2}\right) + V_{\rm CW}(\phi)\,, \tag{2.98}$$

and the Higgs mass is [see eq. (2.90)]

$$m_H^2 = -2m^2 + \frac{\lambda^2 v^2}{8\pi^2}\left[1 - \frac{\mu^2}{\lambda v^2}\ln\left(\frac{M^2}{\mu^2}\right)\right]. \tag{2.99}$$

Naively, it appears that one can make m_H^2 arbitrarily small by increasing m^2. However, as m^2 is increased, the value of the potential at its minimum increases as well. By demanding that $V(v) \leq 0$ (so that the local minimum at $\phi = v$ remains a global minimum), one finds that $m^2 \leq m^2_{\rm max}$. Inserting this value into the formula for m_H^2 gives $m_H^2 \geq m_{LW}^2$, where

$$m_{LW}^2 = \frac{\lambda^2 v^2}{16\pi^2}\left\{1 - \frac{2\mu^2}{\lambda v^2}\left[1 - \frac{\mu^2}{\lambda v^2}\ln\left(\frac{M^2}{\mu^2}\right)\right]\right\}. \tag{2.100}$$

This is the Linde-Weinberg bound. In the case where $\mu = 0$ (so that $M^2 = \lambda v^2$),

$$m_{LW}^2 = \frac{M^4}{16\pi^2 v^2} = \tfrac{1}{2}m_{CW}^2\,. \tag{2.101}$$

Applying this result to the Standard Model, one obtains

$$m_H^2 \geq m_{LW}^2 = \tfrac{1}{2}m_{CW}^2 \simeq (7\ {\rm GeV})^2\left(1 - 1.09\frac{m_t^4}{m_W^4}\right). \tag{2.102}$$

As before, this result is valid only when the right hand side above is positive. For $m_t \gtrsim 78$ GeV, other considerations are required to deduce a lower bound for the Higgs mass.

Renormalization Group (RG) Considerations

In the one-loop corrected potential computed above, we encountered logarithms, such as $\ln(\phi^2/v^2)$. For small deviations of ϕ from $\phi \simeq v$, this logarithm is small and presents no particular problem. However, for values of ϕ far away from the potential minimum, this logarithm becomes large and our perturbative calculation becomes suspect.

The RG is precisely what is needed to sum up the large logarithms. Let us explore this in some detail. As a warm-up exercise, consider a theory of a single real scalar field with no tree-level scalar mass: $V^{(0)}(\phi) = \frac{1}{4}\lambda\phi^4$. Since $\mu = 0$, one must introduce a mass scale $\mathcal{M}$ in defining the renormalized coupling. As previously discussed, a convenient renormalization condition is $(d^4V^{(1)}/d\phi^4)_{\phi=\mathcal{M}} = 0$. The resulting one-loop effective potential is

$$V(\phi) = \tfrac{1}{4}\lambda\phi^4 + \frac{9\lambda^2}{64\pi^2}\phi^4\left[\ln\left(\frac{\phi^2}{\mathcal{M}^2}\right) - \frac{25}{6}\right]. \qquad (2.103)$$

The factor of $-25/6$ above is a minor nuisance and I will drop it from now on. This involves no loss of generality, since I can simply absorb it into the definition of $\mathcal{M}$. Note that since $\mathcal{M}$ is an arbitrary parameter, the coupling constant λ and the field ϕ must depend implicitly on $\mathcal{M}$ in such a way that $V(\phi)$ is independent of $\mathcal{M}^2$.

The statement of the $\mathcal{M}^2$-independence of V is the renormalization group equation (RGE): $dV/d\mathcal{M}^2 = 0$. Using the chain rule,

$$\left(\mathcal{M}^2\frac{\partial}{\partial\mathcal{M}^2} + \beta(\lambda)\frac{\partial}{\partial\lambda} - \gamma\phi\frac{\partial}{\partial\phi}\right)V(\phi) = 0\,, \qquad (2.104)$$

where

$$\begin{aligned}\beta(\lambda) &\equiv \mathcal{M}^2\frac{d\lambda}{d\mathcal{M}^2}\,,\\ \gamma\phi &\equiv -\mathcal{M}^2\frac{d\phi}{d\mathcal{M}^2}\,.\end{aligned} \qquad (2.105)$$

It is convenient to introduce the variable $t \equiv \ln(\phi/\mathcal{M}^2)$. Note that

$$\mathcal{M}^2\frac{\partial}{\partial\mathcal{M}^2} = -(1+\gamma)\frac{\partial}{\partial t}\,, \qquad (2.106)$$

so that the RGE takes the form

$$\left(-\frac{\partial}{\partial t} + \overline{\beta}\frac{\partial}{\partial\lambda} - \overline{\gamma}\phi\frac{\partial}{\partial\phi}\right)V(\phi) = 0, \qquad (2.107)$$

where $\overline{\beta} \equiv \beta/(1+\gamma)$ and $\overline{\gamma} \equiv \gamma/(1+\gamma)$. Since V must be of the form

$$V(\phi) = \tfrac{1}{4}\Lambda(\lambda, t)\phi^4 \tag{2.108}$$

we obtain a RGE for the function $\Lambda(\lambda, t)$

$$\left(-\frac{\partial}{\partial t} + \overline{\beta}\frac{\partial}{\partial\lambda} - 4\overline{\gamma}\right)\Lambda(\lambda, t) = 0\,. \tag{2.109}$$

The solution is

$$\Lambda(\lambda, t) = \lambda(t)\exp\left\{-4\int_0^t dt'\overline{\gamma}\,[\lambda(t')]\right\}, \tag{2.110}$$

where $\lambda(t)$ is the running coupling constant defined by

$$\frac{d\lambda(t)}{dt} = \overline{\beta}[\lambda(t)]\,, \qquad \lambda(0) \equiv \lambda\,. \tag{2.111}$$

In these lectures, I will never go beyond the one-loop approximation so I can put $\overline{\beta} = \beta$ and $\overline{\gamma} = \gamma$. Furthermore, in the pure scalar field theory considered here, there is no scalar wave function renormalization at one-loop, so I may set $\gamma = 0$. One therefore obtains a very simple result for the RG-improved potential

$$V(\phi) = \tfrac{1}{4}\lambda(t)\phi^4\,. \tag{2.112}$$

Comparing with eq. (2.103) [with the $-25/6$ removed], it follows that

$$\beta(\lambda) = \frac{9\lambda^2}{16\pi^2} + \mathcal{O}(\lambda^3)\,. \tag{2.113}$$

Using this result to determine $\lambda(t)$, we obtain

$$V(\phi) = \frac{\frac{1}{4}\lambda\phi^4}{1 - \frac{9\lambda}{16\pi^2}\ln\left(\frac{\phi^2}{\mathcal{M}}\right)}\,. \tag{2.114}$$

If we expand out the denominator in eq. (2.114), we see that the leading logarithm of the one-loop potential [eq. (2.103)] has been correctly reproduced. Note that the minimum of the one-loop RGE-improved potential is at $\phi = 0$ in contrast to the potential of eq. (2.103) which possesses a minimum at $\phi \neq 0$. The RGE analysis has extended the range of ϕ over which the expression for $V(\phi)$ is valid, and we can correctly conclude that the symmetry is not spontaneously broken in the pure scalar theory.

By the same technique, we may derive the one-loop RG-improved Higgs potential in the Standard Model. The result is

$$V(\phi) = \tfrac{1}{2}\mu^2(t)G^2(t)\phi^2 + \tfrac{1}{4}\lambda(t)G^4(t)\phi^4 \tag{2.115}$$

with

$$G(t) \equiv \exp\left\{-\int_0^t dt'\gamma[g_i(t'),\lambda(t')]\right\}, \tag{2.116}$$

$$\begin{aligned}\frac{d\mu^2(t)}{dt} &= \mu^2(t)\beta_{\mu^2}[g_i(t),\lambda(t)]\\ \frac{d\lambda(t)}{dt} &= \beta_\lambda[g_i(t),\lambda(t)]\\ \frac{dg_i(t)}{dt} &= \beta_{g_i}[g_i(t),\lambda(t)]\,.\end{aligned} \tag{2.117}$$

The g_i consist of the gauge couplings and Higgs-fermion Yukawa couplings. To a good approximation, all fermions apart from the top-quark are massless. Hence, we can ignore all Yukawa couplings with the exception of the Higgs-top quark coupling, $g_t \equiv \sqrt{2}m_t/v$.

The beta-functions listed above can be computed by a variety of techniques. Here I will sketch one method which makes use of the effective potential formalism. For simplicity, I shall set $\mu^2 = 0$. The one-loop effective potential can then be expressed in the following form

$$V(\phi) = \tfrac{1}{4}\lambda\phi^4 + \frac{1}{64\pi^2}\,\mathrm{Str}\,M^4(\phi)\ln\left(\frac{M^2(\phi)}{\mathcal{M}^2}\right). \tag{2.118}$$

Since $V(\phi)$ satisfies the RGE given in eq. (2.104), we obtain (up to one-loop accuracy)

$$\left(\beta_\lambda\frac{\partial}{\partial\lambda} - \gamma\phi\frac{\partial}{\partial\phi}\right)\lambda\phi^4 = \frac{1}{16\pi^2}\,\mathrm{Str}\,M^4(\phi)\,. \tag{2.119}$$

It follows that

$$\beta_\lambda = 4\lambda\gamma + \frac{1}{16\pi^2}\,\mathrm{Str}\,M^4(\phi)/\phi^4\,. \tag{2.120}$$

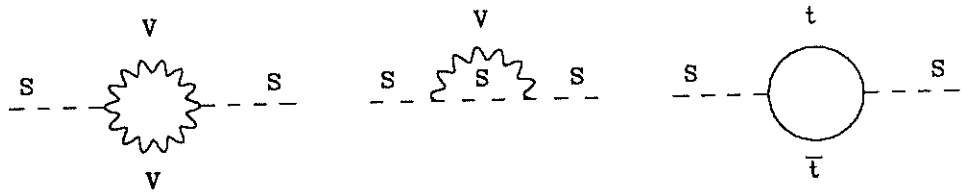
The contributions to Str $M^4(\phi)$ from the $W^\pm$, Z, t-quark, and scalar sector are

$$\begin{aligned}\mathrm{Str}\,M^4(\phi) =&(6)[\tfrac{1}{4}g^2\phi^2]^2 + (3)[\tfrac{1}{4}(g^2+g'^2)\phi^2]^2 + (-12)[\tfrac{1}{2}g_t^2\phi^2]^2\\ &+ [3\lambda\phi^2]^2 + (3)[\lambda\phi^2]^2\,,\end{aligned} \tag{2.121}$$

[Note that the contribution of the scalar sector is obtained by using the results below eq. (2.79) with $\mu = 0$ and $N = 4$.] Adding up the various pieces, we obtain

$$\mathrm{Str}\, M^4(\phi)/\phi^4 = \tfrac{3}{16}\left[2g^4 + (g^2 + g'^2)^2\right] - 3g_t^4 + 12\lambda^2 . \tag{2.122}$$

To compute γ, we need to evaluate the wave function renormalization of the scalar field. The diagrams involved are



and the result is

$$\gamma = \frac{1}{64\pi^2}\left[6g_t^4 - \frac{3}{2}(3g^2 + g'^2)\right] . \tag{2.123}$$

Combining our results,

$$\beta_\lambda = \frac{1}{16\pi^2}\left\{12\lambda^2 + 6\lambda g_t^2 - 3g_t^4 - \tfrac{3}{2}\lambda(3g^2 + g'^2) + \tfrac{3}{16}\left[2g^4 + (g^2 + g'^2)^2\right]\right\} . \tag{2.124}$$

The computation of β_{g_t} is a little more involved, since it requires the evaluation of scalar wave function renormalization, fermion wave function renormalization, and $\phi f \bar{f}$ vertex renormalization. This will be left as an exercise for the reader. The end results of this analysis are the RGE's for λ and g_t:

$$\frac{dg_t}{dt} = \frac{1}{16\pi^2}\left(\tfrac{9}{4}g_t^3 - 4g_s^2 g_t - \tfrac{9}{8}g^2 g_t - \tfrac{17}{24}g'^2 g_t\right) , \tag{2.125}$$

$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2}\left\{12\lambda^2 + 6\lambda g_t^2 - 3g_t^4 - \tfrac{3}{2}\lambda(3g^2 + g'^2) + \tfrac{3}{16}\left[2g^4 + (g^2 + g'^2)^2\right]\right\} . \tag{2.126}$$

These two equations contain information on the allowed domains of the t-quark and Higgs masses, via the relations

$$g_t = \frac{\sqrt{2}m_t}{v}, \qquad \lambda = \frac{m_H^2}{2v} , \tag{2.127}$$

where $v = 246$ GeV.

An Upper Higgs Mass Bound

If λ is large, then I can ignore all but the first term on the right-hand side of eq. (2.126). The solution to this RGE is then

$$\frac{1}{\lambda(v)} - \frac{1}{\lambda(\Lambda)} = \frac{3}{4\pi^2}\ln(\Lambda^2/v^2)\,, \tag{2.128}$$

which exhibits the famous Landau pole: $\lambda(\Lambda) = \infty$ at $\Lambda^2 = v^2\exp[4\pi^2/3\lambda(v)]$. If this behavior were not modified by higher loops, we would say that new physics must enter at an energy scale near Λ.

Alternatively, suppose I demand that no new physics appear until a scale Λ. Clearly, $\Lambda \leq M_P \simeq 10^{19}$ GeV, although it could be much smaller. Assuming that $\Lambda > m_H$ (so that if we integrate out all new physics above Λ we would find an effective theory identical to the Standard Model with a single Higgs boson), what is the maximum value of m_H as a function of Λ? Since m_H is related to λ via $m_H^2 = 2\lambda(v)v^2$, I can derive a bound by setting $1/\lambda(\Lambda) = 0$. Then,

$$m^2_{H,\rm max} = \frac{8\pi^2v^2}{3\ln(\Lambda^2/v^2)}\,. \tag{2.129}$$

Thus, m_H cannot become arbitrarily large without violating the inequality $\Lambda > m_H$. The two extreme cases are: $\Lambda = M_P$ which yields $m_{H,\rm max} \simeq 150$ GeV, and $\Lambda = m_{H,\rm max}$ which implies $m_{H,\rm max} \simeq 800$ GeV. This analysis can be improved by solving numerically the complete set of coupled RGE's and studying the behavior of $m_{H,\rm max}$ as a function of m_t and Λ. This was first done by Cabibbo, Maiani, Parisi and Petronzio (under the assumption that Λ was the grand unification scale) and has been repeated for other values of Λ by numerous authors. A typical output of such a computation is shown in fig. 2.

It must be emphasized that the analysis based on the one-loop RGE's can at best provide a qualitative indication for an upper Higgs mass bound. The reason is simple to understand: for large m_H, the Higgs self-coupling λ is large, and the one-loop RGE's are not reliable (having been derived by perturbative methods). As a result, there has been a heroic effort to provide a non-perturbative version of the above arguments, primarily via lattice techniques—strong coupling expansions and lattice monte carlo computations. Remarkably, numerical results based on lattice computations seem to be in rough agreement with the perturbative results described above, and various groups have quoted upper bounds for the Higgs mass of around 650 GeV (for $\Lambda \sim 2m_H$).

The results of these lattice computations provide strong evidence for the "triviality" of scalar field theories. Triviality is most simply understood in the context of

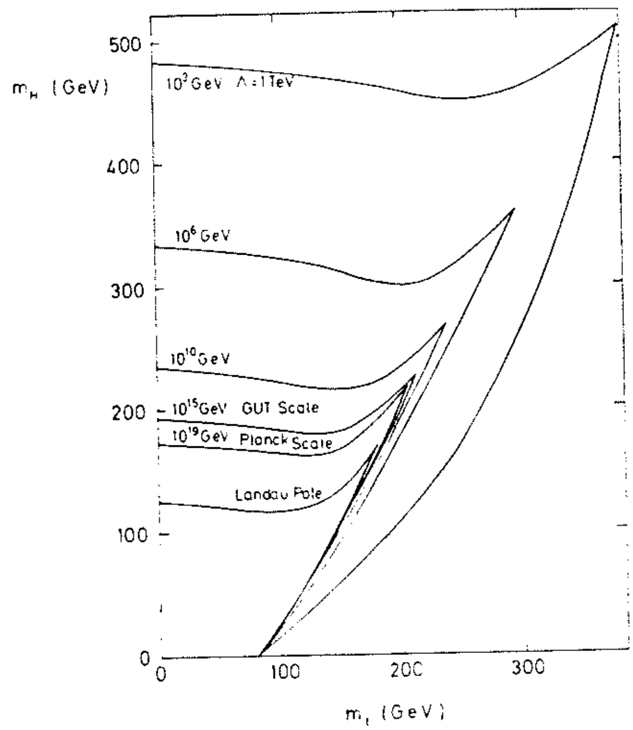


Figure 2 The allowed parameter space of m_H and m_t for various embedding scales Λ. Allowed is the area around the origin bounded by the various curves. The horizontal lines come from avoiding triviality and the vertical lines are determined from $\lambda(t)$ becoming negative at scales lower than Λ. This figure is taken from M. Lindner, *Z. Phys.* **C31**. 295 (1986).

eq. (2.128). Suppose I try to take the cutoff $\Lambda \to \infty$. Stability of the potential requires $\lambda > 0$ at all scales below Λ. Thus, if $\Lambda \to \infty$ [with $\lambda(\Lambda) > 0$], then $\lambda(v) \to 0$. That is, at the low-energy scale (v), the scalar field theory is non-interacting—it is trivial! On the lattice, the inverse lattice spacing plays the role of the cutoff. Thus triviality in this case means that the coupling λ is driven to zero in the continuum limit. That is, scalar field theory is a free field theory.

It is rigorously known that a one-component scalar field theory (in four space-time dimensions) is trivial. Completely rigorous results are not yet available for more complicated field theories. Nevertheless, the numerical lattice work and the strong-coupling expansions of Luscher and Weisz present strong evidence for triviality of the Higgs sector. Operationally, this means that the scalar field theory cannot exist up to arbitrarily high scales. In fact, triviality is not so trivial. A priori, the β_λ-function could have had a zero at a non-zero value of λ (outside the range of validity of perturbation theory), leading to a non-trivial theory! The conclusion of the lattice computations is that no such non-trivial fixed point exists.

A Lower Bound of the Higgs Mass for $m_t \gtrsim m_W$

Let us return to the RGE's for g_t and λ given in eqs. (2.125) and (2.126). These equations can be used to deduce a lower bound for the Higgs mass for $m_t \gtrsim m_W$. (For $m_t \lesssim m_W$, the Linde-Weinberg bound applies.) To see how such a bound arises, take g_t constant and ignore all terms in the RGE proportional to gauge coupling constants (since these tend to be rather small). Then, eq. (2.126) takes the following form:

$$\frac{d\lambda}{dt} = A(\lambda - \lambda_+)(\lambda - \lambda_-)\,, \tag{2.130}$$

with $\lambda_- < 0 < \lambda_+$ and $A > 0$. Thus, if $0 < \lambda(v) < \lambda_+$, then λ will be driven negative at high momentum scales. This would be disastrous, since the sign of the $\lambda\phi^4$ term is governed by $\lambda(\phi)$, as shown in eq. (2.115). Thus, for large values of the scalar field ϕ, the coefficient of the leading term in $V(\phi)$ would be negative; *i.e.*, the Higgs potential would be unstable.

A very crude estimate for the Higgs mass bound can be obtained by demanding that $\lambda(v) > \lambda_+$. This would imply that would imply

$$\begin{aligned} m_H^2 &> m_W^2 + \tfrac{1}{2}m_Z^2 - m_t^2 + \sqrt{(m_W^2 + \tfrac{1}{2}m_Z^2 - m_t^2)^2 + 4m_t^4 - 2m_W^4 - m_Z^4} \\ &\gtrsim (\sqrt{5} - 1)m_t^2\,. \end{aligned} \tag{2.131}$$

In addition, for sufficiently large m_t, the top quark Yukawa coupling would develop a Landau pole. Thus, at fixed Higgs mass, the top quark mass is bounded from above. All these features are confirmed by a more complete analysis, which makes use of the full one-loop RGE's. Typical results are shown in fig. 2. Note that $\lambda(v) < \lambda_+$ is allowed so long as $\lambda(v)$ is not driven negative between the electroweak scale and the scale Λ. As Λ is lowered, there is less room for evolution, and the condition for the stability of the potential is weakened. Likewise, the *upper* limits on m_t and m_H are also weakened as Λ is lowered since there is less room for coupling constant evolution to reach the Landau pole.

2.5 Unitarity Bounds Revisited and the Equivalence Theorem

An Upper Bound for the Higgs Mass?

At the end of Lecture 1, I indicated that by removing the Higgs boson entirely from the theory, tree-level unitarity was violated, due to bad high energy behavior of scattering amplitudes involving longitudinal vector bosons. The bad high energy behavior is removed once the Higgs boson is restored. But if $m_H \gg G_F^{-1/2}$, then unitarity would still be violated over some range of $\sqrt{s}$. Thus, demanding tree-level unitarity for all $\sqrt{s}$ would place an upper limit on the value of m_H.

Consider $W_L^+W_L^- \to W_L^+W_L^-$. For $s, m_H^2 \gg m_W^2$, the amplitude was given previously in eq. (1.42). Let us project out the $J = 0$ partial wave. (It turns out that this leads to the strictest limit.)

$$\begin{aligned}
\mathcal{M}^{J=0} &= \frac{1}{16\pi s}\int_{-s}^{0} dt\,\mathcal{M}(L,L;L,L) \\
&= \frac{-G_F m_H^2}{8\pi\sqrt{2}}\left[2 + \frac{m_H^2}{s - m_H^2} - \frac{m_H^2}{s}\ln\left(1 + \frac{s}{m_H^2}\right)\right].
\end{aligned} \tag{2.132}$$

For $s \gg m_H^2$,

$$\mathcal{M}^{J=0} = \frac{-G_F m_H^2}{4\pi\sqrt{2}}. \tag{2.133}$$

At this point, we could use $|\mathcal{M}^{J=0}| \leq 1$ to obtain the desired bound on m_H. However, the bound can be strengthened as follows. Partial wave unitarity implies

$$|\mathcal{M}^J|^2 \leq |\mathrm{Im}\ \mathcal{M}^J|. \tag{2.134}$$

Indeed, this implies that $|\mathcal{M}^J| \leq 1$. However, it also implies that

$$(\mathrm{Re}\ \mathcal{M}^J)^2 \leq |\mathrm{Im}\ \mathcal{M}^J|\left(1 - |\mathrm{Im}\ \mathcal{M}^J|\right). \tag{2.135}$$

The right hand side cannot be larger than 1/4, so we obtain

$$|\mathrm{Re}\ \mathcal{M}^J| \leq \tfrac{1}{2}. \tag{2.136}$$

Since Born amplitudes are real, this will improve the limits on m_H^2 based on $|\mathcal{M}^J| \leq 1$ by a factor of 2. Hence,

$$m_H^2 \leq \frac{2\pi\sqrt{2}}{G_F} \simeq (850\ \mathrm{GeV})^2. \tag{2.137}$$

The most stringent bound is obtained by performing a full coupled channel analysis for the scattering of longitudinal gauge bosons into $W_L^+W_L^-, Z_LZ_L, Z_LH$

and HH. The largest eigenvalue of the amplitude matrix results in the most restrictive bound

$$m_H^2 \leq \frac{4\pi\sqrt{2}}{3G_F} \simeq (700\ \mathrm{GeV})^2\,. \tag{2.138}$$

Is this really an upper bound? If $m_H \gtrsim 700$ GeV, this analysis simply tells us that perturbation theory is no longer reliable. One-loop corrections show no sign of ameliorating the situation, as they increase the $J = 0$ partial wave amplitude. It is amusing that this bound is roughly the same as the one obtained in the lattice studies—indicating that our naive conclusions from perturbative analysis may in fact be reliable!

Conceptually, it is more useful to consider the limit $m_Z^2 \ll s \ll m_H^2$. Then, tree-level unitarity breaks down for energies above some critical energy $\sqrt{s} \geq \sqrt{s_c}$. For example, in this limit, the amplitude for $W_L^+W_L^- \to W_L^+W_L^-$

$$\mathcal{M}^{J=0} = \frac{G_F s}{16\pi\sqrt{2}} \tag{2.139}$$

violates unitarity at large s. The most restrictive bound arises form the isospin zero channel $\sqrt{\frac{1}{6}}(2W_L^+W_L^- + Z_L Z_L)$, and the critical energy is

$$s_c = \frac{4\pi\sqrt{2}}{G_F} = (1.2\ \mathrm{TeV})^2\,. \tag{2.140}$$

This bound is more meaningful than eq. (2.138) in that it is applicable beyond perturbation theory. The reason is that the regime $m_Z^2 \ll s \ll m_H^2$ is precisely the one where rigorous low-energy theorems apply independent of the dynamics of the electroweak symmetry breaking sector. That is,

$$\mathcal{M}^{J=0}(W_L^+W_L^- \to W_L^+W_L^-) = \frac{G_F s}{16\pi\sqrt{2}}\,. \tag{2.141}$$

is rigorous to all orders in Higgs self-coupling (and to leading order in the gauge coupling g^2).

This tells us that $\sqrt{s_c}$ is a meaningful energy. The bad high energy behavior must be repaired by the dynamics of any proposed mechanism for electroweak symmetry breaking. Thus, by the time $s = s_c$, one of two things must have happened:

(*i*) $\mathcal{M}^{J=0}$ is repaired by an elementary scalar Higgs, or

(*ii*) $\mathcal{M}^{J=0}$ is repaired by some other mechanism for ESB.

That is, *the physics of electroweak symmetry breaking must be exposed at or below the 1 TeV energy scale.*

The Equivalence Theorem

The Equivalence Theorem of spontaneously broken gauge theories is one manifestation of the rigorous low energy theorems. This theorem states that physical S-matrix amplitudes involving on-shell longitudinal vector bosons may be evaluated approximately in the R_ξ-gauge by substituting the corresponding Goldstone bosons. That is,

$$\mathcal{M}(W_L, Z_L, \ldots) = \mathcal{M}(w, z, \ldots)_R + \mathcal{O}\left(\frac{m_W}{E_W}, \frac{m_Z}{E_Z}\right). \tag{2.142}$$

Moreover, for $m_W^2 \ll \sqrt{s} \ll m_H^2$, this is true to all orders in $\lambda = m_H^2/2v$ and to leading order in g^2. Thus, the behavior of longitudinal vector bosons in this energy regime is well approximated by an effective low energy theory of the Goldstone bosons

$$\mathcal{L} = -\lambda(w^+w^- + \tfrac{1}{2}zz + \tfrac{1}{2}H^2 + vH)^2 + \text{kinetic energy terms}. \tag{2.143}$$

Integrating out H (which is formally equivalent to taking $\lambda \to \infty$) turns this linear σ-model into a non-linear σ-model. One then finds

$$\mathcal{M}^{J=0}(w^+w^- \to w^+w^-) = \frac{G_F s}{16\pi\sqrt{2}} \tag{2.144}$$

which is identical to the result obtained in eq. (2.141). This is an illustration of the low-energy theorem—it simply depends on the symmetry properties of the Goldstone bosons and is totally independent of the dynamics of the electroweak symmetry breaking.

Here is a sketch of the proof of the Equivalence Theorem. In the R_ξ-gauge, the following gauge fixing functions are employed:

$$F^a = \begin{cases} \partial^\mu W_\mu^\pm - \xi m_W w^\pm \\ \partial^\mu Z_\mu - \xi m_Z z \\ \partial^\mu A_\mu \end{cases} \tag{2.145}$$

The BRS transformations are nearly the same as in Lecture 1. The two equations we will need are:

$$\begin{aligned} \delta_{\mathrm{BRS}}\eta^{a*}(x) &= -\frac{1}{\xi}F^a(x) \\ \delta_{\mathrm{BRS}}F^a(x) &= -\frac{\delta S}{\delta\eta^{*a}(x)}\,. \end{aligned} \tag{2.146}$$

Using the methods of Lecture 1, the following Ward identities can be derived

$$\begin{aligned} &\langle A,\mathrm{out}|\,F^a(x_1)\,|B,\mathrm{in}\rangle = 0 \\ &\langle A,\mathrm{out}|\,T[F^{a_1}(x_1)F^{a_2}(x_2)]\,|B,\mathrm{in}\rangle = -i\xi\delta(x_1-x_2)\delta_{a_1a_2}\langle A|B\rangle\,. \end{aligned} \tag{2.147}$$

Here we use in and out states of *physical* particles rather than the vacuum states, but one can easily check that the same manipulations employed in Lecture 1 go through (as long as these states do not also contain Faddeev-Popov ghosts associated with η^{a*}).

The second Ward identity in eq. (2.147) is even simpler for *connected* Green's functions

$$\langle A,\mathrm{out}|\,T[F^{a_1}(x_1)F^{a_2}(x_2)]\,|B,\mathrm{in}\rangle_{\mathrm{conn}} = 0\,. \tag{2.148}$$

By induction, it is easy to obtain

$$\langle A,\mathrm{out}|\,T[F^{a_1}(x_1)F^{a_2}(x_2)\ldots F^{a_n}(x_n)]\,|B,\mathrm{in}\rangle_{\mathrm{conn}} = 0 \tag{2.149}$$

which is the basis for the Equivalence theorem. To see this, look at the simplest case:

$$\langle A,\mathrm{out}|\,\partial^\mu W^\pm_\mu - \xi m_W w^\pm\,|B,\mathrm{out}\rangle = 0\,. \tag{2.150}$$

To make things perfectly transparent, set $\xi = 1$ so that $W^\pm_\mu$ and $w^\pm$ have the same mass (m_W). Converting to S-matrix elements, we obtain

$$-\frac{ip^\mu}{m_W}\widehat{S}[b \to A + W^\pm(p,\lambda)] = S[B \to A + w^\pm(p)]\,, \tag{2.151}$$

where the full S-matrix element for $B \to A + W^\pm$ is denoted by

$$\varepsilon^\mu_\lambda(p)\widehat{S}[B \to A + W^\pm(p,\lambda)]\,. \tag{2.152}$$

Finally, by using

$$\varepsilon^\mu_L(p) = \frac{p^\mu}{m_W} + \mathcal{O}\left(\frac{m_W}{E_W}\right)\,, \tag{2.153}$$

we end up with

$$-iS[B \to A + W^\pm(p,\lambda = L)] = S[B \to A + w^\pm(p)] + \mathcal{O}\left(\frac{m_W}{E_W}\right)\,. \tag{2.154}$$

This is the Equivalence Theorem for processes with one external gauge boson. The extension of the proof to processes with multiple gauge bosons is very tedious, and can be found in the literature.

Suggestions for Further Reading and a Brief Guide to the Literature

A discussion of the formalism introduced in this lecture can be found in

1. D. Bailin and A. Love, *Introduction to Gauge Field Theory* (Adam Hilger, Bristol, 1986).
2. Stefan Pokorski, *Gauge Field Theories* (Cambridge University Press, Cambridge, 1987).

The discussion of charged pion decay in the Standard Model is a modern rewriting of work that can be found in

3. M. Weinstein, *Phys. Rev.* **D7**, 1854 (1973); **D8**, 2511 (1973).

Radiative symmetry breaking is discussed in

4. S. Coleman and E. Weinberg, *Phys. Rev.* **D7**, 1888 (1973).
5. M. Sher, *Phys. Rep.* **179**, 273 (1989).

Upper Higgs mass bounds are discussed in

6. N. Cabibbo, L. Maiani, G. Parisi and R. Petronzio, *Nucl. Phys.* **B158**, 295 (1979).
7. M. Lindner, *Z. Phys.* **C31**, 295 (1986).

Modern treatments of the Equivalence Theorem and their implications can be found in

8. M.S. Chanowitz and M.K. Gaillard, *Nucl. Phys.* **B261**, 379 (1985).
9. G.J. Gounaris, R. Kogerler, and H. Neufeld, *Phys. Rev.* **D34**, 3257 (1986).
10. H. Veltman, *Phys. Rev.* **D41**, 2294 (1990).

See also

11. *The Higgs Hunter's Guide*, §2.4–2.6.

3. Phenomenology of the Minimal Higgs Boson of the Standard Model

In this lecture I shall focus on the phenomenology of the Higgs boson of the Standard Model with minimal Higgs structure. This model predicts the existence of one neutral CP-even scalar particle (H^0) which I will call the "minimal Higgs boson". First, I shall review the properties of the minimal Higgs boson, and survey the relevant Higgs couplings needed for Higgs searches. In addition to the couplings of H^0 to quarks, leptons and gauge bosons, we will need to know the coupling of H^0 to hadrons (such as protons and pions) in order to obtain limits on light Higgs bosons. We can obtain information on these couplings with the help of Higgs boson low energy theorems. At present, the best limits on the existence of the minimal Higgs boson are obtained from experiments at LEP. These limits will be summarized, and the prospects for improved limits will be discussed. Finally, I will survey the proposed Higgs searches at future colliders—the hadron supercolliders (LHC and SSC) and a possible very high energy e^+e^- linear collider (sometimes called the NLC for "next linear collider" or TLC for "TeV linear collider").

3.1 Properties of the Minimal Higgs Boson

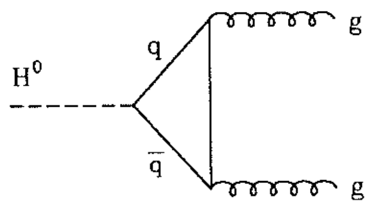
First, let us quickly review the properties of the minimal Higgs boson. The scale of electroweak interactions is set by the vacuum expectation value of the neutral member of the Higgs doublet:

$$v \equiv \frac{2m_W}{g} = 246 \text{ GeV}. \tag{3.1}$$

Of the four degrees of freedom which make up the complex Higgs doublet, three (Goldstone bosons) are absorbed by the $W^\pm$ and Z gauge bosons leaving one physical massive CP-even neutral scalar—the H^0. Although the tree-level couplings of H^0 are all fixed by the theory, the Higgs mass is a free parameter, which is unconstrained (at tree-level). In Lecture 2, I showed that by including higher order radiative corrections (and imposing "triviality constraints"), one can deduce both a lower and upper bound for m_H. The results were summarized in fig. 2. The lower bound is very sensitive to the mass of the top-quark and the upper bound implies that $m_H \lesssim$ 650–800 GeV. (This appears to rule out the possibility of a minimal Higgs boson with a 1 TeV mass. Nevertheless, one can imagine scenarios beyond the Standard Model which could simulate the effects of a 1 TeV Higgs boson. Thus, one should not ignore this mass region in future Higgs boson searches.)

The (tree-level) Higgs couplings to gauge bosons and fermions, and the Higgs boson self-couplings are summarized in fig. 1. Of these, the $H^0W^+W^-$, H^0ZZ and $H^0f\bar{f}$ couplings are the most important for phenomenology. However, there are a number of Higgs couplings which are absent at tree-level but appear at one-loop which are also phenomenologically relevant.

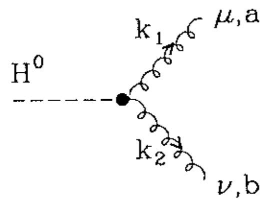
Consider first the one-loop induced Higgs coupling to two gluons.



This diagram leads to an effective Lagrangian

$$\mathcal{L}^{\rm eff}_{H^0gg} = \frac{g\alpha_s N_g}{24\pi m_W}\, H^0 G^a_{\mu\nu} G^{\mu\nu a}\,. \tag{3.2}$$

This effective Lagrangian can be summarized by the following Feynman rule



$$\frac{g\alpha_s N_g}{6\pi m_W}\,(k_2^\mu k_1^\nu - k_1\cdot k_2 g^{\mu\nu})\delta^{ab}$$

where one must remember to include an addition factor of 2 because the two gluons are identical. N_g depends on the quark masses that appear in the loop. Explicitly,

$$N_g \equiv \frac{9}{16}\left|\sum_i F_{1/2}(x_i)\right|^2, \qquad\qquad x_i \equiv \frac{m^2_{q_i}}{m^2_H}\,, \tag{3.3}$$

where $F_{1/2}$ results from the calculation of the loop integral. This function is given in eqs. (2.17–2.19) of the HHG. Two limiting cases are of interest:

$$F_{1/2}(x) \to \begin{cases} -4/3, & x \gg 1\,, \\ -2x\ln^2 x, & x \ll 1\ . \end{cases} \tag{3.4}$$

That is, N_g is roughly equal to the number of quarks which are heavy compared to H^0. It is particularly noteworthy that the effects of heavy quarks do not decouple! This is a feature of spontaneously broken gauge theories where not all couplings and masses are independent. The Appelquist-Carrezone theorem states that the effects of heavy particles decouple in the limit of fixed couplings. However, in this case, the $H^0 q\bar{q}$ coupling is proportional to m_q and thus cannot be held fixed in the large quark mass limit.

Using the $H^0 gg$ vertex above, one finds

$$\Gamma(H^0 \to gg) = \frac{g^2 \alpha_s^2 m_H^3 N_g^2}{288\pi^3 m_W^2}, \tag{3.5}$$

where one must include a factor of 1/2 for identical particles if one integrates over all of two-body phase space. The $H^0 gg$ coupling plays a key role in determining the cross section for Higgs boson production at hadron colliders. The dominant mechanism involved is the fusion of two gluons into a Higgs boson. The resulting cross section is easily computed:

$$\frac{d\sigma}{dy}(AB \to H^0 + X) = \frac{\pi^2 \Gamma(H^0 \to gg)}{8 m_H^3} g_A(x_a, m_H^2) g_B(x_b, m_H^2), \tag{3.6}$$

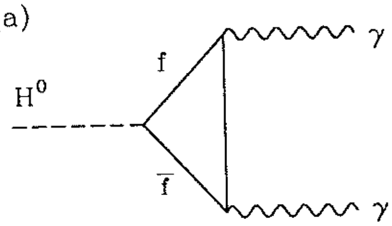
where g_A and g_B are the gluon distribution functions in hadrons A and B respectively, and

$$\begin{aligned} x_a &= \frac{m_H e^y}{\sqrt{s}}, \qquad x_b = \frac{m_H e^{-y}}{\sqrt{s}}, \\ y &= \tfrac{1}{2} \ln \left(\frac{E_H + p_H^{\parallel}}{E_H - p_H^{\parallel}} \right). \end{aligned} \tag{3.7}$$

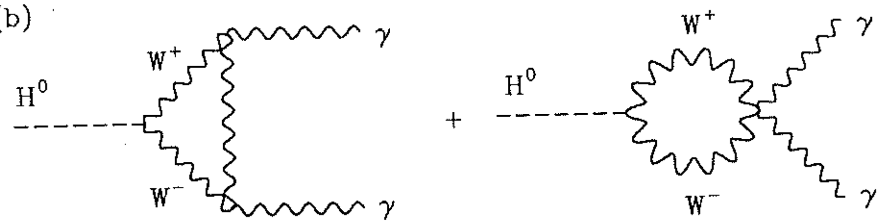
The rapidity y is defined above in terms of the Higgs boson energy and longitudinal momentum, defined in the laboratory frame.

Next, we turn to the one-loop induced Higgs boson coupling to two photons. The contributing diagrams in the Standard Model are the ones shown below.

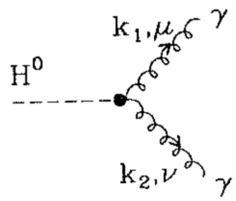
(a)



(b)



The evaluation of these diagrams leads to the following effective interaction expressed as a Feynman rule



$$\frac{g\alpha N_\gamma}{3\pi m_W}\left(k_2^\mu k_1^\nu - k_1 \cdot k_2 g^{\mu\nu}\right)$$

where

$$N_\gamma = \frac{9}{16}\left|\sum_i N_{ci} e_i^2 F_j(x_i)\right|^2 , \qquad x_i \equiv \frac{m_i^2}{m_H^2} . \tag{3.8}$$

In the sum over i, $N_{ci} = 3$ for quarks and 1 for color-singlets, e_i is the electric charge in units of e, m_i is the mass of particle i, and F_j results from the calculation of the loop integral and depends on the spin (j) of particle i. The F_j are given in eqs. (2.17–2.19) of the HHG. In the limit where the mass of particle i is large, the F_j approach a j–dependent constant*

$$F_j(x) \underset{x\gg 1}{\longrightarrow} \begin{cases} -1/3 & j=0 \\ -4/3 & j=1/2 \\ +7 & j=1 \end{cases} . \tag{3.9}$$

Once again, we note that the effects of heavy particles do no decouple in the large mass limit. The numbers on the right-hand side of eq. (3.9) are not random! In fact, the same numbers arise when one computes the electromagnetic beta–function in the Standard Model. If one evaluates the photon two-point function $\Pi_{\mu\nu}(q)$ (say, using dimensional regularization with $\epsilon = 4 - n$ for n spacetime dimensions), then

* Note that although charged scalar loops ($j = 0$) do not occur in the (unitary gauge of the) Standard Model, they will occur in extensions to the Standard Model which generally possess charged Higgs scalars.

one can show that

$$\Pi_{\mu\nu}(q) = (q_\mu q_\nu - g_{\mu\nu}q^2)\left[\frac{\beta(\alpha)}{\alpha}\frac{1}{\epsilon} + \text{finite terms}\right] . \tag{3.10}$$

Summing over charged gauge bosons, fermions [and charged scalars in extended models], I find

$$\beta(\alpha) = \frac{\alpha^2}{2\pi}\left[-7 + \tfrac{4}{3}\sum_{\text{fermions}} N_{ci}e_i^2 + \tfrac{1}{3}\sum_{\text{scalars}} N_{ci}e_i^2\right] . \tag{3.11}$$

The factor of -7 above (arising from the $W^\pm$ in the loop) is understood as follows. The contribution of a massless multiplet of SU(N) gauge bosons is equal to $-11N/3$. If we evaluate $\Pi_{\mu\nu}(q)$ in the Landau gauge, then the contribution of the $W^\pm$ is $-22/3$ (obtained by setting $N = 2$). But we must also add the contribution of the massless Goldstone bosons, which contribute an extra $1/3$ according to eq. (3.11). The sum is gauge invariant (since $\beta(\alpha)$ is gauge invariant in the minimal subtraction scheme), and thus must be equal to the contribution of a massive $W^\pm$ in the unitary gauge.

Thus, we recognize the asymptotic form for $F_j(x)$ [eq. (3.9)] to be intimately related to the coefficients appearing in the divergent part of $\Pi_{\mu\nu}(q)$. The origin of this connection lies in the Higgs boson low energy theorems.

3.2 Higgs Boson Low Energy Theorems

The Higgs boson low energy theorems relate the amplitudes of two processes which differ by the insertion of a zero momentum Higgs boson. They are useful in estimating the properties of very light Higgs bosons in the same way that soft-pion theorems are used to study low-energy pion interactions. A pedestrian approach to the Higgs boson low energy theorems begins by noting that the Higgs boson interactions in the Standard Model can be written in the following form

$$\mathcal{L} = -\left(1+\frac{H}{v}\right)\sum_f m_f\bar{f} - \left(1+\frac{H}{v}\right)^2\left(m_W^2 W_\mu W^\mu + \tfrac{1}{2}m_Z^2 Z_\mu Z^\mu\right) . \tag{3.12}$$

Consider a Higgs field with zero four-momentum: $[P_\mu, H] = i\partial_\mu H = 0$. This implies that H is a constant field. From eq. (3.12), it follows that the effect of a constant field H is equivalent to redefining all mass parameters of the theory

$$m_i \longrightarrow m_i \left(1 + \frac{H}{v}\right) . \tag{3.13}$$

This immediately implies the following low energy theorem

$$\lim_{p_H \to 0} \mathcal{M}(A \to B + H^0) = \frac{1}{v} \left(\sum_f m_f \frac{\partial}{\partial m_f} + \sum_V m_V \frac{\partial}{\partial m_V} \right) \mathcal{M}(A \to B) , \tag{3.14}$$

where the sum over V includes both the W and Z bosons. This theorem is rather trivial when applied to the elementary particles of the model. But its range of applicability is much wider. As a demonstration, let us derive a low-energy theorem for the $H^0 gg$ interaction (g = gluon). At one-loop, the transition amplitude $\mathcal{M}(g \to g)$, which is just the gluon two-point function, depends on m_f due to an intermediate quark loop. One can show that the effect of heavy fermion loops is to add the following piece to the effective QCD Lagrangian

$$\delta\mathcal{L} = \frac{-\alpha_s}{24\pi} G^a_{\mu\nu} G^{\mu\nu a} \sum_f \ln \left(\frac{\Lambda^2_{UV}}{m_f^2} \right) , \tag{3.15}$$

where $G^a_{\mu\nu}$ is the gluon field strength tensor and Λ_{UV} is the ultraviolet cutoff. Using eq. (3.14), we obtain the following effective Lagrangian governing the Higgs-gluon interaction

$$\mathcal{L}_{H^0 gg} = \frac{N_H \alpha_s}{12\pi v} H^0 G^a_{\mu\nu} G^{\mu\nu a} , \tag{3.16}$$

where N_H is the number of heavy quark flavors. Here, "heavy" means the number of quarks heavier than H and the scale of QCD, Λ. As expected, eq. (3.16) gives precisely the same answer as the triangle diagram calculation of $H^0 \longrightarrow gg$ in one-loop perturbation theory [see eq. (3.2)]; namely, the $H^0 gg$ matrix element is constant in the limit of $m_q \longrightarrow \infty$. This technique can also be used to obtain the effective $H^0\gamma\gamma$ interaction in the soft Higgs limit.This is the explanation for why the asymptotic form of eq. (3.9) matches the corresponding coefficients which appear in the electromagnetic β-function, as discussed above.

Consider now the application of the low-energy theorems to the study of Higgs interactions with mesons and baryons at low energy. The mesons and baryons are complicated bound state systems made up of light quarks and gluons. The Higgs bosons can interact with these systems in three distinct ways: (i) interaction with the gluons via eq. (3.16); (ii) direct interactions with the light constituent quarks; and (iii) via a weak interaction process, where the quarks exchange a W or Z boson, and the Higgs interacts with the exchanged vector boson. We can develop low-energy theorems which separate out the interactions via gluons from the direct interactions with fermions and vector bosons as follows. We divide up the quarks into "light" ($m_q < \Lambda$) and "heavy" ($m_q > \Lambda, m_H$). The heavy quarks are important in that they are responsible for the H^0gg interaction; hence, we remove the heavy quarks from eq. (3.14). Instead, we derive a new low-energy theorem as follows. We observe that we can combine eq. (3.16) with the gluon kinetic energy term to obtain

$$\mathcal{L} = \frac{-1}{4g_s^2}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a - f_{abc}A_\mu^b A_\nu^c)^2 \left(1 - \frac{N_H \alpha_s}{3\pi v} H^0\right), \tag{3.17}$$

where $\alpha_s \equiv g_s^2/(4\pi)$, and we have rescaled the gluon field, $A_\mu^a \to g_s^{-1} A_\mu^a$. In the zero momentum limit where H^0 is a constant field, we see that the H^0 interactions can be reproduced simply by rescaling α_s. Thus, if we denote the corresponding change by $\alpha_s \to \alpha_s + \delta\alpha_s$, then to first order,

$$\delta\alpha_s = \frac{N_H \alpha_s^2}{3\pi v} H^0 . \tag{3.18}$$

The following low-energy theorem is thereby obtained:

$$\lim_{p_H \to 0} \mathcal{M}(A \to B + H^0)\big|_{gluons} = \frac{N_H \alpha_s^2}{3\pi v} \frac{\partial}{\partial \alpha_s} \mathcal{M}(A \to B), \tag{3.19}$$

where the subscript "gluons" indicates that we are exhibiting the partial contribution to $\mathcal{M}(A \to B + H^0)$ due to the H^0gg interactions induced by N_H heavy quark loops. By dimensional analysis, it is often possible to deduce the dependence of $\mathcal{M}(A \to B)$ on the intrinsic scale of QCD, Λ. In defining Λ, we employ the following normalization in the definition of the QCD β-function

$$\mu \frac{\partial \alpha_s}{\partial \mu} = \alpha_s \beta(\alpha_s), \tag{3.20}$$

with

$$\beta(\alpha_s) = \frac{-b\alpha_s}{2\pi} + \mathcal{O}(\alpha_s^2), \tag{3.21}$$

where $b \equiv 11 - \frac{2}{3}n_f$, and n_f is the number of quark flavors. Then, Λ is defined as

$$\Lambda = \mu \exp\left\{ - \int \frac{d\alpha_s}{\alpha_s \beta(\alpha_s)} \right\}. \tag{3.22}$$

Note that by using eq. (3.21), $d\Lambda/d\mu = 0$, which implies that Λ is a physical parameter of the theory. It then follows that

$$\frac{\partial\Lambda}{\partial\alpha_s} = \frac{-\Lambda}{\alpha_s\beta(\alpha_s)} \,. \tag{3.23}$$

Thus, in the one-loop approximation, we can replace eq. (3.19) with

$$\lim_{p_H\to 0} \mathcal{M}(A\to B+H^0)\big|_{\text{gluons}} = \frac{2N_H}{3bv}\,\Lambda\,\frac{\partial}{\partial\Lambda}\,\mathcal{M}(A\to B)\,, \tag{3.24}$$

where b should be computed in a theory where the heavy flavors are decoupled, namely $b = 11 - \frac{2}{3}n_L$, with n_L equal to the number of light flavors.

The remaining contributions due to the interactions with the light constituent quarks and weak vector bosons are obtained by deleting the heavy quarks from eq. (3.14)

$$\lim_{p_H\to 0} \mathcal{M}(A\to B+H^0)\big|_{q,W,Z} = \frac{1}{v}\left(\sum_{u,d,s} m_q\frac{\partial}{\partial m_q} + \sum_{V} m_V\frac{\partial}{\partial m_V}\right)\mathcal{M}(A\to B), \tag{3.25}$$

where the sum runs over the light quarks and $V = W$ and Z.

As an example of this formalism, let us work out the Higgs–nucleon coupling using the low energy theorems. The gluons and the strange quark content of the proton determine the magnitude of the H^0NN coupling. (Of course, the effects of the u and d quarks are negligible due to their small masses.) Using the low energy theorems quoted above, we obtain

$$g_{H^0NN} = \frac{2N_H}{3b}\Lambda\frac{\partial m_N}{\partial\Lambda} + m_s\frac{\partial m_N}{\partial m_s}\,. \tag{3.26}$$

By dimensional arguments, the nucleon mass can be written as

$$m_N = c_1\Lambda + c_2 m_s\,. \tag{3.27}$$

In addition, since

$$m_s\,\frac{\partial}{\partial m_s}\,\frac{i}{\not p - m_s} = \frac{i}{\not p - m_s}\,(-im_s)\,\frac{i}{\not p - m_s}\,, \tag{3.28}$$

it follows that $m_s\partial/\partial m_s$ acting on a strange quark line in a Feynman diagram is equivalent to the insertion (with zero momentum) of the operator $m_s s\bar{s}$. Thus, we may conclude that

$$m_s \frac{\partial m_N}{\partial m_s} = \langle N | m_s \bar{s} s | N \rangle \,, \tag{3.29}$$

where I have normalized $|N\rangle$ such that $\langle N | \overline{\psi}_N \psi_N \, |N\rangle = 1$. Thus, we can evaluate $g_{H^0 NN}$ explicitly. The result is

$$g_{H^0 NN} = \frac{g}{2m_W} \left[\left(1 - \frac{2N_H}{3b} \right) \langle N | \, m_s \bar{s} s \, |N\rangle + \frac{2N_H m_N}{3b} \right] . \tag{3.30}$$

It is convenient to parametrize the Higgs–nucleon–nucleon coupling by,

$$g_{H^0 NN} \equiv \frac{g m_N}{2m_W} \eta \tag{3.31}$$

where η is a parameter which expresses the deviation from point coupling. Using $N_H = 3$ and the results of a recent estimate, $\langle p | \, m_s \bar{s} s \, |p\rangle = 334 \pm 132$ MeV, I obtain $\eta \simeq 0.5 \pm 0.1$.

A slightly fancier version of the above derivation makes use of the trace of the energy-momentum tensor, Θ^μ_μ. If we integrate out the heavy quarks from the theory, eq. (3.16) implies that the effective Higgs interaction Lagrangian is

$$\mathcal{L}_{\text{eff}} = \frac{-H}{v} \left(\sum_{i=u,d,s} m_i \overline{\psi}_i \psi_i - \frac{N_H \alpha_s}{12\pi} \, G^a_{\mu\nu} G^{\mu\nu a} \right) . \tag{3.32}$$

We can express $\mathcal{L}_{\text{eff}}$ in terms of Θ^μ_μ by noting that

$$\Theta^\mu_\mu = \frac{-b\alpha_s}{8\pi} \, G^a_{\mu\nu} G^{\mu\nu a} + \sum_{i=u,d,s} m_i \overline{\psi}_i \psi_i \,, \tag{3.33}$$

where the first term on the right hand side is due to the conformal anomaly. Thus, we can write

$$\mathcal{L}_{\text{eff}} = \frac{H}{v} \left[\left(\frac{2N_H}{3b} - 1 \right) \sum_{i=u,d,s} m_i \overline{\psi}_i \psi_i - \frac{2N_H}{3b} \, \Theta^\mu_\mu \right] . \tag{3.34}$$

Finally, noting that we can define the nucleon mass and $H^0 NN$ coupling by

$$\begin{aligned} \langle N | \, \Theta^\mu_\mu \, |N\rangle \, \Big|_{q=0} &= m_N \\ \langle N | \, \mathcal{L}_{\text{eff}} \, |N\rangle = - \langle N | \, \mathcal{H}_{\text{eff}} \, |N\rangle &= -g_{HNN} \,, \end{aligned} \tag{3.35}$$

we recover eq. (3.30).

As a second application of the Higgs boson low energy theorems, consider the Higgs Boson couplings to pions. In this case, it is advantageous to make use of the chiral Lagrangian which neatly summarizes the low energy properties of pions.

$$\mathcal{L} = \tfrac{1}{4} f_\pi^2 \, \mathrm{Tr}\, \partial_\mu \Sigma \partial^\mu \Sigma^\dagger + \tfrac{1}{2} f_\pi^2 (\mathrm{Tr}\, \mu M \Sigma^\dagger + \mathrm{h.c.}) + \dots , \tag{3.36}$$

where we have dropped terms with more than two derivatives. (Higher derivative terms can be neglected as long as typical pion momenta are small compared to $4\pi f_\pi$.) This Lagrangian can be viewed as an effective Lagrangian for QCD consisting of two flavors of quarks (u and d).* The first term of eq. (3.36) describes the properties of the Goldstone bosons arising from spontaneous symmetry breaking of the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ chiral symmetry of the (two flavor) QCD Lagrangian to the diagonal SU(2) isospin subgroup. This symmetry breaking is a result of the strong QCD force, and the Goldstone bosons are identified as the pions

$$\Sigma = \exp\left(\frac{2i\Pi^a T^a}{f_\pi}\right) \tag{3.37}$$

with $f_\pi = 93$ MeV, $T^a = \sigma^a/2$ and

$$\Pi^a T^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{pmatrix} . \tag{3.38}$$

The second term in eq.(3.36) explicitly breaks the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry, where M is the diagonal quark mass matrix. The origin of the nonzero quark masses lies in the electroweak sector of the theory. By inserting eq. (3.38) into eq. (3.36) and examining the term quadratic in the π field, we can determine the relation between the parameter μ and the mass of the pion

$$m_\pi^2 = \mu(m_u + m_d) . \tag{3.39}$$

We can couple the Higgs bosons to the pions by adding all possible operators which are consistent with the chiral symmetry and its explicit breaking via the quark mass matrix. As before, we keep only the terms with the fewest (zero or two) derivatives. Then, $\mathcal{L} \to \mathcal{L} + \Delta\mathcal{L}$ with

$$\Delta\mathcal{L} = \tfrac{1}{4} f_\pi^2 c_1 \frac{H}{v} \mathrm{Tr}\, \partial_\mu \Sigma \partial^\mu \Sigma^\dagger + \tfrac{1}{2} f_\pi^2 c_2 \frac{H}{v} \mathrm{Tr}\, (\mu M \Sigma^\dagger + \mathrm{h.c.}) . \tag{3.40}$$

The c_i are strong interaction parameters which cannot be fixed by appealing to chiral symmetry. However, they can be determined by applying the Higgs low energy

---

$\star$ For simplicity, I shall ignore the strange quarks. A more general discussion can be found in the HHG.

theorems [eqs. (3.24) and (3.25)]. To apply these theorems, we must determine the m_q and Λ dependence of f_π, μ and M (the quark mass matrix). Eq. (3.24) implies that all terms in the chiral Lagrangian which behave like Λ^p will be multiplied by

$$\left(1+\frac{2pN_H}{3b}\frac{H}{v}\right) . \tag{3.41}$$

From eq. (3.25), it follows that all terms in the chiral Lagrangian which are proportional to the quark mass matrix will be multiplied by $(1+H/v)$. Using dimensional arguments, one can deduce that $f, \mu \sim \Lambda$ and $M \sim m_q$. Thus, for $N_H = 3$,

$$\begin{aligned} c_1 &= \frac{4N_H}{3b} = \frac{4}{9} , \\ c_2 &= 1+\frac{2N_H}{b} = \frac{5}{3} . \end{aligned} \tag{3.42}$$

In the above derivation, there is one sleight of hand which needs to be justified. Namely, there is a factor of f hidden in the definition of $\Sigma = \exp(2i\Pi^a T^a/f)$. Nevertheless, even though we stated that $f \sim \Lambda$, we neglected this dependence in Σ when we applied the low energy theorem [eq. (3.24)]. In fact, one can show that this is the correct procedure. First we note that the effective low energy theory is invariant under the combined scaling of dimensionful parameters and the dilatation transformation of the fields

$$\Theta^\mu_\mu = \partial_\mu s^\mu = -\left(\Lambda\frac{\partial}{\partial\Lambda}+\sum_{q=u,d,s} m_q\frac{\partial}{\partial m_q}+\sum_V m_V\frac{\partial}{\partial m_V}\right)\mathcal{L} , \tag{3.43}$$

where s^μ is the scale current. However, the field Σ (and likewise the Π field) do not scale under a dilatation transformation; *i.e.*, they have scale dimension equal to zero. It follows that for the leading term of the chiral Lagrangian, $\mathcal{L} = \frac{1}{4}f^2\mathrm{Tr}\partial^\mu\Sigma\partial_\mu\Sigma^\dagger$, one can derive $\partial_\mu s^\mu = -2\mathcal{L}$. As a result, to be consistent with eq. (3.43), we must use $\partial\Sigma/\partial\Lambda = 0$ in the low energy theorem, which is the justification for the procedure described above.

With these results in hand, we can evaluate the $H\pi\pi$ coupling. Thus, the matrix element for $H^0 \to \pi^+\pi^-$ is

$$\begin{aligned} \mathcal{M}(H^0 \to \pi^+\pi^-) &= \frac{-g}{4m_W}\left[c_1 m_H^2 + 2(c_2-c_1)m_\pi^2\right] \\ &= \frac{-g}{9m_W}\left(m_H^2+\frac{11}{2}m_\pi^2\right) . \end{aligned} \tag{3.44}$$

Once again, we see an enhancement of the H^0 coupling (compared to what one would have obtained based on the direct coupling of H^0 to the u and d quarks), due to the H^0 coupling to gluons via the heavy quark loop. Normalizing to the decay rate for $H^0 \to \mu^+\mu^-$,

$$\frac{\Gamma(H^0 \to \pi^+\pi^-, \pi^0\pi^0)}{\Gamma(H^0 \to \mu^+\mu^-)} = \frac{1}{27}\frac{m_H^2}{m_\mu^2}\left(1 + \frac{11}{2}\frac{m_\pi^2}{m_H^2}\right)\frac{\left(1 - 4m_\pi^2/m_H^2\right)^{1/2}}{\left(1 - 4m_\pi^2/m_H^2\right)^{3/2}}. \qquad (3.45)$$

It is important to note that the above results are consequences of the low-energy theorems. In particular, the $H^0 \to \pi^+\pi^-$ decay rate could deviate substantially from the result obtained above if the $\pi^+\pi^-$ invariant mass is near a resonance. Such information is not contained in the low-energy Lagrangians used above.

Similar techniques can be used to obtain the Higgs boson coupling to $K\pi$. Here, one would make use of the full SU(3) $\times$ SU(3) chiral Lagrangian, and incorporate the weak $\Delta S = 1$ transition. See the HHG for further details.

3.3 Search for the Minimal Higgs Boson

Armed with the knowledge of the properties of the Standard Model Higgs boson, we now proceed to review the various Higgs search strategies which have been proposed. Unfortunately, the Higgs boson mass is *a priori* a completely unknown parameter (apart from the weak set of mass bounds discussed in Lecture 2). Thus, we must devise many strategies which allow us to search the entire allowed range of masses from 0 up to (roughly) 1 TeV. In addition, one can either search for the effects of virtual Higgs exchange (indirect Higgs effects) or direct Higgs production. We now proceed to examine these two possibilities.

Searches for Virtual Effects of Higgs Bosons

The Standard Model is now known to be correct at the Z^0 mass at the one percent level due to precision electroweak measurements at LEP—a remarkable achievement. Some of the electroweak observables exhibit a dependence on the Higgs boson mass, due to virtual Higgs exchange at one-loop. First, consider the ρ-parameter. In Lecture 2, I discussed how the custodial SU(2) symmetry of the Higgs sector of the Standard Model guarantees that $\rho = 1$ at tree-level. This agrees with electroweak measurements, which imply that any deviation of ρ from 1 must be less than 1%. However, the custodial SU(2) symmetry is violated by hypercharge gauge interactions. This effect can be illustrated by considering the effect of a very heavy Higgs boson on ρ. Here, we must be careful to define ρ precisely; numerous definitions exist in the literature. One definition is based on the measurements of charged and neutral current cross sections in low-energy neutrino-nucleon scattering. Define the following ratio of *tree-level* cross sections

$$R_\nu^0 = \frac{\sigma_0(\nu_\mu N \to \nu_\mu X)}{\sigma_0(\nu_\mu N \to \mu^- X)} . \tag{3.46}$$

If R_ν^1 is the corresponding quantity when one-loop radiative corrections are included, then ρ_{NC} is defined via

$$R_\nu^1 = (\rho_{NC})^2 R_\nu^0 . \tag{3.47}$$

In the limit of $m_H \gg m_W$, the leading Higgs mass effect is

$$\rho_{NC} \simeq 1 - \frac{3g^2}{32\pi^2} \tan^2\theta_W \ln\left(\frac{m_H}{m_W}\right) . \tag{3.48}$$

Note that $g'^2 \equiv g^2 \tan^2\theta_W$. Thus, the custodial SU(2) symmetry is indeed broken by the hypercharge gauge interactions as noted above.

Logarithmic sensitivity to a very large Higgs mass can also be found in the prediction of the W mass. The prediction for m_W is obtained by solving the equation

$$m_W^2 \left(1 - \frac{m_W^2}{m_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F} \frac{1}{(1-\Delta r)} , \tag{3.49}$$

where Δr incorporates the one-loop radiative corrections. The contribution of a heavy Higgs boson ($m_H \gg m_W$) to Δr is

$$\Delta r_H \simeq \frac{11g^2}{96\pi^2} \ln\left(\frac{m_H}{m_W}\right) . \tag{3.50}$$

In the two cases just examined, for $m_H \lesssim 1$ TeV, the effect of a heavy Higgs boson on the physical observable is very small, certainly below the level which is accessible to present day experiment. One would need to improve current precision electroweak measurements by at least an order of magnitude to obtain interesting experimental constraints on the Higgs mass. It is interesting to compare the above results with the corresponding effects of a large top quark mass. Both ρ_{NC} and Δr exhibit quadratic sensitivity to m_t at one-loop. It is for this reason that interesting bounds on the top quark mass can be obtained from the present LEP data. The effects of a large Higgs mass proportional to m_H^2 are also present, but these enter only at two-loop order. That is, the leading effects of a large Higgs mass take the form

$$g^2\left[\ln\left(\frac{m_H}{m_W}\right)+\sum_{k=1}^{\infty}c_k g^{2k}\left(\frac{m_H^2}{m_W^2}\right)^k\right], \tag{3.51}$$

where the c_k are dimensionless numbers. That is, the quadratic sensitivity to a large Higgs mass is "screened" by an extra factor of g^2. This is the screening theorem, first noted by Veltman.

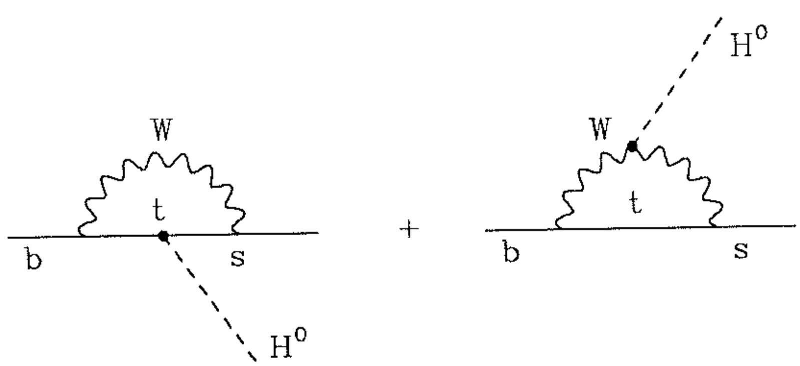
One can conclude that effects of the Higgs boson will almost certainly not be discovered initially through its virtual effects. Thus, we must look to direct production and detection of the Higgs boson at present and future colliders.

Higgs Boson Searches Before LEP

Before LEP began collecting data in the fall of 1989, one had to employ a variety of techniques in order to rule out the existence of a light Higgs boson ($m_H \lesssim 5$ GeV). A detailed description of the various search techniques can be found in the HHG. By far the most effective limits were derived from the non-observation of H^0 in K-decay and B-decay

$$\begin{aligned} &K \to \pi H^0 \\ &B \to K H^0 \\ &B \to H^0 + X \quad (\text{via } b \to sH^0). \end{aligned} \tag{3.52}$$

The amplitudes for these processes display a quadratic sensitivity to the mass of the top-quark! For example, the following two diagrams contribute to $b \to sH^0$



and yields the following effective Lagrangian

$$\mathcal{L}^{\text{eff}}_{bsH} = \frac{3g^3 m_t^2 V_{tb}^* V_{ts}}{256\pi^2 m_W^3}\, m_b \bar{s}(1+\gamma_5)b + \text{h.c.}, \tag{3.53}$$

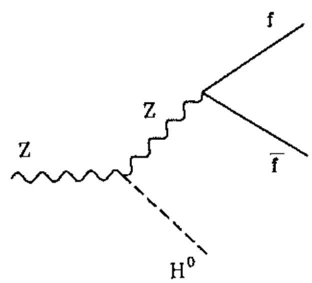
where V_{ij} denotes elements of the Kobayashi-Maskawa (KM) mixing matrix. An analogous expression can be obtained for $\mathcal{L}^{\text{eff}}_{sdH}$. Since $m_t > 89$ GeV, one finds significant rates for these one-loop decays

$$\begin{aligned} BR(K \to \pi H^0) &\sim 10^{-5} \times \text{phase space factor} \\ BR(B \to H^0 + X) &\simeq 0.36 \left(\frac{m_t}{m_W}\right)^4 \left(1 - \frac{m_H^2}{m_b^2}\right)^2 \left|\frac{V_{ts}V_{tb}}{V_{cb}}\right|^2 . \end{aligned} \tag{3.54}$$

The combination of mixing angles which appears in the last formula is of $\mathcal{O}(1)$. Putting in the numbers and comparing with the data, it was possible to conclude (independent of LEP) that $m_H \gtrsim 2m_\mu \sim 3.6$ GeV. We now turn our attention to the LEP data where much stronger Higgs mass limits have since been obtained.

Higgs Bosons Searches at LEP

In 1989–1990, the four LEP detectors collected over half a million Z's. In the Standard Model, the Z can decay into a Higgs boson and a pair of fermions by the following diagram



where $f\bar{f} = \{\nu\bar{\nu}, \ell^+\ell^-(\ell = \mu, e, \tau), q\bar{q}\}$. The theoretical branching ratio is shown in fig. 3. The four LEP detector collaborations have presented Higgs mass limits based on very detailed analyses which examine many different $f\bar{f}$ final states, and carefully treat the low mass Higgs region where many different Higgs decay modes must be taken into account. At present, the ALEPH collaboration has obtained the most stringent Higgs mass limit: $m_H > 48$ GeV (at 95% confidence level). Combining the Higgs search results from all four LEP experiments raises the Higgs mass bound to slightly above 50 GeV.

As LEP proceeds to collect more data, the Higgs mass limits will slowly improve. A high statistics search in Z decays with 10^7 Z's should be sensitive to Higgs masses in the range $m_H \simeq 60$—70 GeV. To improve the Higgs mass limit further, one must push up the LEP center of mass energy. LEP-II is expected to run during the middle of this decade, with $\sqrt{s} \simeq 180$—200 GeV. At these energies, the dominant Higgs production process is $e^+e^- \to H^0 Z$

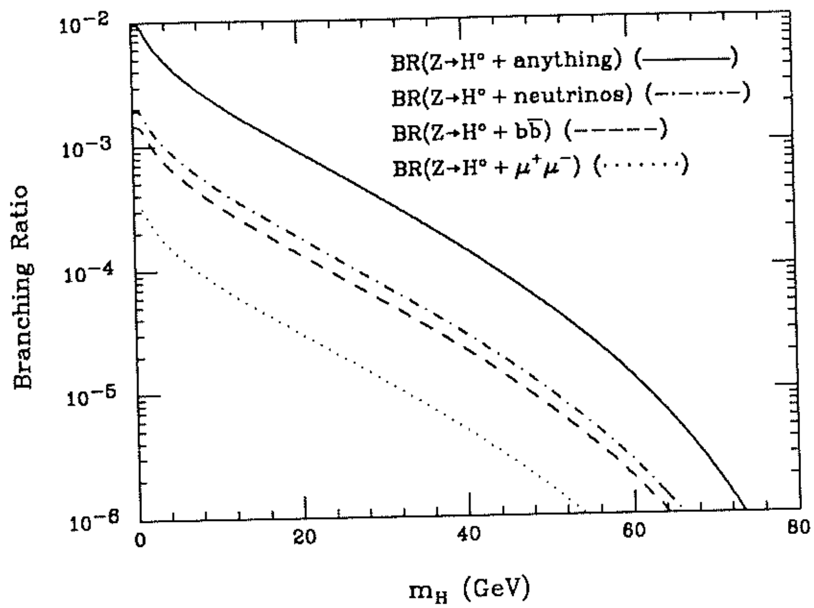


Figure 3 Branching ratios for Higgs production in Z decay are given, $BR(Z \to H^0 + X)$, for various possible final states X. The solid curve depicts X summed over all possible final state quark and lepton pairs (with $t\bar{t}$ omitted, assuming that $m_t > m_Z/2$), the dashed-dot curve depicts $X = \nu\bar{\nu}$ summed over three neutrino generations, the dashed curve depicts $X = b\bar{b}$, and the dotted curve depicts $X = \mu^+\mu^-$ (or any other charged lepton pair).

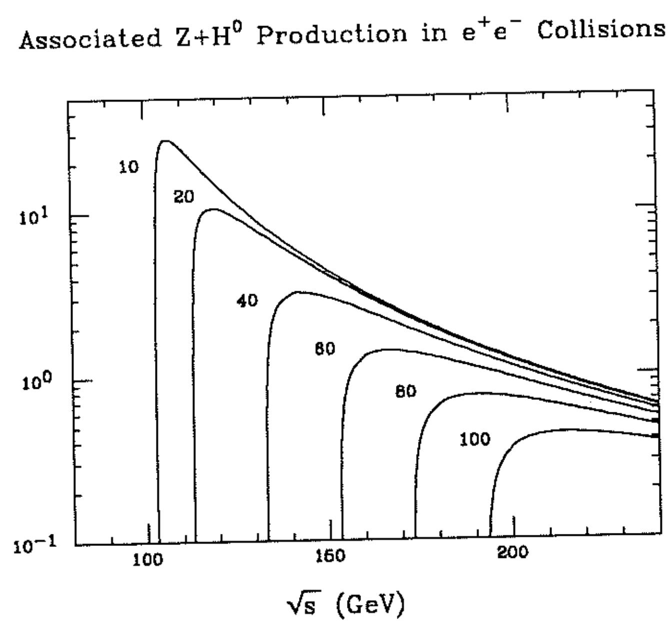
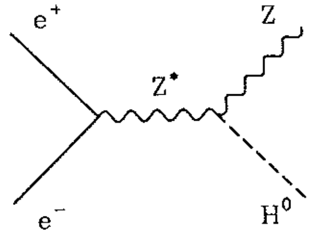


Figure 4 Total cross section for $e^+e^- \to H^0Z$ as a function of $\sqrt{s}$ for the fixed m_{H^0} values indicated by the numbers (in GeV) beside each line.



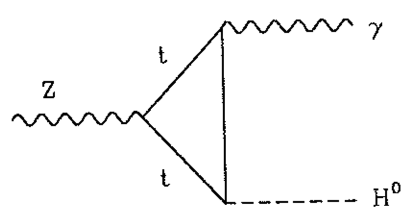
with cross section given by

$$\sigma(e^+e^- \to H^0 Z) = \frac{\pi\alpha^2\lambda^{1/2}(\lambda + 12sm_Z^2)[1 + (1 - 4\sin^2\theta_W)^2]}{192s^2 \sin^4\theta_W \cos^4\theta_W (s - m_Z^2)^2}, \tag{3.55}$$

where $\lambda \equiv (s - m_H^2 - m_Z^2)^2 - 4m_H^2 m_Z^2$. A graph of this cross section for various values of the Higgs mass as a function of $\sqrt{s}$ is shown in fig. 4. One can see that for a fixed value of m_H, the cross section is maximal for $\sqrt{s} \simeq m_Z + \sqrt{2}m_H$. Given 1000 pb^{-1} of data, one expects to be able to observe Higgs masses up to m_Z. Note that the region $m_H \simeq m_Z$ is particularly troublesome due to the $e^+e^- \to ZZ$ background. Both ZZ and $H^0 Z$ lead to four-fermion final states. But in principle, these can be separated by making use of the fact that $BR(H^0 \to b\bar{b}) \simeq 100\%$ as compared to $BR(Z \to b\bar{b}) \simeq 20\%$. Given sufficient energy and luminosity, and a vertex detector which can tag b-quark jets with high efficiency (say 30–50%), it should be possible to discover a Higgs boson if it is degenerate in mass with the Z.

So far, the Higgs searches at LEP and LEP-II described above depend on the ZZH^0 vertex. It is of interest to probe Higgs production mechanisms with sensitivity to the Higgs-fermion coupling. Of these, only the $H^0 t\bar{t}$ coupling is appreciable. We have already seen that the amplitudes for K and B decay exhibit a quadratic sensitivity to the top-quark mass. We now examine whether a similar behavior can be found in Z decays to the Higgs boson.

Consider first the decay $Z^0 \to H^0\gamma$, which is a one-loop decay. One of the contributing diagrams involves a t-quark loop



In addition, we must add the corresponding diagram with a W-loop. The final result is

$$BR(Z \to H^0\gamma) \simeq 2.3 \times 10^{-8} \left(1 - \frac{m_H^2}{m_Z^2}\right)^3 |A_W + A_f|^2, \tag{3.56}$$

where A_W and A_f are the corresponding W-loop and fermion loop contributions

$$A_W \simeq -\left(9.5 + 0.65\frac{m_H^2}{m_W^2}\right)$$

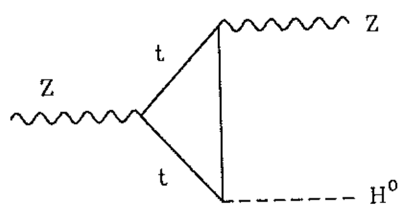
$$\lim_{m_f \to \infty} A_f = \frac{2N_c e_f(T_{3f} - 2e_f \sin^2\theta_W)}{3\sin\theta_W \cos\theta_W} = \begin{cases} 0.063 & L^- \\ 0.613 & U \\ 0.549 & D \end{cases} \tag{3.57}$$

Unfortunately, A_f is completely negligible! There is no quadratic sensitivity to the quark mass, and in this instance, the W-loop term dominates. Nevertheless, it is instructive to understand why A_f approaches a constant in the large fermion mass limit. Gauge invariance implies that the effective $ZH^0\gamma$ interaction must be of the form

$$\mathcal{L}_{ZH^0\gamma} \simeq \frac{\lambda_t}{m_t} F^{\gamma}_{\mu\nu} F^{Z\mu\nu} H^0 \tag{3.58}$$

in the limit of large m_t. Here, $\lambda_t \equiv g m_t/(2m_W)$ is the $H^0 t\bar{t}$ coupling. The factor of m_t^{-1} must arise for dimensional reasons when $m_t \to \infty$, since $F^{\gamma}_{\mu\nu} F^{Z\mu\nu} H^0$ is a dimension-five operator. But λ_t/m_t is independent of m_t. Thus A_f must be independent of m_f in the large fermion mass limit.

Consider next the radiative corrections to $Z \to H^0 Z$, where either the initial or final Z is off-shell. Again, the top-quark mass can enter in the loop



This is a radiative correction to the LEP-I process $Z \to H^0 Z^*$ and to the LEP-II process $e^+e^- \to Z^* \to H^0 Z$. In both cases, the amplitude exhibits a quadratic sensitivity to m_t. The leading term in the effective ZZH^0 interaction (as $m_t \to \infty$) induced by the above graph is

$$\mathcal{L}_{ZZH^0} \simeq \lambda_t m_t Z_\mu Z^\mu H^0 \,. \tag{3.59}$$

In contrast to the previous case, $Z_\mu Z^\mu H$ is a dimension-3 operator so the factor of m_t must occur in the numerator. Since $\lambda_t m_t \sim m_t^2$, we conclude that the one-loop radiative correction will grow with m_t^2. An explicit calculation shows that the leading heavy fermion effect is negative, and therefore decreases the width (or cross section) from the tree-level prediction.

The Equivalence Theorem was used by Sally Dawson and me to obtain the large m_t behavior of $\mathcal{M}(e^+e^- \to ZH^0)$. We computed instead $\mathcal{M}(e^+e^- \to zH^0)$, where z is the neutral Goldstone boson. This gives us the correct result up to terms of $\mathcal{O}(m_Z/\sqrt{s})$. But I have already argued that the one-loop correction to $\mathcal{M}(e^+e^- \to ZH^0)$ grows as $G_F m_t^2$. Dimensional analysis implies that the leading behavior of both amplitudes is the same. Thus, we can match the leading $G_F m_t^2$ of both amplitudes. I shall denote $R_f \equiv (\sigma - \sigma_0)/\sigma_0$, where σ_0 is the tree-level cross section and σ includes the one-loop radiative correction. Then, the contribution of a heavy fermion doublet (U, D) to R_f in the limit $s \gg m_Z^2,\, m_H^2$ is given by

$$R_f = \begin{cases} -\frac{2G_F N_c m_U^2}{3\pi^2\sqrt{2}}\,, & m_U^2 = m_D^2 \gg s\,, \\ -\frac{G_F N_c m_U^2}{3\pi^2\sqrt{2}}\left[1 + \frac{3\cos 2\theta_W}{4\sin^2\theta_W} + \frac{3\cos^2\theta_W(1-4\sin^2\theta_W)}{1+(1-4\sin^2\theta_W)^2}\right]\,, & m_U^2 \gg m_D^2\,, s\,. \end{cases} \tag{3.60}$$

If $m_D^2 \gg m_U^2\,, s$, simply replace m_U with m_D in the above equation. An analogous calculation gives (for $m_U, m_D \gg m_Z$)

$$\Gamma(Z \to H^0 \nu\bar{\nu}) = \Gamma_0 \left\{1 - \frac{G_F N_c}{3\pi^2\sqrt{2}}\left[m_U^2 + m_D^2 + \frac{3\cos 2\theta_W}{4\sin^2\theta_W} f(m_U^2, m_D^2)\right]\right\}\,, \tag{3.61}$$

where

$$f(x, y) \equiv x + y - \frac{2xy}{x-y}\ln\left(\frac{x}{y}\right)\,, \tag{3.62}$$

and Γ_0 is the tree-level decay rate for $Z \to H^0\nu\bar{\nu}$.

If we assume that (U, D) above is the (t, b) doublet, then we find a modest suppression (roughly 5%) of the cross section and the Z partial width into the Higgs boson. However, we also must include the effects of W boson loops which enter as a positive contribution to the one-loop corrections. The net effect of the complete one-loop corrections is rather small for reasonable values of the top quark mass ($m_t \lesssim 200$ GeV).

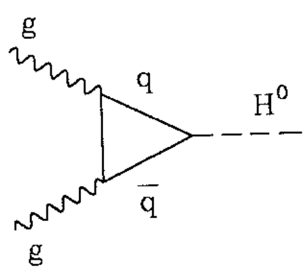
Finally, let me briefly mention the possibility of discovering the Higgs boson in quarkonium decay. Based on current Higgs mass bounds from LEP, only a $t\bar{t}$ bound state could provide a source for Higgs bosons via the toponium decay $\Theta \to H^0\gamma$. Unfortunately, for $m_t > 89$ GeV, it is almost certain that toponium will not be produced at LEP-II. At a future higher energy e^+e^- collider, even if toponium lived long enough to be considered a bound state, the branching ratio $BR(\Theta \to H^0\gamma)$ would be too small to be observed, due to the prominence of the single quark decays (in which one of the constituent t-quarks decays directly to bW). Nevertheless, the Higgs boson could play an interesting role in the precise shape of the $e^+e^- \to t\bar{t}$ cross section near the $t\bar{t}$ threshold. Precision measurements could see the effects of a Higgs boson of moderate mass (one that could be detected directly via $e^+e^- \to H^0 Z$), and could provide a determination of the strength of the $H^0 t\bar{t}$ coupling.

Higgs Boson Searches at Hadron Supercolliders

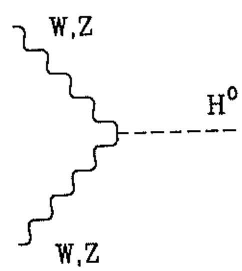
After LEP-II (and before a future high energy e^+e^- linear collider), attention will shift to the hadron supercolliders—the LHC at CERN (with a projected $\sqrt{s} =$ 17 TeV) and the SSC in Texas (with $\sqrt{s} = 40$ TeV). The design luminosity of the SSC is 10^{33} cm^{-2} sec^{-1}. The LHC luminosity is not yet fixed, although luminosities up to 50 times larger have been considered in order to extend the LHC discovery reach (and to make up for its lower center-of-mass energy as compared to the SSC).

The basic mechanisms for Higgs production at a hadron collider are

(*i*) gluon–gluon fusion



(*ii*) vector boson fusion



The corresponding cross sections are shown in fig. 5. In discussing Higgs searches at hadron supercolliders, three mass regions have been distinguished: "intermediate" ($m_W \lesssim m_H \lesssim 2m_Z$), "heavy" ($2m_Z \lesssim m_H \lesssim 800$ GeV) and "obese" ($m_H \gtrsim 800$ GeV). The boundaries between these regions are not sharp. The dominant Higgs production mechanisms are gluon-gluon fusion and WW fusion. Gluon-gluon fusion dominates for the lighter Higgs masses; WW fusion may become dominant for heavier Higgs masses (depending on the value of m_t which governs the size of the gluon-gluon fusion contribution), as shown in fig. 5. If the design luminosity of the SSC is achieved, one would expect an integrated luminosity of 10^4 pb^{-1} in a one year run. This implies that between 10^6 and 10^4 Higgs bosons will be produced per year at the SSC, if 100 GeV $\lesssim m_H \lesssim 1$ TeV. Although this may seem like a large number of events, the existence of large Standard Model backgrounds usually will require experimentalists to make substantial cuts on their data samples in order to expose an unambiguous Higgs signal.

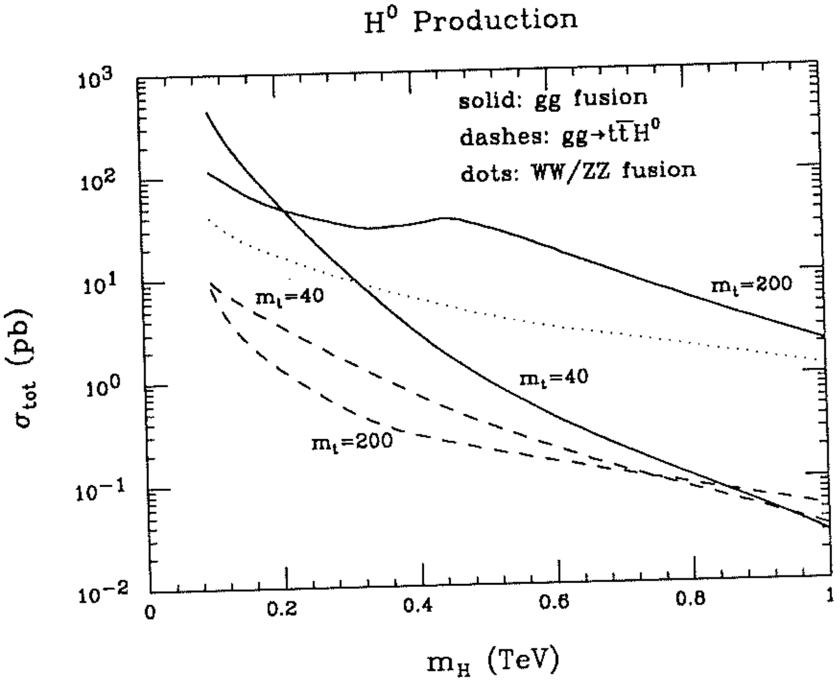


Figure 5 Cross sections for H^0 production at the SSC due to the partonic subprocesses WW fusion, gg fusion, and $gg \to t\bar{t}H^0$ are given as a function of the Higgs mass for two extreme values of the top quark mass, $m_t = 40$ and $m_t = 200$ GeV.

The intermediate mass regime ($m_W \lesssim m_H \lesssim 2m_Z$) is the most problematical. Since $m_t \gtrsim m_Z$, the Higgs boson in this mass range cannot decay into $t\bar{t}$. Thus, the dominant decay mode of the Higgs boson is into $b\bar{b}$ pairs. Such a final state is extremely difficult to observe due to the presence of large Standard Model jet backgrounds. It will be necessary to consider rarer production or decay modes with more distinguishing characteristics. Various mechanisms have been studied in the literature, including (i) $gg \to H^0 \to ZZ^*$, where both on-shell and off-shell Z's decay to e^+e^- or $\mu^+\mu^-$; (ii) $gg \to H^0 \to \gamma\gamma$, (iii) $gg \to t\bar{t}H^0$, where the associated $t\bar{t}$ production is used to trigger on the H^0 production; (iv) $q\bar{q} \to W^* \to WH^0$; and (v) $gg \to H^0 \to \tau^+\tau^-$. Each of these mechanisms may provide signatures which allow experimenters to probe part of the intermediate mass regime. Mechanism (i) provides the cleanest signature and should allow for the discovery of the Higgs boson if $m_H \gtrsim 130$ GeV. For smaller Higgs masses, $BR(H^0 \to Z\ell^+\ell^-)$ has decreased to the point that too few Higgs events survive in a typical SSC year. It may be possible to extend the range of observable Higgs masses by searching for the rare $H^0 \to \gamma\gamma$ mode. This will require specially designed SSC detectors which can distinguish effectively between high energy photons and neutral hadronic jets and can measure the $\gamma\gamma$ invariant mass with extremely good resolution (to better than 1%). Recently, there has been a suggestion that the $t\bar{t}H^0$ and WH^0 final states where $H^0 \to \gamma\gamma$ may be the best way to probe the lower masses of the intermediate mass Higgs region. Clearly, it will require a diligent effort to discover or rule out an intermediate mass Higgs at a hadron supercollider.

In the heavy Higgs mass regime ($2m_Z \lesssim m_H \lesssim 800$ GeV), the Higgs boson decays dominantly into gauge bosons. For example, away from threshold (*e.g.*, for $m_H \gg 2m_Z$),

$$\Gamma(H^0 \to W^+W^-) = 2\Gamma(H^0 \to ZZ) \simeq \frac{G_F m_H^3}{8\pi\sqrt{2}}\,, \tag{3.63}$$

The m_H^3 behavior above is a consequence of the longitudinal polarization states of the W and Z. As m_H gets large, so does the coupling of H^0 to the Goldstone bosons which have been eaten by the W and Z. The same effect explains growing importance of the WW/ZZ fusion mechanism to the Higgs production cross section as m_H becomes large. In contrast, the Higgs decay width to a pair of heavy quarks

$$\Gamma(H^0 \to Q\overline{Q}) = \frac{3G_F m_Q^2 m_H}{4\pi\sqrt{2}}\left(1 - \frac{4m_Q^2}{m_H^2}\right)^{3/2} \tag{3.64}$$

grows only linearly in the Higgs mass. As a result, for $m_H \gg 2m_Z,\ 2m_Q$,

$$\frac{\Gamma(H \to Q\overline{Q})}{\Gamma(H \to W^+W^-, ZZ)} \simeq \left(\frac{2m_Q}{m_H}\right)^2 \ll 1\,. \tag{3.65}$$

Thus, for Higgs masses sufficiently above $2m_Z$, the *total* Higgs width is well approximated by ignoring the Higgs decay to $t\bar{t}$ and including only the two gauge boson decay modes. One then obtains the following convenient mnemonic

$$\Gamma_{\text{total}}(H) \simeq 0.48 \text{ TeV}\left(\frac{m_H}{1 \text{ TeV}}\right)^3\,. \tag{3.66}$$

In order to detect a heavy Higgs boson at the LHC and SSC, one must make use of the two gauge boson decays. As indicated above, these modes dominate over the $t\bar{t}$ final state. In any case, it is very doubtful that the Higgs boson could be observed in the $t\bar{t}$ final state due to the much larger background due to the direct production of $t\bar{t}$ (via gg fusion), as well as the misidentification of t-quark jets in the even larger two-jet Standard Model background.

Among the two gauge boson decays of the heavy Higgs boson, there is one "gold-plated" signature: $H^0 \to ZZ$, where both Z's decay into electron or muon pairs. This should allow for the discovery of a Higgs boson with mass up to 600 GeV with the canonical 10^4 pb^{-1} integrated luminosity. If the luminosity can be increased by a factor of 10 in a high luminosity running mode of the SSC, it may be possible to detect Higgs bosons with masses as high as 800 GeV. The LHC, running at a center-of-mass energy which is a factor of 3 to 4 lower that of the SSC will have a somewhat smaller Higgs discovery reach. A second "silver-plated" mode is: $H^0 \to ZZ$ where one Z decays into e^+e^- or $\mu^+\mu^-$ and the other Z decays into neutrinos. Although this signature has larger backgrounds to contend with, it should be possible to reject the background with an appropriate set of cuts. Because of the larger branching ratio of Z into neutrino pairs, this signature has the potential of reaching higher Higgs masses than the gold-plated signature. In principle, one would also like to have access to the W^+W^- modes of the Higgs boson, which are (about) twice as prolific as ZZ and have larger leptonic branching fractions. However, the possible signatures (*e.g.*, $H^0 \to W^+W^-$, where one W decays leptonically and the second W decays either leptonically or hadronically) have very large Standard Model backgrounds which are generally difficult to deal with. Nevertheless, interesting techniques have been suggested, and the study of such signatures may eventually be quite fruitful.

Finally, in the "obese" region ($m_H \gtrsim 800$ GeV), the Higgs boson width is becoming rather large [see eq. (3.66)], with Γ_H/m_H close to $\mathcal{O}(1)$. This corresponds to the regime where the WW and ZZ scattering is becoming strong (at least in the scalar channel). Such effects may be able to be observed at the SSC in vector boson production processes which occur via the two gauge boson fusion mechanism. Experimentalists should be prepared to search for evidence of such strong interaction phenomena in WW and ZZ scattering if no evidence for the Higgs boson is found.

Higgs Bosons Searches at future e^+e^- colliders

In general, the e^+e^- collider provides a much cleaner environment than the hadron colliders for discovering and examining the detailed properties of new physics phenomena. Unfortunately, there are many engineering and accelerator physics problems to overcome before such a machine can be built. One expects that progress in the design and construction of a future high energy e^+e^- linear collider would occur in two stages. The next linear collider (NLC) would presumably possess a center-of-mass energy in the range $\sqrt{s} \simeq 300$–500 GeV. In the second stage, one can imagine a future high energy e^+e^- linear collider in the TeV range (TLC) with $\sqrt{s} \simeq 1$–2 TeV.

At the NLC, the Higgs search at LEP-II could be extended over the entire intermediate mass Higgs region via $e^+e^- \to ZH$. The only real subtlety of the analysis would be whether a Higgs boson which is degenerate in mass with the Z could be seen. As discussed earlier, for $m_H \simeq m_Z$ one would have to remove the $e^+e^- \to ZZ$ background in order to discover the H^0. Given sufficient luminosity (expected at these machines), it is generally believed that this would not present an obstacle to discovering or ruling out a Higgs boson in this mass range. Thus, the NLC could play a very important role in exploring the intermediate Higgs mass regime.

At higher center-of-mass energies, a new mechanism for Higgs boson production begins to enter: $e^+e^- \to \nu\bar{\nu}H^0$ via W^+W^- fusion. (The rate for $e^+e^- \to e^+e^-H^0$ via ZZ and $\gamma\gamma$ fusion is significantly smaller.) The rate of the W^+W^- fusion process grows (logarithmically) with energy for fixed Higgs boson mass, and is therefore the dominant mechanism for Higgs boson production at the TLC. In contrast to the LHC and SSC, the $W^\pm$ and Z gauge bosons can be detected at an e^+e^- linear collider via their *hadronic* decay modes. The reason for this is that the Standard Model QCD backgrounds, which are so severe at a hadronic supercollider, are much smaller (relatively speaking) at an e^+e^- collider. That is, in e^+e^- annihilation, the production cross-sections for fermion pairs, gauge boson pairs, and Higgs bosons are all (very roughly) of the same order of magnitude. As a result, at the TLC, one can make full use of the hadronic decay modes of the $W^\pm$ to detect the W^+W^- decays of the Higgs boson. Thus, a 2 TeV e^+e^- collider (with an integrated luminosity of 10^4 pb^{-1}) could effectively explore the entire Higgs mass range up to 1 TeV.

Suggestions for Further Reading and a Brief Guide to the Literature

A discussion of the phenomenology of the minimal Higgs boson can be found in

1. *The Higgs Hunter's Guide*, §2.1–2.3, Chapter 3.
2. Vernon D. Barger and Roger J.N. Phillips, *Collider Physics* (Addison Wesley Publishing Co., Redwood City, CA, 1987), Chapter 12.

The calculation of large fermion mass effects in $e^+e^- \to ZH$ and in $Z \to H\nu\bar{\nu}$ is given in

3. S. Dawson and H.E. Haber, SCIPP 90/08, to be published in Phys. Rev. D.

A detailed discussion of Higgs phenomenology at LEP and LEP-II can be found in

4. John Ellis and Roberto Peccei, editors, *Physics at LEP*, vols. 1 and 2, CERN Yellow Report 86-02 (1986).
5. G. Altarelli, R. Kleiss and C. Verzegnassi, editors, *Physics at LEP*, vol. 1: Standard Physics, CERN Yellow Report 89-08 (1989).

The published (minimal) Higgs mass limits from the four LEP experiments are:

6. D. Decamp *et al.* [ALEPH Collaboration] *Phys. Lett.* **B245**, 289 (1990); **B246**, 306 (1990); P. Abreu *et al.* [DELPHI Collaboration], *Nucl. Phys.* **B342**, 1 (1990); B. Adeva *et al.* [L3 Collaboration] *Phys. Lett.* **B248**, 203 (1990); **B252**, 518 (1990); M. Akrawy *et al.* [OPAL Collaboration] *Phys. Lett.* **B236**, 224 (1990); **B251**, 211 (1990).

The most recent Higgs mass limits of the ALEPH collaboration has been presented in

7. D. Decamp *et al.*, CERN-PPE/91-19 (1991), presented at the Aspen, La Thuile and Moriond conferences (Winter 1991).

The basics of supercollider physics, with some of the early discussions of Higgs searches at hadron supercolliders can be found in

8. E. Eichten, I. Hinchliffe, K. Lane and C. Quigg, *Rev. Mod. Phys.* **56**, 579 (1984) [E: **58**, 1065 (1986)].

Higgs searches at future e^+e^- linear colliders are discussed in

9. C. Ahn *et al.*, SLAC-Report-329 (1988); J.H. Mulvey, editor, *Proceedings of the Workshop on Physics at Future Accelerators*, vols. 1 and 2, CERN 87-07 (1987).

4. Beyond the Minimal Model: Extended Higgs Sectors

In the last lecture, I described the phenomenology of the minimal Higgs boson of the Standard Model. Although this minimal choice is completely arbitrary (as far as we know), we have seen that it provides an important benchmark for assessing our ability to detect a Higgs boson at present and future accelerators. However, given the fact that the present experimental information concerning the Higgs sector is rather limited, it is clearly prudent to explore the implications of more complicated Higgs models, both in the context of the Standard Model and in extended theories.

To go beyond the Standard Model with minimal Higgs content, there are a number of possible directions. One can expand the Higgs sector by either introducing multiple copies of the $\mathrm{SU}(2)_L$ doublet, $|Y| = 1$ Higgs multiplets, and/or add additional Higgs singlets, triplets or even more complicated multiplets of Higgs scalars. One can introduce a larger gauge symmetry beyond $\mathrm{SU}(2) \times \mathrm{U}(1)$ along with the necessary Higgs structure to generate gauge boson and fermion masses. Examples include left-right symmetric gauge groups such as $\mathrm{SU}(2)_L\times\,\mathrm{SU}(2)_R\times\,\mathrm{U}(1)$, and models with new U(1)'s such as $\mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)$. Finally, one can extend the Standard Model by introducing "low-energy" supersymmetry. In this lecture, I will only consider the possibility of an extended Higgs sector in an $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge theory. The implication of low-energy supersymmetry for Higgs boson phenomenology will be discussed in Lecture 5.

4.1 Constraints on Extended Higgs Sectors

Despite the limited experimental information on the Higgs sector, there are some constraints on extended Higgs models. First, it is an experimental fact that $\rho = m_W^2/(m_Z^2\cos^2\theta_W)$ is very close to 1 (precision electroweak measurements tell us that $\delta\rho \equiv \rho - 1 \lesssim 1\%$). In the Standard Model, the ρ–parameter at tree-level is determined by the Higgs structure of the theory. We also noted in Lecture 2 that deviations from $\rho = 1$ can be generated by radiative corrections if there is a source of custodial SU(2) breaking. To distinguish between these potential sources for $\rho \neq 1$, I shall denote the value of ρ computed at tree level (for a given extended Higgs sector) by ρ_0.

For an arbitrary Higgs sector, the general formula for ρ_0 is

$$\rho_0 \equiv \frac{m_W^2}{m_Z^2\cos^2\theta_W} = \frac{\sum_{T,Y}[4T(T+1) - Y^2]\mid V_{T,Y}\mid^2 c_{T,Y}}{\sum_{T,Y} 2Y^2 \mid V_{T,Y}\mid^2}, \tag{4.1}$$

where $\langle\phi(T,Y)\rangle = V_{T,Y}$ defines the vacuum expectation values of each neutral Higgs field, and T and Y specify the total $\mathrm{SU}(2)_L$ isospin and the hypercharge of the Higgs representation to which it belongs. Y is normalized such that the electric charge of the scalar field is $Q = T_3 + Y/2$, and

$$c_{T,Y} = \begin{cases} 1, & (T,Y) \in \text{complex representation}, \\ \frac{1}{2}, & (T, Y=0) \in \text{real representation}. \end{cases} \tag{4.2}$$

Here, I have used a rather narrow definition of a real representation as consisting of a real multiplet of fields with integer weak isospin and $Y = 0$. The requirement that $\rho_0 = 1$ for arbitrary $V_{T,Y}$ values is

$$(2T+1)^2 - 3Y^2 = 1. \tag{4.3}$$

This is a well known type of Diophantine equation called Pell's equation. It possesses an infinite number of solutions where $(2T, Y)$ is a pairs of integers. (Of course, T must be non-negative.) The smallest nontrivial solution is $T = 1/2$ and $|Y| = 1$ which corresponds to the minimal Higgs model. The solutions to eq. (4.3) with larger values of T and Y are usually discarded since the representations involved are rather complicated (the simplest example is a Higgs representation with weak isospin 3 and $|Y| = 4$). On the other hand, a Higgs sector with multiple copies of a multiplet satisfying eq. (4.3) also leads to $\rho_0 = 1$. Thus, the simplest extended Higgs sector with $\rho_0 = 1$ consists of two (or more) $T = 1/2$, $|Y| = 1$ doublets.* Henceforth, I shall refer to such models as "multi-doublet models", although I shall always implicitly assume that $|Y| = 1$.

There are also other ways to satisfy the $\rho_0 \simeq 1$ constraint. First, I can choose the vacuum expectation values $V_{T,Y}$ for fields that do not satisfy $(2T+1)^2 - 3Y^2 = 1$ to be small enough so that $\delta\rho \lesssim 1\%$. For example, if the deviation of ρ from 1 is due entirely to the tree-level VEV's of precisely one extra Higgs triplet of hypercharge Y, then an analysis of Amaldi *et al.* from 1987 found

$$\frac{|V_{1,0}|}{v} \leq 0.047 \qquad \frac{|V_{1,2}|}{v} \leq 0.081 \tag{4.4}$$

with $v = 246$ GeV. Clearly, with precision electroweak data from LEP, better limits can be obtained.

* There is a difference between the minimal model and the extended multi-doublet models. In the minimal model, a custodial SU(2) in the Higgs sector guarantees that $\rho_0 = 1$. In multi-Higgs-doublet models, the custodial SU(2) symmetry is violated by the Higgs self couplings, which account for the mass splitting between charged and neutral Higgs bosons. Although this violation does not alter $\rho_0 = 1$, it does lead to a (finite) shift in ρ at one loop which is quadratically sensitive to Higgs masses (but vanishes if the charged and neutral Higgs bosons are degenerate in mass).

Second, I could choose a combination of Higgs fields, with $V_{T,Y}$ chosen precisely to give $\rho_0 = 1$. Since the minimum of the Higgs potential fixes the $V_{T,Y}$, one would have to fine-tune the parameters of the Higgs potential in general to ensure that $\rho_0 = 1$. Nevertheless, there are examples where by clever choice of multiplets, one can find an appropriate Higgs potential that respects a custodial SU(2) symmetry which fixes the $V_{T,Y}$ such that $\rho_0 = 1$. The simplest example of such a model (due to Chanowitz and Golden and independently due to Georgi and Machacek) involves one complex $Y = 1$ doublet, one complex $Y = 2$ triplet and one real $Y = 0$ triplet Higgs multiplet, with $\langle\phi(\frac{1}{2}, 1)\rangle = a/\sqrt{2}$ and $\langle\phi(1,2)\rangle = \langle\phi(1,0)\rangle = b$. One can construct a Higgs potential with the appropriate custodial SU(2) symmetry which guarantees this pattern of VEV's. It is easy to check using eq. (4.1) that $\rho_0 = 1$. The CGGM model is still rather unnatural in the following sense. Since the custodial SU(2) symmetry is violated by the hypercharge gauge interactions, the coefficients of custodial SU(2) symmetry violating terms of the Higgs potential will be infinitely renormalized, and thus must be "fine-tuned" to zero in order to preserve $\rho_0 = 1$. Note that this does not happen in the minimal Higgs model, where the most general Higgs potential permitted by the gauge symmetry also automatically respects the custodial SU(2) symmetry. Moreover, in multi-doublet models, custodial SU(2) violating terms in the Higgs potential simply lead to finite radiative corrections to ρ. Therefore, for the rest of this lecture, I shall focus primarily on multi-doublet extensions of the minimal Higgs model.

The second major theoretical constraint on the Higgs sector comes from the severe experimental limits on the existence of flavor-changing neutral currents (FCNC's). In the minimal Higgs model, tree-level flavor changing neutral currents are automatically absent. This result is easily understood by examining the Higgs-fermion Yukawa interactions

$$-\mathcal{L}_Y = g^U_{ij}\overline{Q}^i_L \Phi u^j_R + g^D_{ij}\overline{Q}^i_L \widetilde{\Phi} d^j_R + \text{h.c.}, \tag{4.5}$$

where Φ is the complex doublet Higgs field, $\widetilde{\Phi} \equiv i\sigma_2\Phi^*$ and i, j label the quark generations. After symmetry breaking, $\Phi^0 \to \Phi^0 + v/\sqrt{2}$, one sees that quark masses are generated. *A priori*, the quark mass matrices are not diagonal in "generation space"

$$M^U_{ij} = g^U_{ij}\frac{v}{\sqrt{2}}, \qquad M^D_{ij} = g^D_{ij}\frac{v}{\sqrt{2}}\,. \tag{4.6}$$

However, when the quark mass matrices are diagonalized, the Yukawa couplings are automatically diagonalized at the same time. Thus, there are no Higgs mediated FCNC's at tree-level in the minimal model. In general, this ceases to be true in non-minimal Higgs models. If I now replicate the Higgs doublets by adding an index k, and define the corresponding Yukawa couplings as g_{ijk}, then in this more general case I would find

$$M_{ij}^U = \sum_k \frac{g_{ijk}^U v_k}{\sqrt{2}}, \qquad M_{ij}^D = \sum_k \frac{g_{ijk}^D v_k}{\sqrt{2}}. \tag{4.7}$$

Clearly, in a basis where M_{ij} is diagonal, the g_{ijk} (for all k) need not be simultaneously diagonal. Hence, in general I expect Higgs mediated FCNC's at tree-level in models with a non-minimal Higgs sector. One then has two choices. First, by arranging the parameters of the model so that the Higgs masses are large (typically of order 1 TeV), tree-level FCNC's mediated by Higgs exchange can be suppressed sufficiently so as not to be in conflict with known experimental limits. The second choice is more elegant, and is based on a theorem of Glashow and Weinberg. The theorem states that tree-level FCNC's mediated by Higgs bosons will be absent if all fermions of a given electric charge couple to no more than one Higgs doublet. If we require this theorem to be satisfied, the Higgs couplings to fermions are constrained, but not unique. For example, there are (at least) two ways to satisfy the Glashow-Weinberg theorem in two-Higgs doublet models. One possibility [Model I] is a model in which one Higgs doublet does not couple to fermions at all (due to a discrete symmetry) and the other Higgs doublet couples to fermions in the same way as in the minimal Higgs model. A second possibility [Model II] is a model in which one Higgs doublet couples to down quarks (but not to up quarks) while the second Higgs doublet couples to up quarks (but not down quarks). Such a coupling pattern can be arranged by imposing either a discrete symmetry or supersymmetry. As we shall see in Lecture 5, the Higgs sector of the minimal supersymmetric extension of the Standard Model is a model of this type. If we also consider the Higgs couplings to leptons, there are additional possibilities. However, it is simplest to assume that the Higgs-lepton couplings follow the same pattern as the Higgs-quark couplings. The resulting phenomenology of Higgs-fermion interactions in Models I and II can differ significantly.

The main conclusion to draw from the above discussion is that there is still plenty of freedom for the Higgs sector in the Standard Model, although the choice is not totally arbitrary. It is also clear that models with multiple doublets are the preferred models for non-minimal Higgs structures, although there is still room for investigation outside this framework. Finally, the two-Higgs-doublet version of the Standard Model is particularly attractive because:

1. It is an extension of the minimal model which adds new phenomena (*e.g.*, physical charged Higgs bosons).
2. It is a minimal extension in that it adds the fewest new arbitrary parameters.

3. It satisfies theoretical constraints of $\rho \simeq 1$ and does not introduce tree-level FCNC's (if the Higgs-fermion couplings are appropriately chosen).
4. Such a Higgs structure is required in "low-energy" supersymmetric models.

4.2 Two-Higgs-Doublet Models: Theory

Let us investigate the minimal extension of the Higgs sector—the Standard Model with two complex Higgs doublets. Let ϕ_1 and ϕ_2 denote two complex $Y = 1$, $\mathrm{SU}(2)_L$ doublet scalar fields. The Higgs potential which spontaneously breaks $\mathrm{SU}(2)_L\times \mathrm{U}(1)_Y$ down to $\mathrm{U}(1)_{\mathrm{EM}}$ is

$$\begin{aligned} V(\phi_1,\phi_2) = {} & \lambda_1(\phi_1^\dagger\phi_1 - v_1^2)^2 + \lambda_2(\phi_2^\dagger\phi_2 - v_2^2)^2 \\ & + \lambda_3\left[(\phi_1^\dagger\phi_1 - v_1^2) + (\phi_2^\dagger\phi_2 - v_2^2)\right]^2 \\ & + \lambda_4\left[(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) - (\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)\right] \\ & + \lambda_5\left[\mathrm{Re}\,(\phi_1^\dagger\phi_2) - v_1v_2\right]^2 \\ & + \lambda_6\left[\mathrm{Im}\,(\phi_1^\dagger\phi_2)\right]^2 , \end{aligned} \tag{4.8}$$

where the λ_i are all real parameters. This potential is the most general one subject to the $\mathrm{SU}(2)\times\mathrm{U}(1)$ gauge symmetry and a discrete symmetry, $\phi_1 \to -\phi_1$, which is only softly violated (by dimension-two terms). The latter constraint is a technical one which is related to insuring that flavor changing neutral currents are not too large. For simplicity, I have also assumed that the Higgs sector is CP-invariant. The above potential guarantees the correct pattern of electroweak symmetry breaking over a large range of parameters. For example, if all the λ_i are non-negative, then the minimum of the potential is manifestly:

$$\langle\phi_1\rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle\phi_2\rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \tag{4.9}$$

which breaks the $\mathrm{SU}(2)_L\times \mathrm{U}(1)_Y$ down to $\mathrm{U}(1)_{\mathrm{EM}}$, as desired. In fact, the allowed range of the λ_i corresponding to this desired minimum is somewhat larger (and can be deduced by requiring that all squared Higgs masses are positive).

A key parameter of the model is the ratio of the vacuum expectation values

$$\tan\beta = v_2/v_1 . \tag{4.10}$$

It is straightforward to remove the Goldstone bosons and determine the physical

Higgs states. In the charged sector, the charged Goldstone boson is

$$G^{\pm} = \phi_1^{\pm} \cos\beta + \phi_2^{\pm} \sin\beta, \tag{4.11}$$

and the physical charged Higgs state is orthogonal to $G^{\pm}$:

$$H^{\pm} = -\phi_1^{\pm} \sin\beta + \phi_2^{\pm} \cos\beta, \tag{4.12}$$

with mass $m_{H^{\pm}}^2 = \lambda_4(v_1^2 + v_2^2)$. Due to the CP-invariance assumed above, the imaginary parts and the real parts of the neutral scalar fields do not mix. In the imaginary (CP-odd) sector, the neutral Goldstone boson is:

$$G^0 = \sqrt{2}\left(\mathrm{Im}\,\phi_1^0 \cos\beta + \mathrm{Im}\,\phi_2^0 \sin\beta\right), \tag{4.13}$$

and the orthogonal neutral physical state is:

$$A^0 = \sqrt{2}\left(-\mathrm{Im}\,\phi_1^0 \sin\beta + \mathrm{Im}\,\phi_2^0 \cos\beta\right), \tag{4.14}$$

with mass $m_{A^0}^2 = \lambda_6(v_1^2 + v_2^2)$. The real (CP-even) sector contains two physical Higgs scalars which mix through the following squared mass matrix:

$$\mathcal{M} = \begin{pmatrix} 4v_1^2(\lambda_1+\lambda_3) + v_2^2\lambda_5 & (4\lambda_3+\lambda_5)v_1v_2 \\ (4\lambda_3+\lambda_5)v_1v_2 & 4v_2^2(\lambda_2+\lambda_3)+v_1^2\lambda_5 \end{pmatrix}. \tag{4.15}$$

The physical mass eigenstates are:

$$\begin{aligned} H^0 =& \sqrt{2}\left[(\mathrm{Re}\,\phi_1^0 - v_1)\cos\alpha + (\mathrm{Re}\,\phi_2^0 - v_2)\sin\alpha\right], \\ h^0 =& \sqrt{2}\left[-(\mathrm{Re}\,\phi_1^0 - v_1)\sin\alpha + (\mathrm{Re}\,\phi_2^0 - v_2)\cos\alpha\right]. \end{aligned} \tag{4.16}$$

The corresponding masses are:

$$m_{H^0,h^0}^2 = \tfrac{1}{2}\left[\mathcal{M}_{11} + \mathcal{M}_{22} \pm \sqrt{(\mathcal{M}_{11} - \mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}\,\right], \tag{4.17}$$

and the mixing angle α is obtained from:

$$\begin{aligned} \sin 2\alpha =& \frac{2\mathcal{M}_{12}}{\sqrt{(\mathcal{M}_{11} - \mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}}, \\ \cos 2\alpha =& \frac{\mathcal{M}_{11} - \mathcal{M}_{22}}{\sqrt{(\mathcal{M}_{11} - \mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}}. \end{aligned} \tag{4.18}$$

Note that according to eq. (4.17), $m_{H^0} \geq m_{h^0}$ as suggested by the notation.

To summarize, this model possesses five physical Higgs bosons: a charged pair ($H^{\pm}$); two neutral CP-even scalars (H^0 and h^0, where, by convention, $m_{H^0} > m_{h^0}$); and a neutral CP-odd scalar (A^0), often called a pseudoscalar. Instead of the one free parameter of the minimal model, this model has six free parameters: four Higgs masses, the ratio of vacuum expectation values, $\tan\beta$, and a Higgs mixing angle, α. Note that $v_1^2 + v_2^2$ is fixed by the W mass: $m_W^2 = g^2(v_1^2 + v_2^2)/2$.

Higgs Boson Couplings

It is important to examine the couplings of the physical Higgs bosons to vector bosons and fermion pairs, since these couplings control the production and decay of the Higgs bosons. First, consider the couplings to vector bosons. To understand the pattern of couplings, consider the fact that the Standard Model, *in the absence of the quarks and leptons*, separately conserves C and P. Thus we can assign unique J^{PC} quantum numbers to all the bosons of the theory, if the fermions are ignored. The quantum number assignments are displayed in Table 1. To see how some of the entries have been derived, note that the existence of a ZH^+H^- coupling implies that Z is a 1^{--} vector boson, and the $H^0h^0h^0$ vertex implies that H^0 is a 0^{++} scalar. It then follows from the existence of a ZH^0A^0 coupling that A^0 is both C-odd and CP-odd as indicated in Table 1. Similarly, h^0 is a 0^{++} scalar. Thus, CP-invariance forbids the ZH^0h^0 coupling [the ZH^0H^0 and Zh^0h^0 coupling are forbidden by Bose symmetry as well], and the ZZA^0 and $W^+W^-A^0$ couplings are forbidden by C-invariance.

The C and P assignments for A^0 and G^0 may appear surprising. Formally, they have been obtained by noting that the couplings in the bosonic sector of the Lagrangian which are missing are consistent with the C, P choices of Table 1. Moreover, the quantum number assignments of Table 1 are unique. Physically, one can understand the results as follows. In a one doublet model, the imaginary part of the neutral Higgs field is the Goldstone boson which is "eaten" and becomes the longitudinal component of the Z. This field, like all Goldstone boson fields, is derivatively coupled and is, therefore, CP-odd. Since the bosonic sector conserves C and P separately, the Goldstone boson must in fact have the $C = -1$ quantum number of the Z. Its $P = +1$ quantum number is the same as that of the other scalar components, and is opposite in sign from the parity of the vector bosons due to the one unit difference in spin. In a two-doublet model, there are two neutral Higgs fields with imaginary components. One linear combination of the imaginary components is the Goldstone boson, and the other linear combination is A^0. Both these fields must have the same C and P quantum numbers; hence, the 0^{+-} assignment for A^0 given in Table 1.

A second argument can be given for the absence of a tree-level coupling of the A^0 to vector boson pairs. First, let us recall that the coupling of the CP-even

Table 1

Quantum numbers of Higgs and Gauge Bosons

When C and P are separately conserved			
	J^{PC}		J^P
γ	1^{--}	$W^{\pm}$	1^-
Z	1^{--}	$H^{\pm}$	0^+
H^0	0^{++}	$G^{\pm}$	0^+
h^0	0^{++}		
A^0	0^{+-}		
G^0	0^{+-}		
When C and P are violated but CP is conserved			
	J^{PC}		J^P
γ	1^{--}	$W^{\pm}$	$1^-, 1^+$
Z	$1^{--}, 1^{++}$	$H^{\pm}$	$0^+, 0^-$
H^0	$0^{++}, 0^{--}$	$G^{\pm}$	$0^+, 0^-$
h^0	$0^{++}, 0^{--}$		
A^0	$0^{+-}, 0^{-+}$		
G^0	$0^{+-}, 0^{-+}$		

scalar Higgs boson(s) to a pair of massive vector bosons arises from the covariant derivative $(D_\mu\phi)^\dagger(D^\mu\phi)$ terms in the Lagrangian after replacing one of the ϕ's by its vacuum expectation value. However, in a CP conserving theory this mechanism does not generate a coupling for the CP-odd A^0. This is because in the convention, adopted here, where the vacuum expectation value of ϕ is taken to be real, the A^0 originates from the imaginary component of ϕ. More formally, since the A^0 is CP-odd, a gauge invariant interaction must take the form: $\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}A^0$. However, this is a dimension-five term which cannot appear in the fundamental Lagrangian, and is only generated by loop graphs.*

In common parlance, the A^0 is usually referred to as a *pseudoscalar*. This is technically incorrect, since we have seen above that in the absence of fermions, the A^0 has $P = +1$ (and $C = -1$). Incorporating the fermions into the theory, C and P are no longer separately conserved, although CP remains a good quantum number (to a very good approximation). Thus, it is more precise to refer to A^0 as being CP-odd. When C and P are violated (with CP conserved), the Higgs and vector bosons can be thought of as admixtures of two eigenstates of definite C and P as indicated in Table 1. (The photon couplings in the Standard Model still respect C and P separately.) Consider the coupling of the neutral Higgs boson to a fermion-antifermion pair. It is well known that an $f\overline{f}$ pair has $P = (-1)^{L+1}$ and $C = (-1)^{L+S}$ for total spin S and orbital angular momentum L, and thus cannot couple to 0^{--} and 0^{+-}. Therefore, in the interactions of the neutral Higgs bosons with $f\overline{f}$, the H^0 and h^0 behave as pure 0^{++} scalars, whereas A^0 behaves as a pure 0^{-+} pseudoscalar. In addition, the ZZA^0 and $W^+W^-A^0$ couplings, which were previously forbidden by C-invariance in the bosonic sector of the theory, can now be generated at one-loop due to the triangle diagram with fermions running around the loop. This diagram generates the effective dimension-five operator $\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}A^0$ referred to above.

There are a few other couplings forbidden at tree level for other reasons. Couplings involving neutral particles only and one or two photons clearly vanish at tree level, although they are generated at one-loop. The same is true for the coupling of all neutral Higgs bosons to a pair of gluons. The radiatively generated A^0gg, H^0gg, and h^0gg vertices are important since two-gluon fusion is one of the major production mechanisms for neutral Higgs bosons at a hadron collider. Two other vertices, $H^+W^-\gamma$ and H^+W^-Z, also vanish at tree level. The $H^+W^-\gamma$ tree-level vertex is zero as a consequence of the conservation of the electromagnetic current. The vanishing of the H^+W^-Z vertex is more model dependent; it turns out to be a general feature of multi-doublet models. To prove this, consider a model with N doublets (with $Y = 1$). In principle, all neutral fields can acquire a VEV. Let us redefine the Higgs fields (*i.e.*, perform a rotation) so that only the first doublet ϕ_1 has a nonvanishing VEV. The HVV coupling (where $V = W^\pm$, Z, or γ) arises from the ϕ_1 kinetic energy term (with ∂_μ replaced by the covariant derivative)

$$\left[(\partial_\mu - igW_\mu^a T^a)\phi_1^\dagger\right](\partial^\mu + igW^{\mu a}T^a)\phi_1\,, \tag{4.19}$$

---

* By a similar argument, one can also deduce that there is no tree-level coupling of the CP-even Higgs bosons to a massless vector boson pair ($\gamma\gamma$ or gg). These couplings are generated at one-loop, corresponding to a dimension-five interaction of the form $F^{\mu\nu}F_{\mu\nu}h$ (where $h = H^0$ or h^0).

when $\phi_1 \to \phi_1 + \langle\phi_1\rangle$. Since we have redefined the scalar fields so that $\langle\phi_i\rangle = 0$ for $i \neq 1$, it follows that $\phi_1^\pm$ must be the charged Goldstone bosons eaten by the $W^\pm$ and Im ϕ_1^0 is the neutral Goldstone boson eaten by the Z^0. Hence, the physical $H^\pm$ must be linear combinations of the $\phi_j^\pm$ $(j = 2, 3, \ldots, N)$ so that there cannot be any $H^\pm VV$ vertices. Again, these vertices are radiatively generated at one-loop, and lead to interesting rare decays of the charged Higgs boson.

The pattern of the allowed tree-level HVV and HHV couplings can be understood by examining the cancellation of bad high energy behavior in scattering processes involving two or more longitudinal vector bosons. This cancellation is guaranteed by the gauge structure of the theory as described in section 1.3. In order to make the notation transparent, I shall denote the Higgs boson of the minimal model by ϕ^0.

Probably the most important vertices for phenomenology are couplings of the CP-even scalars to W^+W^- and ZZ. The analysis of section 1.3 showed that the tree-level unitarity of $W_L^+W_L^- \to W_L^+W_L^-$ requires a Higgs exchange diagram with $g_{\phi^0W^+W^-} = gm_W$. A similar analysis of $Z_LZ_L \to Z_LZ_L$ yields

$$g_{\phi^0 ZZ} = \frac{gm_Z}{\cos\theta_W}\,. \tag{4.20}$$

In multi-doublet models, it is clear that in order to reproduce the unitarity cancellation, the sum of Higgs exchange diagrams must reproduce the results of the minimal model. Thus, it follows that

$$\begin{aligned}\sum_i g^2_{H_i^0W^+W^-} &= g^2_{\phi^0W^+W^-} = g^2m_W^2\,,\\ \sum_i g^2_{H_i^0ZZ} &= g^2_{\phi^0ZZ} = \frac{g^2m_Z^2}{\cos^2\theta_W}\,,\end{aligned} \tag{4.21}$$

where only the CP-even scalars contribute to the sum over i. In the two-Higgs doublet model, we have

$$g^2_{H^0VV} + g^2_{h^0VV} = g^2_{\phi^0VV} \tag{4.22}$$

which holds separately for $V = W$ or Z. In terms of the angles α and β defined earlier, we have

$$\begin{aligned}g_{h^0VV} &= g_Vm_V\sin(\beta-\alpha)\\ g_{H^0VV} &= g_Vm_V\cos(\beta-\alpha)\,,\end{aligned} \tag{4.23}$$

where

$$g_V = \begin{cases} g, & V = W\,,\\ g/\cos\theta_W, & V = Z\,.\end{cases} \tag{4.24}$$

Without specific predictions for α and β, one might be tempted to say that the scalar Higgs coupling to vector boson pairs should be somewhat suppressed compared to their values in the minimal-Higgs model (perhaps reduced by a factor of $\sqrt{2}$ in an "average" model). However, we will see in Lecture 5 that in the supersymmetric model this expectation is generally false; in particular $\cos(\beta-\alpha)$ tends to be quite small, and $\sin(\beta-\alpha)$ is near 1.

There are many other possible sum rules for Higgs couplings which can be derived by similar considerations. Here are a few more examples which can be obtained in a general multi-doublet model. From the cancellation of bad high energy behavior in $A^0Z \to VV$ (where $VV = W^+W^-$ or ZZ), and in $ZA^0 \to ZA^0$, it follows that

$$\begin{aligned}\sum_i g_{H_i^0 AZ}\, g_{H_i^0 VV} &= 0\,,\\ \sum_i g^2_{H_i^0 A^0 Z} &= \tfrac{1}{2} g_{ZZA^0A^0}\,,\end{aligned} \tag{4.25}$$

where only the CP-even scalars contribute to the sum over i. In the two-Higgs doublet model, we find

$$g^2_{h^0A^0Z} + g^2_{H^0A^0Z} = \frac{g^2}{4\cos^2\theta_W}\,. \tag{4.26}$$

In terms of the angles α and β,

$$\begin{aligned}g_{h^0A^0Z} &= \frac{g\cos(\beta-\alpha)}{2\cos\theta_W}\,,\\ g_{H^0A^0Z} &= \frac{-g\sin(\beta-\alpha)}{2\cos\theta_W}\,,\end{aligned} \tag{4.27}$$

which clearly satisfy the sum rules above.

The couplings of Z to a pair of Higgs bosons are also phenomenologically important. For example, at LEP, in addition to searching for h^0 in $Z \to hf\overline{f}$, one can also search for $Z \to h^0A^0$. Although the angle factors can suppress the h^0ZZ and h^0A^0Z couplings, we note that eqs. (4.23) and (4.27) imply

$$g^2_{h^0ZZ} + 4m_Z^2 g^2_{h^0A^0Z} = \frac{g^2 m_Z^2}{\cos^2\theta_W} \tag{4.28}$$

which guarantees that both vertices cannot be simultaneously suppressed.

Let us now consider the Higgs-fermion couplings. As discussed above, in the two-Higgs doublet model, the Higgs-fermion coupling is model dependent. Even if one imposes the theorem of Glashow and Weinberg to forbid tree-level FCNC's induced by Higgs exchange, one still has a number of choices for how to couple the quarks and leptons to the two Higgs doublets. A set of discrete symmetries can always be concocted to make a particular choice natural (in the technical sense).* In Model I, the quarks and leptons do not couple to the first Higgs doublet (ϕ_1), but couple to the second Higgs doublet (ϕ_2) in a manner analogous to the minimal Higgs model. In Model II, ϕ_1 couples only to down-type quarks and charged leptons and ϕ_2 couples only to up-type quarks and neutrinos.

Consider a three-generation model with diagonal (positive) quark matrices M_U and M_D (for the charge 2/3 and $-1/3$ quarks respectively) and Kobayashi-Maskawa mixing matrix K. Then, in Model I, the Higgs-fermion interaction takes the following form:

$$
\begin{aligned}
\mathcal{L}^{I}_{Hf\bar{f}} = &-\frac{g}{2m_W \sin\beta}\,\overline{D}M_D D(H^0\sin\alpha + h^0\cos\alpha) - \frac{ig\cot\beta}{2m_W}\,\overline{D}M_D\gamma_5 D A^0 \\
&-\frac{g}{2m_W \sin\beta}\,\overline{U}M_U U(H^0\sin\alpha + h^0\cos\alpha) + \frac{ig\cot\beta}{2m_W}\,\overline{U}M_U\gamma_5 U A^0 \\
&+\frac{g\cot\beta}{2\sqrt{2}m_W}\left(H^+\overline{U}\left[M_U K(1-\gamma_5) - K M_D(1+\gamma_5)\right]D + \text{h.c.}\right) .
\end{aligned}
\tag{4.29}
$$

In this case, $\tan\beta \equiv v_2/v_1$, where v_2 is the vacuum expectation value of the Higgs field which couples to *both* up and down-type quarks (whereas the other Higgs field is decoupled from the quarks). In contrast, the Model II interaction is:

$$
\begin{aligned}
\mathcal{L}^{II}_{Hf\bar{f}} = &-\frac{g}{2m_W \cos\beta}\,\overline{D}M_D D(H^0\cos\alpha - h^0\sin\alpha) + \frac{ig\tan\beta}{2m_W}\,\overline{D}M_D\gamma_5 D A^0 \\
&-\frac{g}{2m_W \sin\beta}\,\overline{U}M_U U(H^0\sin\alpha + h^0\cos\alpha) + \frac{ig\cot\beta}{2m_W}\,\overline{U}M_U\gamma_5 U A^0 \\
&+\frac{g}{2\sqrt{2}m_W}\left(H^+\overline{U}\left[\cot\beta M_U K(1-\gamma_5) + \tan\beta K M_D(1+\gamma_5)\right]D + \text{h.c.}\right) .
\end{aligned}
\tag{4.30}
$$

This time, we define $\tan\beta \equiv v_2/v_1$, where v_1 (v_2) is the vacuum expectation value of the Higgs field which couples only to down-type (up-type) quarks. In the two equations above, U and D are column matrices consisting of three generations of quark fields. [In both Models I and II, the Higgs-lepton couplings can be read

---

* The required discrete symmetry is violated by dimension-two terms of the Higgs potential given in eq. (4.8). However, this violation is "soft" and only generates FCNC's at the loop-level which are not phenomenologically dangerous.

off from the expressions above by replacing (U, D) with the corresponding lepton fields, replacing quark mass matrices with the corresponding *diagonal* lepton mass matrices, and setting $K = 1$.] Note that as previously advertised, the neutral Higgs interactions are flavor diagonal. In addition, the structure of the charged Higgs interactions involving the Kobayashi-Maskawa matrix is analogous to that of the ordinary charged current mediated by the W.

We will see in Lecture 5 that the Model II choice for the Higgs-fermion couplings is the required structure for the Higgs sector of the minimal supersymmetric model. Thus, for later convenience, the couplings of the neutral Higgs bosons in Model II [given in eq. (4.30)] relative to the canonical Standard Model values are summarized below (using 3rd family notation):

$$\begin{array}{llll} H^0 t\bar{t}: & \dfrac{\sin\alpha}{\sin\beta} & H^0 b\bar{b}: & \dfrac{\cos\alpha}{\cos\beta} \\[2ex] h^0 t\bar{t}: & \dfrac{\cos\alpha}{\sin\beta} & h^0 b\bar{b}: & \dfrac{-\sin\alpha}{\cos\beta} \\[2ex] A^0 t\bar{t}: & \cot\beta & A^0 b\bar{b}: & \tan\beta, \end{array} \tag{4.31}$$

where we must keep in mind that A^0 is coupled via a γ_5 to a $q\bar{q}$ pair. Note that the $A^0 u\bar{u}$ ($A^0 d\bar{d}$) coupling is suppressed (enhanced) if $\tan\beta > 1$, and vice versa if $\tan\beta < 1$. Similar results hold for H^0 and h^0, although these couplings also involve the mixing angle α which can reduce the size of the couplings somewhat. The charged Higgs boson of Model II has a coupling to the tb channel given by:

$$g_{H^- t\bar{b}} = \frac{g}{2\sqrt{2} m_W} \left[m_t \cot\beta (1 + \gamma_5) + m_b \tan\beta (1 - \gamma_5) \right]. \tag{4.32}$$

Note that the t-quark-mass piece is suppressed for $\tan\beta > 1$. Finally, we note that the pattern of suppressed and enhanced couplings of Model II is quite distinct from that of Model I. In the latter case, the couplings of the pseudoscalar and charged Higgs bosons to *all* fermion types are uniformly suppressed (enhanced) if $\tan\beta > 1$ ($\tan\beta < 1$). A similar remark can be made concerning the neutral scalar Higgs-fermion couplings, although one must also take the dependence on the mixing angle α into account [as indicated in eq. (4.29)].

One can check that the pattern of Higgs-fermion couplings exhibited above respects the conditions imposed by unitarity of $f\bar{f} \to V_L V_L$ scattering. Clearly, the cancellation of bad high energy behavior in the minimal model due to the Higgs exchange graph must be reproduced by summing over the corresponding

Higgs exchange diagrams of the extended Higgs model. It follows that

$$\sum_i g_{H_i^0 VV} g_{H_i^0 f\bar{f}} = g_{\phi^0 VV} g_{\phi^0 f\bar{f}} , \tag{4.33}$$

where only CP-even scalars contribute to the sum over i. The right hand side of the above equation can be obtained from fig. 1. Thus, in the two-doublet model,

$$g_{h^0 W^+W^-}\ g_{h^0 f\bar{f}} + g_{H^0 W^+W^-}\ g_{H^0 f\bar{f}} = \tfrac{1}{2} g^2 m_f , \tag{4.34}$$

which is indeed satisfied by eqs. (4.23) and (4.31).

4.3 CP Violation in Multi-Higgs-Doublet Models

Up until now, I have arbitrarily imposed CP conservation on the Higgs potential. This is probably a reasonable thing to do if one is interested in direct observation of Higgs bosons. Presumably, CP-violating effects are small and will have little effect on Higgs boson phenomenology. However, CP-violating effects can be induced by virtual Higgs exchange and could arise in processes such as K and B decay and the electric dipole moment of the neutron. In such cases, the Higgs sector can play a crucial role in determining the phenomenology of observable CP violation.

Originally, Weinberg proposed a three-Higgs doublet model in which the *only* source of CP-violation in the Standard Model derives from the Higgs sector (*i.e.*, the KM matrix is real). Three Higgs doublets were required if one wanted to avoid Higgs-mediated tree-level FCNC processes at tree level.* (That is, a CP-violating two-Higgs-doublet model must necessarily possess FCNC's at tree-level.) Unfortunately, Weinberg's model is not phenomenologically viable; CP-violation in the kaon system is not compatible with a real KM matrix. Of course, in models with an extended Higgs sector (and three generations of quarks), one should expect *both* a complex KM matrix and the Higgs sector to contribute to CP-violation.

There is one type of CP-violating observable which is particularly sensitive to a Higgs source of CP-violation: the electric dipole moment of the neutron (d_n) or the electron (d_e). There has been a great deal of attention recently given to the computation of d_n, so I will briefly consider this case. The computation of d_n is a three step process. First, one starts with the standard SU(3) × SU(2) × U(1) gauge theory at the electroweak scale. Integrate out the heavy fields ($W^\pm$, Z, the

---

⋆ One can also construct a Higgs sector with CP-violation and avoid Higgs mediated tree-level FCNC's by using two Higgs doublets and one Higgs singlet.

top-quark, and any heavy Higgs bosons) to derive an effective field theory at ~ 100 GeV:

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm QED} + \sum_i C_i \mathcal{O}_i\,, \tag{4.35}$$

where the $\mathcal{O}_i$ are a set of nonrenormalizable operators. Second, one uses the renormalization group to evolve the parameters of this effective field theory from 100 GeV down to $\mu \equiv 1$ GeV. Finally (and this is the tricky part!), one attempts to match the resulting 1 GeV theory onto a theory consisting of hadrons and photons. For example, one approach is to use the chiral Lagrangian to describe the theory of hadrons and photons, with the matching performed at $M \equiv 4\pi f_\pi \simeq 1.2$ GeV. The size of the unknown coefficients of the chiral Lagrangian are determined by a procedure known as "naive dimensional analysis". Typically, the result of such a procedure is

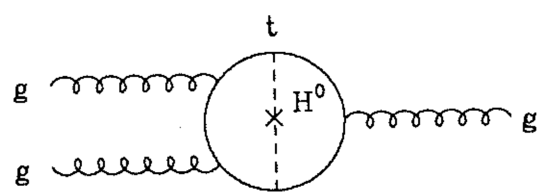
$$d_n \approx \frac{eM}{4\pi}\left[\frac{g_s(\mu)}{g_s(100\ {\rm GeV})}\right]^\gamma C\,. \tag{4.36}$$

Here, I have assumed that the CP-violating effect is dominantly due to one operator with anomalous dimension γ and its associated coefficient C.

In the fall of 1989, Weinberg observed that the dimension-6 three-gluon operator:

$$\mathcal{O} = -f_{abc}\epsilon_{\alpha\beta\gamma\delta}G_a^{\alpha\rho}G_{b\rho}^{\beta}G_c^{\gamma\delta}\,, \tag{4.37}$$

could be the most important operator contributing to the calculation of d_n! The Higgs physics enters in the computation of the coefficient function of this operator. Namely, one must evaluate diagrams of the form



where $---\!\times\!---$ is the CP-violating Higgs propagator which is proportional to some unknown phase, $\mathrm{Im}\,Z$, of the model. There are also contributions from charged Higgs bosons as well. A computation of the neutral Higgs contribution yields (roughly)

$$d_n \simeq \frac{e\sqrt{2}G_F M}{16\pi^2}\,h(m_t/m_H)\,\zeta_{\rm QCD}(\mu)\,{\rm Im}\,Z \sim 10^{-26}\ ({\rm Im}\ Z)\ {\rm e\ cm}\,, \tag{4.38}$$

where

$$\zeta_{\rm QCD}(\mu) = \left(\frac{g_s(\mu)}{4\pi}\right)^3 \left(\frac{g_s(\mu)}{g_s(100\ {\rm GeV})}\right)^{-108/23} \sim 10^{-4}\,. \tag{4.39}$$

The function h results from the evaluation of a two-loop integral corresponding to the diagram above. For typical values of m_t/m_H, $h \sim 0.1$. The factor Im Z is unknown and depends on the parameters of the extended Higgs Model. Weinberg showed that unitarity imposes a bound on this parameter: Im $Z \leq 1/2$, and that it is not hard to construct examples in which this upper bound is saturated. For the charged Higgs contribution, d_n could even be a factor of 10 larger than the estimate given in eq. (4.38). These results should be compared with the 1989 experimental measurement from Grenoble: $d_n = -3 \pm 5 \times 10^{-26}$ e cm. Thus, the next round of improved experiments will certainly impose interesting constraints on the CP-violating parameters of the Higgs sector.

4.4 Multi-Scalar Models with a High Energy Scale

In the absence of any definite information regarding the Higgs sector, it appears necessary to explore arbitrary Higgs sectors which are consistent with the constraints discussed in section 4.1. Nevertheless, it may be useful to stop a moment and contemplate what we expect to be revealed about the Higgs sector in future experiments. I would like to argue that the most "natural" circumstance (after imposing the required fine-tuning of parameters to fix the electroweak scale at its observed value) is a "low-energy" effective electroweak theory with precisely one Higgs doublet. Moreover, should additional Higgs scalars be discovered, this would be strongly suggestive of addition symmetries beyond those contained in the minimal Standard Model.*

To exhibit the reasoning behind these remarks, let me suppose that there is unspecified new physics at a scale $\Lambda \gg v = 246$ GeV. For simplicity, I parametrize the new physics by an SU(2) $\times$ U(1) singlet scalar N, with $\langle N \rangle \sim \mathcal{O}(\Lambda)$. When N is coupled to an arbitrary multi-Higgs-doublet model, all dimensionful parameters become of $\mathcal{O}(\Lambda)$. I impose the gauge hierarchy $v \ll \Lambda$, which requires one fine-tuning, but I demand that no additional fine-tuning be performed. The resulting theory has the following properties:

1. The low energy scalar sector contains precisely one light neutral Higgs h^0 whose couplings differ from those of the minimal Higgs model by terms of

---

* In low-energy supersymmetric models, similar arguments would imply that the "low-energy" Higgs sector must correspond precisely to the two-doublet Higgs sector of the minimal supersymmetric model.

$\mathcal{O}(v^2/\Lambda^2)$. All other Higgs bosons of the model are "heavy", with masses of $\mathcal{O}(\Lambda)$.

2. The Higgs bosons can couple arbitrarily to the quarks and leptons (violating the conditions of the Glashow-Weinberg theorem). The h^0 couplings to fermions are approximately flavor conserving, with violations of $\mathcal{O}(v^2/\Lambda^2)$.
3. If we demand CP-conserving Higgs–fermion Yukawa couplings, but allow CP-violation to enter via the Higgs sector, the resulting low-energy theory possesses precisely the usual KM matrix with one non-trivial phase. The CP-violating h^0 interactions are suppressed by $\mathcal{O}(v^2/\Lambda^2)$.
4. Unitarity constraints on h^0 interactions are the same as those of the minimal Higgs model, up to $\mathcal{O}(v^2/\Lambda^2)$ corrections; the heavy neutral Higgs boson couplings to W^+W^- or ZZ are suppressed by $\mathcal{O}(v/\Lambda)$ terms.
5. The Linde-Weinberg bound for h^0 is the same as in the minimal model [up to $\mathcal{O}(v^2/\Lambda^2)$ corrections].

I shall briefly illustrate the last two points in the two-Higgs-doublet model. Using the first point above, it follows that h^0 remains light, whereas the masses $H^0, A^0, H^\pm$ are driven up to the $\mathcal{O}(\Lambda)$ mass scale. This implies that

$$\begin{aligned} \cos(\beta-\alpha) &= \mathcal{O}(v^2/\Lambda^2)\,, \\ \sin(\beta-\alpha) &= \mathcal{O}(1)\,. \end{aligned} \tag{4.40}$$

Comparing with eq. (4.23), we see that point 4 is verified.

The Linde-Weinberg bound in the two-Higgs-doublet model is most easily obtained as follows. First, define new scalar fields ξ and η by rotating the fields Re ϕ_1^0, Re ϕ_2^0 by angle β (where $\tan\beta = v_2/v_1$). In this new basis, $\langle\eta\rangle = 0$ and $\langle\xi\rangle = (v_1^2+v_2^2)^{1/2}$. The radiative corrections are then evaluated in the direction of field space where $\eta = 0$; *i.e.*, one computes $V_{\text{eff}}(\xi)$. Only in this direction can the radiative corrections be significant. The problem is now reduced to one where the analysis of Lecture 2 applies. One can show that the L-W bound is

$$m_{H^0}^2\cos^2(\beta-\alpha) + m_{h^0}^2\sin^2(\beta-\alpha) \geq m_{LW}^2 \tag{4.41}$$

with $v^2 \equiv 2(v_1^2+v_2^2)$ and

$$m_{LW}^2 = \operatorname{Str}\frac{\lambda_i^2 v^2}{16\pi^2}\left\{1 - \frac{2\mu_i^2}{\lambda_i v^2}\left[1 - \frac{\mu_i^2}{\lambda_i v^2}\ln\left(\frac{\mu_i^2+\lambda_i v^2}{\mu_i^2}\right)\right]\right\}\,, \tag{4.42}$$

assuming that the tree-level particle spectrum contains particles i of mass squared $M_i^2 \equiv \mu_i^2 + \lambda_i v^2$. I now divide the particle spectrum into two classes:

type j: particles with $M_i^2 = \lambda_i v^2$ (*i.e.*, $\mu_i = 0$), whose masses are protected by the electroweak scale. These particles include $W^\pm, Z^0$, quarks and leptons, and one light Higgs boson.

type k: particles with $M_i^2 = \mu_i^2 + \lambda_i v^2$ such that $\mu_i \gg v$ [$\mu_i = \mathcal{O}(\Lambda)$]. These particles include all the heavy Higgs bosons.

It then follows from eq. (4.42) that

$$m_{LW}^2 = \frac{1}{16\pi^2 v^2}\,\mathrm{Str}\,M_j^4 + \frac{v^4}{24\pi^2}\,\mathrm{Str}\,\frac{\lambda_k^3}{\mu_k^2}\,. \tag{4.43}$$

The first term on the right hand side is precisely the Linde-Weinberg bound of the minimal model, and the second term is explicitly of $\mathcal{O}(v^2/\Lambda^2)$ times the first term. In particular, note that the heavy scalars to not appear in Str M_j^4. Thus, point 5 above is verified.

4.5 Two-Higgs-Doublet Models: Phenomenology

First, let us briefly review the present experimental limits. There are some weak constraints in the $m_{H^\pm}$ vs. $\tan\beta$ plane from the consideration of B–$\overline{B}$ mixing and rare B-decays. Charged Higgs exchange can contribute to the $\Delta B = 2$ box diagram. If we assume that the Higgs contribution can be no larger than the result obtained in the minimal Standard Model, one can derive some constraints on $m_{H^\pm}$ if $\tan\beta < 1$. Similarly, based on calculations of the decay rates of various rare B-decays: $b \to s\gamma, b \to sg$, and $b \to s\mu^+\mu^-$, one can obtain weak limits on m_{H^+} if $\tan\beta < \frac{1}{2}$.

The best limits come from direct particle searches. The direct limits on $m_{H^\pm}$ are the least model-dependent, and depend only on $m_{H^\pm}$ and the ratio of branching fractions:

$$\frac{BR(H^+ \to \tau^+\nu)}{BR(H^+ \to c\bar{s})} \simeq \frac{m_\tau^2 \tan^4\beta}{3m_c^2}\,. \tag{4.44}$$

Experiments at LEP search for both $\tau\nu$ and purely hadronic final states, and thus can present limits on $m_{H^\pm}$ that are independent of $\tan\beta$. All four experiments quote similar limits on the charged Higgs mass. The most conservative limit is $m_{H^\pm} \geq 36.5$ GeV (at 95% confidence level) if the hadronic decay modes dominate. The limit improves if some fraction of the charged Higgs bosons decay to $\tau\nu$, reaching $m_{H^\pm} \geq 43$ GeV if $BR(H^+ \to \tau^+\nu) = 100\%$.

The present limits on the neutral Higgs bosons are more model dependent. Light scalars or pseudoscalars, if they exist, would have been seen (modulo some experimental and theoretical uncertainty) in (i) $K \to \pi+$ nothing, if H^0 lives long enough to escape the detector; (ii) $K \to \pi e^+e^-$, if $m_H < 2m_\mu$; (iii) $K \to \pi\mu^+\mu^-$, if $2m_\mu < m_H < m_K - m_\pi$; (iv) $B \to KX$ ($X = \mu^+\mu^-, \pi^+\pi^-, K^+K^-$), if $m_H < 2m_\tau$; and (v) $\Upsilon \to \gamma(h^0$ or $A^0)$. At LEP, one can search simultaneously for $Z \to h^0 f\bar{f}$ and $Z \to h^0 A^0$ and rule out certain areas of Higgs parameter space. In particular, note that eq. (4.28) relates the ZZh^0 and Zh^0A^0 couplings which govern the two decay rates. For example, one can deduce limits on m_{h^0} and m_{A^0} as a function of $\sin(\beta - \alpha)$. In the minimal supersymmetric model, $\sin(\beta - \alpha)$ is determined by m_{h^0} and m_{A^0}, so one can derive stronger limits in this case from the LEP data. We will briefly mention these limits in Lecture 5.

Note that the Higgs mass limits from LEP are logically independent from the Higgs mass limits derived from K, B and Υ decays. The LEP limits on m_{h^0} depend on the ZZh^0 coupling, whereas Higgs production in K and B decay [Υ decay] depend primarily on the $h^0t\bar{t}$ [$h^0b\bar{b}$] coupling. But, we have seen that in the two-Higgs-doublet model, g_{ZZh^0} and $g_{f\bar{f}h^0}$ are independent couplings. Note that this last observation is particularly relevant in more general approaches to electroweak symmetry breaking, where the origin of fermion masses may not even be related to the origin of the vector boson masses. Thus, it is important to obtain independent experimental constraints on $g_{h^0f\bar{f}}$ and g_{ZZh^0}, wherever possible.

Let us now briefly consider Higgs searches at LEP, LEP-II and future e^+e^- supercolliders. I will focus here on the changes in Higgs phenomenology as compared with the minimal Higgs boson of Lecture 3. If one (or more) of the neutral Higgs bosons is lighter than some quarkonium state, then the decays of a $V(1^{--})$ quarkonium state to $h^0\gamma$, $H^0\gamma$, or $A^0\gamma$ (if allowed) can be either enhanced or suppressed by the square of the relevant coupling given in eq. (4.31). Next, let us reconsider the Higgs search at a Z factory. For the scalar bosons, the rate for $Z \to h\ell^+\ell^-$ ($h = h^0$ or H^0) will generally be somewhat suppressed. Nonetheless, both scalar Higgs would probably be detectable in this mode (presuming both are light enough) unless one Higgs completely saturates the sum rule of eq. (4.22). In this latter case the other scalar will not be detectable in this mode. The neutral pseudoscalar has no tree-level couplings to VV. Hence, the rate for $Z \to A^0\ell^+\ell^-$ (which occurs only at one-loop) is extremely small. However, new decays of the Z are possible in the two doublet model:

$$Z \to A^0h^0, \qquad Z \to A^0H^0, \qquad Z \to H^+H^-, \tag{4.45}$$

which lead to simultaneous production of a scalar Higgs boson and the pseudoscalar Higgs boson or of a charged Higgs pair. The decay rates normalized to the partial width of Z into one generation of neutrinos are:

$$\begin{aligned}\frac{\Gamma(Z\to A^0h^0)}{\Gamma(Z\to\nu\bar{\nu})}&=\tfrac{1}{2}\cos^2(\beta-\alpha)B^3\\ \frac{\Gamma(Z\to H^+H^-)}{\Gamma(Z\to\nu\bar{\nu})}&=\tfrac{1}{2}\cos^2 2\theta_W B^3\end{aligned}\tag{4.46}$$

where $B=2|\vec{p}|/m_Z$ and $|\vec{p}|$ is the magnitude of the three momentum of one of the final Higgs particles in the Z rest frame. For $\Gamma(Z\to A^0H^0)$, replace $\cos(\beta-\alpha)$ with $\sin(\beta-\alpha)$ in the first expression above.

At higher energy e^+e^- machines, appropriate for discovering more massive Higgs bosons, the observability of the main neutral Higgs production mechanisms: $e^+e^-\to ZH^0$ and $e^+e^-\to\nu\bar{\nu}H^0$ are dependent upon substantial VV couplings. Thus, so long as the two neutral scalars share fairly equally the allowed VV couplings [see eq. (4.22)], their detection should be quite straightforward at a machine with adequate energy and luminosity. In contrast, the A^0 may be particularly difficult to find at an e^+e^- machine, since it has no VV couplings. The main pseudoscalar production mode that is available is $Z^*\to A^0h^0$ or A^0H^0. The cross sections for these processes are easily computed, and we find

$$\sigma(e^+e^-\to A^0h^0)=\left(\frac{8\sin^4\theta_W-4\sin^2\theta_W+1}{\cos^4\theta_W}\right)\frac{g^4\cos^2(\beta-\alpha)\kappa^3}{192\pi\sqrt{s}\left[(s-m_Z^2)^2+\Gamma_Z^2m_Z^2\right]},\tag{4.47}$$

where κ is the center-of-mass momentum of one of the final state Higgs bosons. For $\sigma(e^+e^-\to A^0H^0)$, replace $\cos(\beta-\alpha)$ with $\sin(\beta-\alpha)$ in eq. (4.47). The detection of this process may be possible if $m_{A^0}+m_{h^0}$ (or $m_{A^0}+m_{H^0}$) is not too large compared to the machine energy. For instance, if the ZA^0H^0 coupling saturates the strength allowed by eq. (4.26), then the cross section for $e^+e^-\to Z^*\to A^0+H^0$ can be as large as one-tenth of a unit of R, where one unit of $R\equiv\sigma(e^+e^-\to\gamma^*\to\mu^+\mu^-)=4\pi\alpha^2/3s$. Finally, we consider charged Higgs production at an e^+e^- supercollider. If $m_t>m_{H^\pm}$, then $e^+e^-\to t\bar{t}$ followed by $t\to bH^+$ will provide a source of charged Higgs bosons. In addition, H^+H^- can be directly produced in e^+e^- annihilation via virtual γ and Z exchange (irrespective of the value of m_t). The asymptotic result for $\sigma(e^+e^-\to H^+H^-)$ in units of R (for $s\gg m_Z^2, 4m_{H^\pm}^2$) is given by:

$$\frac{1+4\sin^4\theta_W}{8\sin^4 2\theta_W}\sim 0.308\,.\tag{4.48}$$

(This should be compared with 0.25 units of R if Z-exchange is not included.) Charged Higgs masses up to about $0.4\sqrt{s}$ will be detectable at an e^+e^- collider with an integrated luminosity of 10^3 inverse units of R.

Let us now turn to Higgs searches at future hadron colliders. Here, I will focus my attention on search strategies at the SSC. First, let us consider whether search techniques that worked for the minimal Standard Model Higgs will also be appropriate in a two-doublet model.

The CP-even Higgs bosons (H^0 and h^0) can be detected in the same manner as the minimal Higgs boson of the Standard Model, so long as they share relatively equally the VV coupling strength. If a Higgs scalar has a mass between about $2m_Z$ and 800 GeV *and its couplings to WW and ZZ are similar to Standard Model strength*, then it should be possible to detect this Higgs boson at the SSC by observing its decay into a pair of vector bosons (followed by subsequent decay of the vector bosons into lepton pairs). On the other hand, for masses less than $2m_Z$, we are in the regime of the "intermediate mass Higgs", in which the Higgs boson mainly decays into $b\bar{b}$. Thus, the same techniques discussed in Lecture 3 for the minimal intermediate mass Higgs boson, apply here as well.

Consider next the CP-odd Higgs boson (A^0). We have already noted that A^0 does not couple to vector boson pairs at tree level. The phenomenological implications of this fact are potentially devastating. First, the important vector boson fusion mechanism for production of a Higgs boson is absent. Thus, the primary production mechanism will be via gg fusion. Second, the dominant decay of A^0 will probably be into the heaviest quark pair available, independent of the Higgs mass. This decay is a poor signature for Higgs production due to large Standard Model backgrounds, so we must examine other possible decay modes. The decay branching ratio of $A^0 \to \gamma\gamma$ is smaller compared to the $\gamma\gamma$ decay of the Standard Model Higgs boson, due to the absence of W boson loop graphs for $A^0 \to \gamma\gamma$. (The absence of VVA^0 couplings also implies that the ZZ^* mode is absent at tree-level in A^0 decays.) There are other possible decay modes such as $A^0 \to Zh^0$, $A^0 \to ZH^0$ and $A^0 \to W^\pm H^\mp$ which may be useful in identifying an A^0 signal. If a scalar with the above properties could be found, and were shown to have a mass larger than $2m_W$, then the absence of decays into vector boson pairs would be strong evidence for the CP-odd nature of this scalar. (An exception to this conclusion occurs in supersymmetric models, which predict that the heavy Higgs *scalar*, H^0, has suppressed couplings to the vector boson channels. Nevertheless, such an observation would be definitive evidence for a non-minimal Higgs sector.)

Finally, consider the charged Higgs boson. There is no coupling of the charged Higgs boson to vector boson pairs (WZ and $W\gamma$) at tree level, so that its decays are likely to be dominated by the heaviest allowed quark channel. In addition, the single particle inclusive cross section for production of the charged Higgs boson is smaller than that typical of a neutral Higgs boson. The gluon-gluon-fusion and vector-boson-fusion mechanisms are not available in this case, so that inclusive production of $H^{\pm}$ must occur by other mechanisms. If the top quark has a moderate mass, but $m_t > m_{H^{\pm}} + m_b$, then the rate for $gg \to t\bar{t}$ followed by $t \to H^+ b$ and $\bar{t} \to H^- \bar{b}$ is very large. Relative to the t decay rate to charged W's we have

$$\frac{\Gamma(t \to H^+ b)}{\Gamma(t \to W^+ b)} = \frac{p_{H^+}}{p_{W^+}} \frac{m_t^2 (m_t^2 - m_{H^+}^2)}{(m_t^2 + 2m_W^2)(m_t^2 - m_W^2)} \cot^2 \beta, \tag{4.49}$$

where p_{H^+} and p_{W^+} are the center-of-mass momenta of the H^+ and W^+ for the respective decays. This is illustrated in fig. 6, where we also exhibit the various charged Higgs branching ratios that are relevant for the charged Higgs search. Thus, the H^+ channel is fully competitive with the W^+ mode. As a result, it may be possible to discover the charged Higgs boson via $t\bar{t}$ production followed by $t \to bH^+$. In fact, in this circumstance, the main challenge would be to discover the top-quark itself. Once the t-quark is found, anomalous t decays might point in the direction of a charged Higgs boson. This may be the only case where one could find evidence for Higgs bosons at the Tevatron!

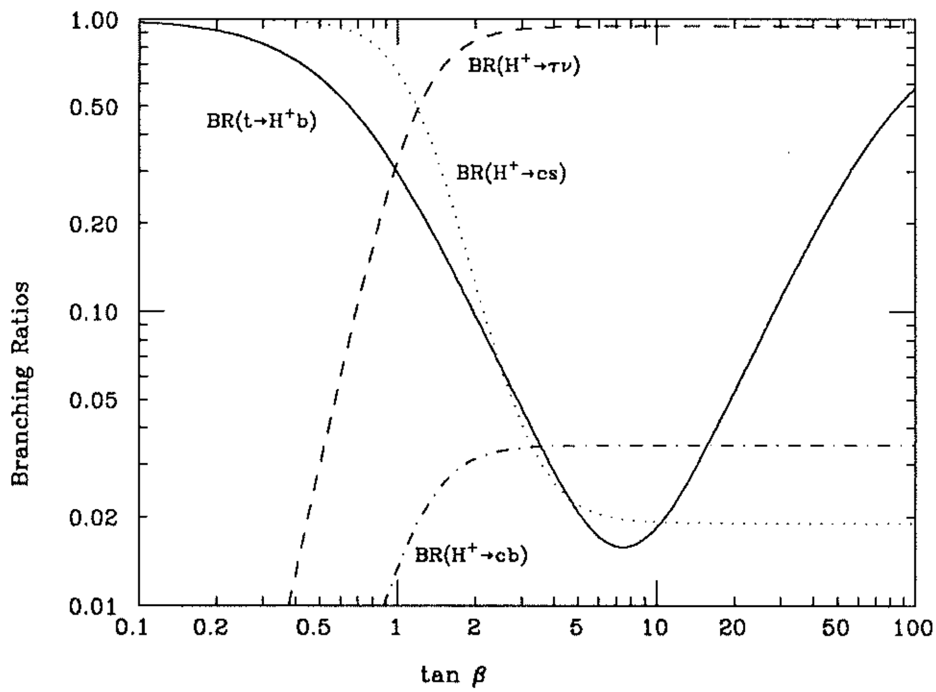


Figure 6 Branching fractions for the decays $t \to H^+ b$ and $H^+ \to \tau^+ \nu$, $c\bar{s}$ and $c\bar{b}$ as a function of $\tan\beta$, for $m_t = 250$ GeV and $m_{H^{\pm}} = 150$ GeV.

If $m_t < m_{H^{\pm}} + m_b$, then the most important production mechanisms for the $H^{\pm}$ derive from the subprocesses $gb \to H^- t$ and $g\bar{b} \to H^+ \bar{t}$. The charged Higgs

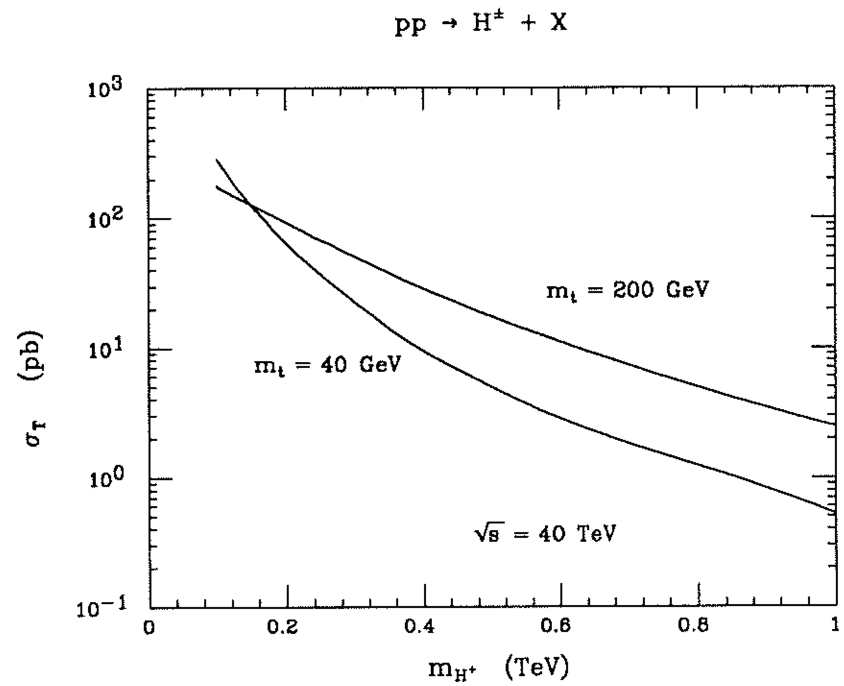


Figure 7 The cross section at the SSC for single charged Higgs production (summed over both charges) coming from $gb \to H^- t$ and $g\bar{b} \to H^+ \bar{t}$ as a function of $m_{H^\pm}$. Two extreme values of the top quark mass are considered: $m_t = 40$ GeV and $m_t = 200$ GeV.

boson cross-section is comparable in magnitude to the gg fusion cross section of the Standard Model Higgs at the same Higgs mass, as illustrated in fig. 7.

Even with such a large sample of charged Higgs bosons, it will be extremely difficult to isolate a signal above Standard Model backgrounds (if $m_{H^\pm} > m_t + m_b$). It is clear that QCD backgrounds to observing the H^+ via its $t\bar{b}$ decay are very large. Thus, one must concentrate on the search for the charged Higgs boson via rarer decay modes. Among the various possibilities are: $H^\pm \to W^\pm \gamma$, $H^\pm \to W^\pm +$ quarkonium, $H^\pm \to W^\pm h^0$, and $H^\pm \to \tau^\pm \nu$. Unfortunately, one would have to have an anomalously large branching ratio for one of these rare decays in order to have a feasible method for detecting the charged Higgs boson. By considering realistic branching ratios and the signatures of the possible rare decays, one comes to the conclusion that the charged Higgs boson will be very difficult to detect at a hadron collider if $m_{H^\pm} > m_t + m_b$.

4.6 Beyond Multi-Higgs-Doublet Models

As described at the beginning of this lecture, in order to contemplate Higgs sectors with Higgs triplets (or higher multiplets), one must first confront the problem of ensuring that $\rho_0 = 1$. Assuming that this task is completed successfully, some new features arise that were not encountered in our study of multi-Higgs doublet models. First, these extended Higgs models usually contain doubly charged Higgs bosons. This allows for spectacular decays, $H^{++} \to W^+W^+$, as well as a new mechanism for Higgs production via W^+W^+ fusion at hadron colliders. At e^+e^- colliders, the detection of $e^+e^- \to H^{++}H^{--}$ would be straightforward. Second, the Higgs boson sum rules which arise from unitarity constraints are more complicated. Two examples are:

$$g^2(4m_W^2 - 3m_Z^2\cos^2\theta_W) \simeq g^2 m_W^2 = \sum_k g^2_{W^+W^-H_k^0} - \sum_k g^2_{W^+W^+H_k^{--}}\,,$$
$$\frac{g^2 m_Z^4\cos^2\theta_W}{m_W^2} \simeq g^2 m_Z^2 = \sum_k g_{W^+W^-H_k^0}\, g_{ZZH_k^0} - \sum_k g^2_{W^+ZH_k^-}\,. \tag{4.50}$$

Thus, if one were to discover that H_1^0 possessed Standard Model coupling strengths: $g_{W^+W^-H_1^0} = gm_W$ and $g_{ZZH_1^0} = gm_Z/\cos\theta_W$ then there would still be room for a non-minimal Higgs sector if either $g_{VVH_i^0} \ll g_{VVH_1^0}$ $(i > 1)$ or if doubly charged Higgs bosons and a (tree-level) $W^\pm H^\mp Z^0$ vertex exist.

The existence of a tree-level $H^\pm W^\mp Z$ vertex is common in models with extended Higgs sectors which include triplets (or higher multiplets). Thus, the $H^\pm W^\mp Z$ vertex is a probe of exotic Higgs sectors. Such models provide a new $H^\pm$ production mechanism (WZ fusion) and decay signature: $H^\pm \to W^\pm Z$. Consider the $H^\pm W^\mp Z$ coupling in a general Higgs model. Previously, I showed that there is no $H^\pm W^\mp Z$ tree-level vertex in models with only Higgs doublets (and singlets). For Higgs bosons in an arbitrary representation, a straightforward calculation gives

$$\begin{aligned}\mathcal{L}_{H^\pm W^\mp V^0} = {} & em_W(W_\mu^+ A^\mu G^- + \text{h.c.}) + gm_Z\Bigg\{ W_\mu^+ Z^\mu \Big[G^-\cos^2\theta_W \\ & - \frac{g}{\sqrt{2}m_W}\sum_k Y_k\left(\phi_k^\dagger\, T^+ v_k + (T^- v_k)^\dagger\phi_k\right)\Big] + \text{h.c.}\Bigg\}\,,\end{aligned} \tag{4.51}$$

where $T^\pm = T^1 \pm iT^2$, and the Goldstone field is given by

$$G^- = \frac{g}{\sqrt{2}m_W}\left\{\sum_k\left[\phi_k^\dagger T^+ v_k - (T^- v_k)^\dagger\phi_k\right] + \sum_k \eta_i^T T^+ u_i\right\}\,. \tag{4.52}$$

In the equations above, I have used the following notation: ϕ_k denotes complex scalar fields, with hypercharge Y_k, and η_i denotes real scalar fields, with integer weak isospin and zero hypercharge. The Lagrangian in eq. (4.51) has been obtained after shifting the scalar fields by their VEVs: $\phi_k \to \phi_k + v_k$ and $\eta_i \to \eta_i + u_i$; the vacuum is assumed to preserve $\mathrm{U(1)_{EM}}$. Note that there is no $H^\pm W^\mp \gamma$ vertex, as expected (since the electromagnetic current is conserved). In general, there is a $H^\pm W^\mp Z$ vertex, which can be written in the following form

$$\mathcal{L}_{H^\pm W^\mp Z} = -g m_Z \xi (W_\mu^+ Z^\mu H^- + \text{h.c.})\,, \tag{4.53}$$

where

$$\xi^2 = \frac{\sum_{T,Y} Y^2[4T(T+1)-Y^2]\;|V_{T,Y}|^2}{\sum_{T,Y} c_{T,Y}[4T(T+1)-Y^2]\;|V_{T,Y}|^2} - \frac{1}{\rho_0^2} \tag{4.54}$$

and $c_{T,Y}$ is given by eq. (4.2). As a check, it is easy to verify $\xi = 0$ for a multi-doublet Higgs model. However, one should note that it is possible to have $\rho_0 = 1$ and $\xi \neq 0$. This can be illustrated with the CGGM model which consists of one complex $Y = 1$ doublet, one complex $Y = 2$ triplet and one real $Y = 0$ triplet Higgs multiplet, with $\langle\phi(\frac{1}{2},1)\rangle = a/\sqrt{2}$ and $\langle\phi(1,2)\rangle = \langle\phi(1,0)\rangle = b$. At the beginning of this lecture, I noted that $\rho_0 = 1$ in this model. It is easy to check that $\xi \neq 0$. Explicitly,

$$g_{H^+W^-Z^0} = \frac{-g m_W \sin\theta_H}{\cos\theta_W}\,, \qquad \sin\theta_H = \left(\frac{8b^2}{a^2+8b^2}\right)^{1/2} \tag{4.55}$$

and $a^2 + 8b^2 = (246\ \mathrm{GeV})^2$. For completeness, I note that this model also possesses a doubly charged Higgs boson with $g_{H^{++}W^-W^-} = \sqrt{2} g m_W \sin\theta_H$.

Although models with exotic Higgs representations are fun, they seem to me to be rather contrived. Multi-doublet Higgs models are the most elegant among the models with extended Higgs sectors. But do we really need an extended Higgs sector at all? The answer to this question may lie in the supersymmetric extension of the Standard Model. We now turn our attention in this direction.

Suggestions for Further Reading and a Brief Guide to the Literature

A detailed discussion of non-minimal Higgs models, with a large collection of Feynman rules can be found in

1. *The Higgs Hunter's Guide*, Chapter 4 and Appendices A and B.

A review of recent developments in CP-violation in gauge theory and an up-to-date bibliography, with emphasis on the calculations of the electric dipole moments of the neutron and electron can be found in

2. Darwin Chang, "Mechanisms of CP-Violation in Gauge Theory and the Recent Developments," Fermilab-Conf-90/265-T (1990), to appear in The Proceedings of the 9th Symposium on Theoretical Physics at Mt. Sorak, South Korea.

The material of section 4.4 is based on

3. H.E. Haber and Y. Nir, *Nucl. Phys.* **B335**, 363 (1990).

The most recent published limits on the charged Higgs boson mass from LEP are

4. D. Decamp *et al.* [ALEPH Collaboration], *Phys. Lett.* **B241**, 623 (1990); P. Abreu *et al.* [DELPHI Collaboration], *Phys. Lett.* **B241**, 449 (1990); B. Adeva *et al.* [L3 Collaboration] *Phys. Lett.* **B251**, 299 (1990); M.Z. Akrawy *et al.* [OPAL Collaboration], *Phys. Lett.* **B242**, 299 (1990).

A complete enumeration of sum rules for Higgs boson couplings based on unitarity constraints in arbitrary non-minimal Higgs models is given in

5. J.F. Gunion, H.E. Haber and J. Wudka, *Phys. Rev.* **D43**, 904 (1991).

5. The Higgs Sector in Models of Low-Energy Supersymmetry

Despite the simplicity of the Higgs mechanism, the existence of fundamental scalars in field theory is problematical. If the electroweak model is embedded in a more fundamental structure characterized by a much larger energy scale (*e.g.*, the Planck scale, which must appear in any theory including gravity), the Higgs boson would tend to acquire mass of order the large scale due to radiative corrections. Only by adjusting (*i.e.*, "fine-tuning") the parameters of the Higgs potential "unnaturally" can one arrange a large hierarchy between the Planck scale and the scale of electroweak symmetry breaking. Equivalently, one can check that in the Standard Model, a calculation of the first order correction to the Higgs boson mass squared yields a quadratically divergent expression arising from Standard Model particle loop graphs. This implies that it is not "natural" to have a Higgs boson that is relatively light unless this divergence can be controlled by the structure of the theory. The Standard Model provides no mechanism for this. Two classes of solutions of this "hierarchy" problem have been advanced. In one class of models, the Higgs bosons are replaced by composite bound states of fundamental fermions. Technicolor models and a variety of composite models fall into this class. This class of models will be addressed in Lecture 6. In a second class of models, supersymmetry is invoked to solve the hierarchy problem. In a supersymmetric theory the quadratic divergence is naturally cancelled by related loop graphs involving the supersymmetric partners of the Standard Model particles which appear in the divergent loops. As a result, the tree level mass squared of the Higgs boson receives corrections that are limited by the extent of supersymmetry breaking. In order that the naturalness and hierarchy problems be resolved, it is necessary that the scale of supersymmetry breaking not exceed $\mathcal{O}(1\ \text{TeV})$. Such "low-energy" supersymmetric theories are especially interesting in that, to date, they provide the only theoretical framework in which the problems of naturalness and hierarchy are resolved while retaining the Higgs bosons as truly elementary spin-0 particles. In this lecture, I will study the structure and phenomenology of Higgs bosons in supersymmetric extensions of the Standard Model.

5.1 Higgs Sector of the Minimal Supersymmetric Model (MSSM)

In the minimal supersymmetric extension of the Standard Model (MSSM), one simply associates a supersymmetric partner to all Standard Model particles. In addition, one must enlarge the Higgs sector to contain two Higgs doublets: a $Y = -1$ Higgs doublet (H_1) which couples to down-type quarks and charged leptons, and a $Y = +1$ Higgs doublet (H_2), which couples to up-type quarks and neutrinos. The Higgs fields are the scalar components of two chiral superfields: $\widehat{H}_1$ and $\widehat{H}_2$. In supersymmetric models, one introduces a superpotential

$$W = -\mu\epsilon_{ij}\widehat{H}_1^i\widehat{H}_2^j + \epsilon_{ij}[f\widehat{H}_1^i\widehat{L}^j\widehat{R} + f_1\widehat{H}_1^i\widehat{Q}^j\widehat{D} + f_2\widehat{H}_2^j\widehat{Q}^i\widehat{U}], \qquad (5.1)$$

where μ is a supersymmetric Higgs mass parameter, $\widehat{Q}$ and $\widehat{L}$ are $\mathrm{SU}(2)_L$ doublet quark and lepton superfields, $\widehat{U}$ and $\widehat{D}$ are $\mathrm{SU}(2)_L$ singlet quark superfields and $\widehat{R}$ is an $\mathrm{SU}(2)_L$ singlet lepton superfield. (The hypercharges are the same as those of the corresponding fermions in the Standard Model.) Thus the last three terms on the right hand side of eq. (5.1) are the supersymmetric analogs of the Higgs-fermion Yukawa couplings. One can now see why two Higgs doublet superfields are needed in the MSSM. If $\widehat{H}_2$ were deleted from the theory, one could not generate a Yukawa coupling between the Higgs superfield and $\widehat{Q}\widehat{U}$. One may be tempted to mimic the Standard Model and couple $i\sigma_2\widehat{H}_1^*$ to $\widehat{Q}^i\widehat{U}$. However, such a term would violate supersymmetry.* Thus, in order to be able to generate mass for both up and down type quarks in a supersymmetric model, one needs both the $\widehat{H}_1\widehat{Q}\widehat{D}$ and $\widehat{H}_2\widehat{Q}\widehat{U}$ terms in eq. (5.1). The requirement of two Higgs doublet superfields in the MSSM also follows from another argument. If I delete, say, $\widehat{H}_2$ from the theory, then I am deleting the fermionic partners of $\widehat{H}_2$ as well. One can check that such a theory would contain anomalies. In the MSSM, the anomalies due to the $Y = -1$ fermionic partners of $\widehat{H}_1$ precisely cancel those due to the $Y = +1$ fermionic partners of $\widehat{H}_2$. Thus the MSSM must be a two-Higgs doublet model.†

We proceed to examine the two-Higgs doublet sector of the MSSM in some detail. We can use many of the results of Lecture 4. But first, we must relate the Higgs fields H_1 and H_2 to the $Y = 1$ fields ϕ_1 and ϕ_2 introduced earlier:

$$\begin{aligned} H_1 &= \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} \phi_1^{0\,*} \\ -\phi_1^- \end{pmatrix} \\ H_2 &= \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}, \end{aligned} \qquad (5.2)$$

since $H_1 = i\sigma_2\phi_1^*$ has hypercharge $Y = -1$. Supersymmetry imposes strong constraints on the form of the Higgs potential. Even allowing for the most general

---

⋆ In supersymmetric models, the superpotential W must be a holomorphic function of chiral superfields. That is, it cannot be a function of chiral superfields and their complex conjugates. For this reason, the sign of the hypercharge of a chiral Higgs superfield is meaningful. One could not have constructed the MSSM with two $Y = +1$ Higgs superfields.

† By requiring that anomalies cancel, it follows that the supersymmetric extension of the N-Higgs-doublet model must contain N Higgs doublets with $Y = -1$ and N Higgs doublets with $Y = +1$. Hence, such models contain an *even* number of Higgs doublets.

soft-supersymmetry breaking in the model, the dimension-four terms of the Higgs potential must respect the supersymmetry. These requirements impose relations among the λ_i of eq. (4.8). The resulting Higgs potential in the MSSM is

$$
\begin{aligned}
V = & \left(m_1^2 + |\mu|^2\right) H_1^{i*} H_1^i + \left(m_2^2 + |\mu|^2\right) H_2^{i*} H_2^i - m_{12}^2 \left(\epsilon_{ij} H_1^i H_2^j + h.c.\right) \\
& + \tfrac{1}{8}\left(g^2 + g'^2\right)\left[H_1^{i*} H_1^i - H_2^{j*} H_2^j\right]^2 + \tfrac{1}{2} g^2 |H_1^{i*} H_2^i|^2,
\end{aligned}
\tag{5.3}
$$

where μ is a supersymmetric Higgs mass parameter and m_1^2, m_2^2, m_{12}^2 are soft-supersymmetry-breaking masses. It is convenient to reexpress the doublet fields in terms of the physical Higgs boson degrees of freedom and the Goldstone boson fields. The relations are:

$$
\begin{aligned}
H_2^1 = & H^+ \cos\beta + G^+ \sin\beta \\
H_1^2 = & H^- \sin\beta - G^- \cos\beta \\
H_1^1 = & v_1 + \frac{1}{\sqrt{2}}(H^0 \cos\alpha - h^0 \sin\alpha + iA^0 \sin\beta - iG^0 \cos\beta) \\
H_2^2 = & v_2 + \frac{1}{\sqrt{2}}(H^0 \sin\alpha + h^0 \cos\alpha + iA^0 \cos\beta + iG^0 \sin\beta),
\end{aligned}
\tag{5.4}
$$

where α is the mixing angle that arises in the process of diagonalizing the 2×2 neutral scalar Higgs mass matrix [see eqs. (4.15) and (4.16)], and $\tan\beta \equiv v_2/v_1$. It is possible to choose a phase convention in which v_1 and v_2 are real and positive. This implies that $0 \leq \beta \leq \pi/2$. In this convention, the Higgs potential is manifestly CP-conserving. In addition, due to the constraints imposed on the supersymmetric model, we find that $-\pi/2 \leq \alpha \leq 0$. Supersymmetry imposes strong constraints on an arbitrary two-Higgs doublet model potential. As a result, there are relations among the six free parameters of the general two-Higgs doublet model ($m_{H^+}, m_{H^0}, m_{h^0}, m_{A^0}, \alpha$, and $\tan\beta \equiv v_2/v_1$). The MSSM Higgs sector possesses two free parameters; if we fix $\tan\beta$ and one of the Higgs masses (or α), then all parameters and couplings of the MSSM Higgs sector are fixed.

One property of eq. (5.3) is particularly noteworthy: all quartic Higgs self-couplings are gauge couplings. This should be contrasted with the Standard Model where the Higgs self coupling is a free parameter. In the MSSM, gauge invariance forbids a trilinear coupling of Higgs superfields which could have given rise to a quartic Higgs self-coupling which is independent of the gauge couplings.‡ As a

‡ If one introduces a singlet Higgs superfield $\widehat{N}$, then the term $\widehat{H}_1 \widehat{H}_2 \widehat{N}$ in the superpotential would produce such a quartic Higgs self-coupling. Such terms do arise in certain extended (non-minimal) supersymmetric models.

result, the Higgs self-couplings in the MSSM are fixed in size and cannot grow arbitrarily large. It follows that tree-level unitarity is automatically satisfied independent of the Higgs masses in the MSSM. In addition, one CP-even scalar mass must be bounded (to be less than some number of order m_Z). Again, these results are in stark contrast to the Standard Model, where tree-level unitarity can be violated and the tree-level Higgs mass is unbounded if one takes the Higgs quartic self-coupling to be arbitrarily large.

For pedagogical purposes, I will analyze the MSSM Higgs potential in three easy steps and derive formulae for the tree-level Higgs masses and mixing angles. Without loss of generality, I will absorb $|\mu|^2$ into the definitions of m_1^2 and m_2^2. First, let us analyze the CP-even scalars. To do this, simply insert $H_1^1 = v_1$, $H_2^2 = v_2$, and $H_1^2 = H_2^1 = 0$ into eq. (5.3). Then

$$V = m_1^2 v_1^2 + m_2^2 v_2^2 - 2m_{12}^2 v_1 v_2 + \tfrac{1}{8}(g^2 + g'^2)(v_1^2 - v_2^2)^2 . \tag{5.5}$$

The minimum conditions are obtained by setting $\partial V/\partial v_i = 0$. Thus,

$$\begin{aligned} m_1^2 &= m_{12}^2 \frac{v_2}{v_1} - \tfrac{1}{4}(v_1^2 - v_2^2)(g^2 + g'^2) \\ m_2^2 &= m_{12}^2 \frac{v_1}{v_2} + \tfrac{1}{4}(v_1^2 - v_2^2)(g^2 + g'^2) . \end{aligned} \tag{5.6}$$

The scalar mass matrix is

$$M_{ij}^2 = \tfrac{1}{2}\frac{\partial^2 V}{\partial v_i \partial v_j} , \tag{5.7}$$

where the factor of 1/2 is due to the normalization of the quadratic terms in V. Thus, the CP-even scalar mass matrix is

$$M_{ij}^2 = \frac{1}{v_1^2 + v_2^2}\begin{pmatrix} m_A^2 v_2^2 + m_Z^2 v_1^2 & -(m_A^2 + m_Z^2)v_1 v_2 \\ -(m_A^2 + m_Z^2)v_1 v_2 & m_A^2 v_1^2 + m_Z^2 v_2^2 \end{pmatrix} , \tag{5.8}$$

where

$$\begin{aligned} m_A^2 &\equiv \frac{m_{12}^2}{v_1 v_2}(v_1^2 + v_2^2) , \\ m_Z^2 &= \tfrac{1}{2}(g^2 + g'^2)(v_1^2 + v_2^2) . \end{aligned} \tag{5.9}$$

Note that

$$\mathrm{Tr}\, M_{ij}^2 \equiv m_{H^0}^2 + m_{h^0}^2 = m_A^2 + m_Z^2 . \tag{5.10}$$

We can easily evaluate the eigenvalues of M_{ij}^2. These are the squared masses of the two CP-even Higgs scalars

$$m^2_{H^0,h^0} = \tfrac{1}{2}\left[m_A^2 + m_Z^2 \pm \sqrt{(m_A^2+m_Z^2)^2 - 4m_Z^2 m_Z^2\cos^2 2\beta}\right]\,. \tag{5.11}$$

The diagonalizing angle is α, with

$$\cos 2\alpha = -\cos 2\beta\left(\frac{m_A^2 - m_Z^2}{m^2_{H^0} - m^2_{h^0}}\right)\,, \qquad \sin 2\alpha = -\sin 2\beta\left(\frac{m^2_{H^0} + m^2_{h^0}}{m^2_{H^0} - m^2_{h^0}}\right)\,. \tag{5.12}$$

Finally, the following inequalities are easily established:

$$\begin{aligned} &m_{h^0} \leq m_A \\ &m_{h^0} \leq m|\cos 2\beta| \leq m_Z\,, \qquad \text{with } m \equiv min(m_Z, m_A) \\ &m_{H^0} \geq m_Z\,. \end{aligned} \tag{5.13}$$

Second, we analyze the CP-odd scalars. Write $H_1^1 = v_1 + i\xi_1$, $H_2^2 = v_2 + i\xi_2$ and set $H_1^2 = H_2^1 = 0$. We now compute the mass matrix $M^2_{ij} = \frac{1}{2}\partial^2 V/\partial\xi_i\partial\xi_j$, using the formulae for m_1^2 and m_2^2 obtained in eq. (5.6). The result is

$$M^2_{ij} = m^2_{12}\begin{pmatrix} v_2/v_1 & 1 \\ 1 & v_1/v_2 \end{pmatrix}\,. \tag{5.14}$$

The zero eigenvalue corresponds to the Goldstone boson. Hence,

$$m^2_{A^0} = \mathrm{Tr}\, M^2_{ij} = m^2_{12}\left(\frac{v_1}{v_2} + \frac{v_2}{v_1}\right) \equiv m_A^2\,. \tag{5.15}$$

That is, we now recognize the parameter m_A defined in eq. (5.9) to be the mass of the CP-odd scalar.

Third, we analyze the charged scalars. Write $H_1^1 = v_1$, $H_2^2 = v_2$, and compute the H_1^2–H_2^1 mass matrix. The result is

$$M^2_{ij} = \left(\frac{m^2_{12}}{v_1 v_2} + \tfrac{1}{2}g^2\right)\begin{pmatrix} v_2^2 & v_1v_2 \\ v_1v_2 & v_1^2 \end{pmatrix}\,, \tag{5.16}$$

which again possesses a zero mass Goldstone boson eigenstate. Hence,

$$\begin{aligned} m^2_{H^\pm} = \mathrm{Tr}\, M^2_{ij} &= \frac{m^2_{12}(v_1^2+v_2^2)}{v_1v_2} + \tfrac{1}{2}g^2(v_1^2+v_2^2) \\ &= m_A^2 + m_W^2\,. \end{aligned} \tag{5.17}$$

It follows that

$$m_{H^\pm} \geq m_W\,. \tag{5.18}$$

This completes the analysis of the MSSM Higgs potential.

To summarize, we have computed the tree-level Higgs boson masses (and mixing angle) in the MSSM. The result of our calculation is that two parameters are sufficient to fix the properties of the Higgs sector. For example, one can adopt $\tan\beta$ and m_{A^0} as the two independent parameters. Once these two quantities are specified all the tree-level Higgs masses can be computed according to

$$\begin{aligned} m_{H^\pm}^2 &= m_{A^0}^2 + m_W^2 \\ m_{H^0,h^0}^2 &= \tfrac{1}{2}\left[m_{A^0}^2 + m_Z^2 \pm \sqrt{(m_{A^0}^2 - m_Z^2)^2 + 4m_Z^2 m_{A^0}^2 \sin^2 2\beta}\,\right], \end{aligned} \tag{5.19}$$

where I rewritten eq. (5.11) in an equivalent form. The spectrum of Higgs boson masses in the minimal supersymmetric model is illustrated in fig. 8. From this graph, we note an interesting limiting case of this model when $m_{A^0} \to \infty$ (at fixed $\tan\beta$). In this limit, A^0, H^0 and $H^\pm$ decouple from the theory and we are left with a Higgs sector (consisting of a single physical CP-even scalar, h^0) which is identical to the Higgs boson of the minimal Standard Model. Moreover, in this limit, the interactions of h^0 with the Standard Model gauge bosons and fermions are equivalent to those of the minimal Higgs boson of the (nonsupersymmetric) Standard Model. I will return to this point in section 5.3.

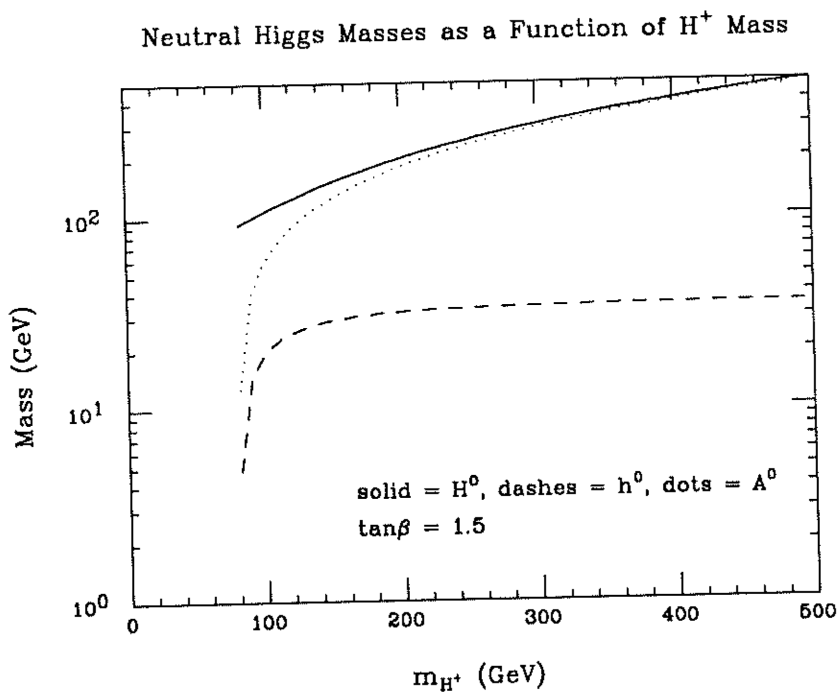


Figure 8 The masses of H^0, h^0 and A^0 are plotted as functions of $m_{H^\pm}$ at fixed $\tan\beta = 1.5$ for the minimal supersymmetric model.

5.2 Radiative Corrections to Higgs Masses in the MSSM

One of the most important predictions of the MSSM is that $m_{h^0} \leq m_Z$. This prediction is particularly important for future experiments at LEP-II. In principle, experiments running at LEP-II operating at design luminosity could either discover the Higgs boson or rule out the MSSM. (Whether this is possible to do in practice depends on whether these experiments can rule out a Higgs boson with $m_{h^0} \approx m_Z$.) However, it is important to emphasize that the bounds of eq. (5.13) are tree-level bounds. In particular, $m_{h^0} \leq m_Z$ need not be respected when radiative corrections are incorporated. In general, one might expect the size of the (electroweak) radiative corrections to the Higgs squared masses to be of order $g^2 m_Z^2/4\pi$, which would shift the Higgs masses by (at most) a few GeV. However, we encounter a surprise when we consider the question: what is the upper bound for the mass of the lightest Higgs scalar h^0 when one includes the full one-loop radiative corrections of the MSSM? An important clue can be found in a paper by M. Berger, who considered the radiative corrections to the Higgs mass sum rule given in eq. (5.10). Ralf Hempfling and I decided to isolate explicitly the radiative correction to the value of m_{h^0} and deduce its maximal value. I will now briefly sketch our computation.

First, consider the model in which the tree-level bound $m_{h^0} \leq m_Z$ is saturated. To achieve this, we must take either $\tan\beta = \pi/2$ or $\tan\beta = 0$, *i.e.*, either $v_1 = 0$ or $v_2 = 0$ (respectively). Clearly, in a realistic model, neither VEV can be zero, otherwise, all charge -1/3 (or 2/3) quarks would be massless. Nevertheless, the approximation $v_1 = 0$ in which all charge -1/3 quarks are massless is not too bad an approximation, considering that the top-quark is so much heavier than all other quarks and leptons. Thus we start by considering the $v_1 = 0$ model. This model is obtained by setting the soft-supersymmetry-breaking mass parameter m_{12} in eq. (5.3) to zero. In this model, the tree-level Higgs mass spectrum consists of $m_{h^0} = m_Z$, $m_{H^0} = m_{A^0} \geq m_Z$, and $m_{H^\pm} = (m_W^2 + m_{A^0}^2)^{1/2}$. [Had we chosen $m_{A^0} < m_Z$, we would have found $m_{h^0} = m_{A^0}$, and $m_{H^0} = m_Z$ (at tree-level), which is not relevant for the computation of the upper bound for m_{h^0}.] Note that the mass degeneracy of the CP-even and CP-odd scalars holds to all orders in perturbation theory due to an extra continuous U(1) global symmetry which is present when $m_{12} = 0$. We can now compute corrections to the value of m_{h^0} and derive an expression for $\Delta m_h^2 \equiv m_{h^0}^2 - m_Z^2$.

There are two corrections that I will compute here. The first consists of the one-loop radiative corrections to the model specified above. This will be denoted by $(\Delta m_h^2)_{\beta=\pi/2}$, where the subscript emphasizes that we have computed this quantity in the model where $v_1 = 0$ (*i.e.*, $\beta = \pi/2$). The second correction consists of computing the shift in m_{h^0} due to the fact that any realistic model must have two non-vanishing VEVs. We incorporate this correction *at tree-level*. This is easy to do by employing the exact tree-level formula [eq. (5.19)]. Thus, the final result for the squared mass shift is

$$\Delta m_h^2 = \left(\Delta m_h^2\right)_{\beta=\pi/2} - \tfrac{1}{2}\left[\sqrt{(m_{A^0}^2 - m_Z^2)^2 + 4m_{A^0}^2 m_Z^2 \sin^2 2\beta} - (m_{A^0}^2 - m_Z^2)\right]. \tag{5.20}$$

As long as $\tan\beta$ is not close to 1, the correction due to the second term above will be small and it is consistent to ignore new one-loop corrections which arise when $\beta \neq \pi/2$.

I now turn to the computation of $(\Delta m_h^2)_{\beta=\pi/2}$. The tree-level potential of the $v_1 = 0$ model is

$$\mathcal{V}_h = m_0^2\left(\frac{h}{\sqrt{2}} + v_0\right)^2 + \tfrac{1}{8}(g_0^2 + g_0'^2)\left(\frac{h}{\sqrt{2}} + v_0\right)^4, \tag{5.21}$$

where $v \equiv v_2$ and the 0 subscripts indicate the bare parameters. We have only kept the relevant terms involving the light scalar field, h. The Z mass term arises from

$$\mathcal{V}_Z = \tfrac{1}{2}m_{Z0}^2 Z_\mu Z^\mu, \tag{5.22}$$

where $m_{Z0}^2 = \frac{1}{2}(g_0^2 + g_0'^2)v_0^2$. We do not need to renormalize the fields, so bare fields will not be indicated explicitly. Minimizing $\mathcal{V}_h$, it follows that $m_0^2 = -m_{Z0}^2/2$. We now introduce the renormalized parameters by shifting the corresponding bare parameters: $m_0^2 \equiv m^2 - \delta m^2$, $v_0 = v - \delta v$, *etc.* Then, we find

$$\mathcal{V}_h = (t - \delta t)h + \tfrac{1}{2}(m_h^2 - \delta m_h^2)h^2 + \mathcal{O}(h^3), \tag{5.23}$$

$$\mathcal{V}_Z = \tfrac{1}{2}(m_Z^2 - \delta m_Z^2)Z_\mu Z^\mu, \tag{5.24}$$

where $m_Z^2 = \frac{1}{2}(g^2 + g'^2)v^2$ and

$$t = \left(\tfrac{1}{2}m_Z^2 + m^2\right)v\sqrt{2}, \tag{5.25}$$

$$m_h^2 = \tfrac{3}{2}m_Z^2 + m^2. \tag{5.26}$$

By making use of the tree-level minimum condition, δv drops out, and we obtain

$$\delta m_h^2 = \delta m_Z^2 + \frac{g}{2m_W}\delta t, \tag{5.27}$$

where $m_W = gv/\sqrt{2}$. At this point in the analysis, m_h and m_Z are renormalized (and finite) parameters, but not yet physical parameters. The physical masses of

h^0 and Z (indicated below with a subscript P) are identified in the usual way as the poles in the corresponding propagators. Let the sum of all one-loop Feynman graphs contributing to the Z-boson and h^0 two-point functions be denoted by $iA_{ZZ}(q^2)g^{\mu\nu} + iB_{ZZ}(q^2)q^\mu q^\nu$ and $-iA_{hh}(q^2)$, respectively, where q is the four-momentum of one of the external legs. The physical masses are then given by

$$m_{ZP}^2 = m_Z^2 + \text{Re } A_{ZZ}(m_Z^2) - \delta m_Z^2\,, \tag{5.28}$$

$$m_{hP}^2 = m_h^2 + \text{Re } A_{hh}(m_h^2) - \delta m_h^2\,. \tag{5.29}$$

We now demand that v is the true vacuum expectation value at one-loop. This means that $t = 0$ and $\delta t = A_h(0)$, where $-iA_h(0)$ is the sum of all one-loop Feynman graphs contributing to the h^0 one-point function (tadpole). That is, the tadpole counterterm cancels the one-loop tadpole graphs, so the full tadpole vanishes. This choice is convenient since there will be no tadpole contributions to the calculation of A_{ZZ} and A_{hh}. It follows that $m_h = m_Z$, and we end up with:

$$(\Delta m_h^2)_{\beta=\pi/2} \equiv m_{hP}^2 - m_{ZP}^2 = \text{Re } \left[A_{hh}(m_Z^2) - A_{ZZ}(m_Z^2)\right] - \frac{g}{2m_W}A_h(0)\,. \tag{5.30}$$

Note that each term in eq. (5.30) is separately divergent. The divergences will cancel only when one sums over a complete supersymmetric multiplet. This is true because m_{h^0} is calculable in the supersymmetric model, in contrast to the Standard Model where the Higgs mass is an infinitely renormalized parameter. The dominant contribution to the Higgs mass shift comes from the quark and squark loop contributions, so I will only consider here the effects from this sector of the model. The parameters of the squark sector include common soft-supersymmetry breaking masses: $M_{\widetilde{Q}}$, $M_{\widetilde{U}}$ and $M_{\widetilde{D}}$, corresponding to $\widetilde{q}_L \equiv (\widetilde{u}_L, \widetilde{d}_L)$, $\widetilde{u}_R$ and $\widetilde{d}_R$ respectively. (Generation labels will be suppressed. For the sleptons, the definitions are similar, except that there is no $\widetilde{\nu}_R$.) In addition, there is a $\widetilde{q}_L$–$\widetilde{q}_R$ mixing mass parameter. The exact expressions for the squark/quark and slepton/lepton contributions to the h^0 mass shift can be computed in a straightforward manner. Here I shall simply quote a convenient approximate formula which is valid in the limit where $m_Z < m_t \ll M_{\widetilde{Q}}$, and where $\widetilde{q}_L$–$\widetilde{q}_R$ mixing is neglected. Summing over six flavors of quarks/squarks and leptons/sleptons and assuming that the common soft-supersymmetry breaking squark and slepton masses are all equal to $M_{\widetilde{Q}}$, the

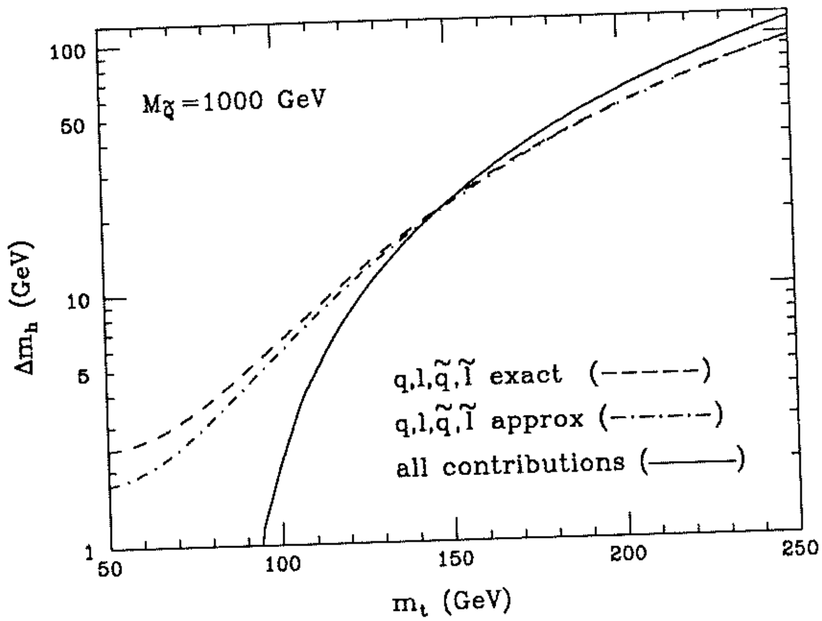


Figure 9 Higgs mass shift due to one-loop radiative corrections. The dashed line denotes the contribution to $\Delta m_h \equiv m_{h^0} - m_Z$ due to three generations of quarks, leptons, and their supersymmetric scalar partners. The squarks and sleptons are taken to have a common soft-supersymmetry breaking mass of $M_{\widetilde{Q}} = 1$ TeV and $\widetilde{t}_L$–$\widetilde{t}_R$ mixing is neglected (*i.e.*, $A = 0$). The dot-dashed line is a plot of eq. (5.31), and provides a good approximation to the dashed line. The solid line represents a sum of *all* contributions to the exact one-loop calculation of Δm_h for a choice of supersymmetric parameters: $\tan\beta = 20$ and $M_{\widetilde{Q}} = A = m_{A^0} = \mu = M = 1$ TeV (where M and μ determine the neutralino/chargino spectrum).

resulting formula is:

$$
\begin{aligned}
\left(\Delta m_h^2\right)_{\beta=\pi/2} \simeq \frac{3g^2 m_Z^4}{16\pi^2 m_W^2} \Bigg\{ & \ln\left(\frac{M_{\widetilde{Q}}^2}{m_t^2}\right)\left[\frac{2m_t^4 - m_t^2 m_Z^2}{m_Z^4} + \tfrac{1}{6}\left(1 - \tfrac{8}{3}s_W^2 + \tfrac{32}{9}s_W^4\right)\right] \\
& + \ln\left(\frac{M_{\widetilde{Q}}^2}{m_Z^2}\right)\left[\tfrac{1}{3}\left(1 - \tfrac{8}{3}s_W^2 + \tfrac{32}{9}s_W^4\right) + \tfrac{1}{2}\left(1 - \tfrac{4}{3}s_W^2 + \tfrac{8}{9}s_W^4\right)\right. \\
& \left. + \tfrac{1}{3}\left(1 - 2s_W^2 + 4s_W^4\right)\right] + \frac{m_t^2}{3m_Z^2}\Bigg\},
\end{aligned}
\tag{5.31}
$$

where $s_W \equiv \sin\theta_W$.

The corrections to the tree-level formula increase as the fourth power of m_t, and therefore can be quite large. In fig. 9, we plot the contribution of the quarks and leptons and their supersymmetric scalar partners to the linear mass shift

$$\Delta m_h \equiv m_{h^0} - m_Z = \left[\Delta m_h^2 + m_Z^2\right]^{1/2} - m_Z\,. \tag{5.32}$$

for $M_{\widetilde{Q}} = 1$ TeV. Clearly, the mass shift can be very significant as m_t becomes large. It is evident from eq. (5.31) that the dependence of Δm_h^2 on $M_{\widetilde{Q}}$ is logarithmic. Thus, even if $M_{\widetilde{Q}}$ is significantly smaller than 1 TeV, the Higgs mass shift can be appreciable if m_t is sufficiently large. For example, if $M_{\widetilde{Q}} = 400$ GeV, an exact numerical computation yields $\Delta m_h = 4$ GeV for $m_t = 100$ GeV and $\Delta m_h = 30$ GeV for $m_t = 200$ GeV. Fig. 9 also exhibits the results of a calculation where all supersymmetric sectors are included (including the possibility of squark mixing), and eq. (5.20) has been used to allow for a value of $\beta \neq \pi/2$. The supersymmetric parameters chosen are $M_{\widetilde{Q}} = A = m_{A^0} = \mu = M = 1$ TeV (where A parameterizes $\widetilde{t}_L$–$\widetilde{t}_R$ mixing) and $M' \simeq M/2$. The result is the solid line plotted in fig. 9, and provides a realistic indication of the true upper bound for the mass of h^0 in the MSSM.

5.3 Higgs Boson Couplings in the MSSM

Let us now turn to the couplings of the various Higgs bosons of the minimal supersymmetric model. A complete summary of all Higgs boson couplings in the minimal supersymmetric model is given in Appendix A of the HHG. Here I shall discuss only the couplings to Standard Model particles, in particular to quarks, W's and Z's. It is these couplings which are crucial in determining the production of the Higgs bosons, and in the absence of light supersymmetric particles would also completely determine their decays.

We have already seen that the A^0 has no VV couplings, The coupling of h^0 and H^0 to VV are given by eq. (4.23), and to A^0Z are given by eq. (4.27). Each of these couplings are either proportional to $\sin(\beta-\alpha)$ or $\cos(\beta-\alpha)$ as indicated below

$\cos(\beta-\alpha)$	$\sin(\beta-\alpha)$
$H^0W^+W^-$	$h^0W^+W^-$
H^0ZZ	h^0ZZ
ZA^0h^0	ZA^0H^0
$W^\pm H^\mp h^0$	$W^\pm H^\mp H^0$

In the supersymmetric model these couplings can be computed in terms of two

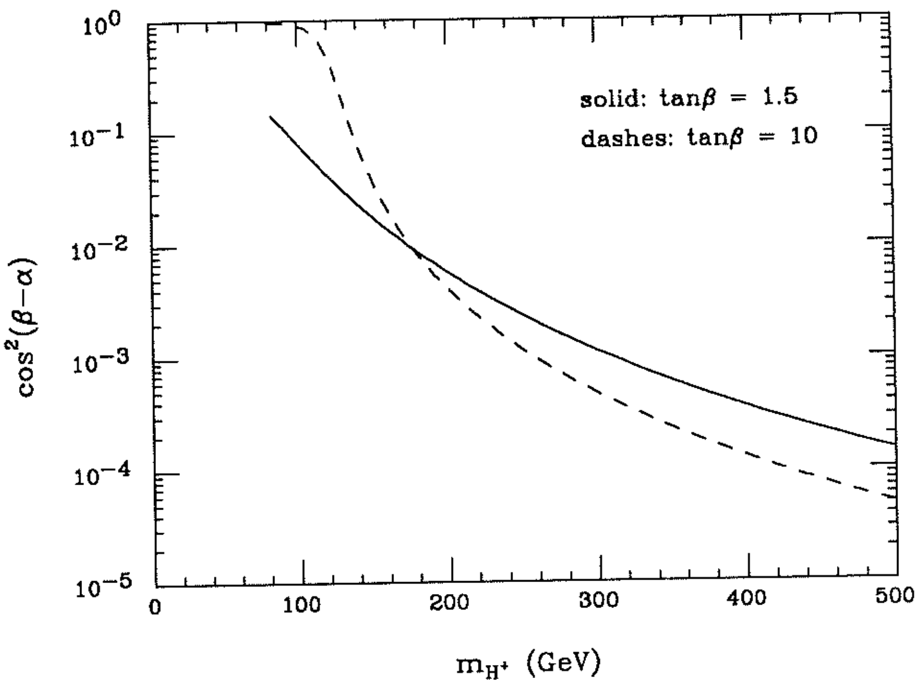


Figure 10 $\cos^2(\beta-\alpha)$ as a function of the charged Higgs mass, $m_{H^\pm}$, at $\tan\beta = 1.5$ and 10. Note that $m_{H^+} \geq m_W$ and $\cos^2(\beta-\alpha) = \cos^2 2\beta$ at $m_{H^+} = m_W$.

Higgs masses (and m_Z). For example,

$$\cos^2(\beta-\alpha) = \frac{m_{h^0}^2(m_Z^2 - m_{h^0}^2)}{(m_{H^0}^2 - m_{h^0}^2)(m_{H^0}^2 + m_{h^0}^2 - m_Z^2)}. \tag{5.33}$$

A close examination of this formula shows a dramatic suppression over a very large region of parameter space. It is easily verified that the maximum possible value for $\cos^2(\beta-\alpha)$ at fixed $\tan\beta$ (which occurs in the limit $m_{H^\pm} \to m_W$) is $\cos^2 2\beta$. Note that as m_{h^0} decreases, so does the maximum possible value of $\cos^2(\beta-\alpha)$. In addition, as $m_{H^\pm}$ increases, $\cos^2(\beta-\alpha)$ decreases further; for instance, once $m_{H^\pm}$ is large enough that $m_{H^0} \gtrsim 2m_W$ then $\cos^2(\beta-\alpha)$ vanishes as $1/m_{H^0}^4$. This behavior is illustrated in fig. 10 for the cases of $\tan\beta = 1.5$ and $\tan\beta = 10$. For example, when $\tan\beta = 1.5$, $\cos^2(\beta-\alpha)$ is $\lesssim 0.15$ at $m_{H^\pm} = m_W$, and is $\lesssim 0.01$ by the time $m_{H^0} > 2m_Z$. Thus, according to eq. (4.23), one should generally expect the coupling of W^+W^- and ZZ to the heavier Higgs scalar (H^0) to be greatly suppressed. Using eq. (4.22), this also implies that the coupling of W^+W^- and ZZ to the lighter Higgs scalar (h^0) should be roughly equal in strength to the corresponding couplings of the minimal Higgs boson.

To understand the origin of the large $\cos(\beta-\alpha)$ suppression, consider the MSSM in the limit of $m_{12} \gg m_Z$. Since m_{12} is a soft-supersymmetry-breaking parameter, increasing m_{12} is equivalent to raising the effective scale of supersymmetry breaking. This limit is equivalent to taking $m_{A^0} \gg m_Z$. It then follows that

(*i*) $m_{A^0} \simeq m_{H^+} \simeq m_{H^0} \gg m_Z, m_{h^0}$;

(*ii*) $\cos(\beta - \alpha) = \mathcal{O}(m_Z^2/m_{A^0}^2)$;

(*iii*) The couplings of h^0 to vector bosons and fermions approach the values of the minimal (one-doublet) Higgs model.

That is, the Higgs sector of the "low-energy" theory (far below m_{12}) is precisely that of the minimal Higgs model. This is consistent with observations made in section 4.4. What we see here is that it does not take a very large value of m_{12} (or m_A) to produce a "low energy" Higgs sector which is nearly identical to that of the minimal Higgs model.

The couplings of the various Higgs bosons to quarks were given in eqs. (4.31) and (4.32) as a function of α and β. A survey of the parameter space reveals that when $\tan\beta > 1$ the H^0 and h^0 couplings to $t\bar{t}$ are somewhat suppressed relative to the Standard Model H^0, while the $b\bar{b}$ couplings are somewhat enhanced, and vice versa for $\tan\beta < 1$. In general, supersymmetric model builders favor models in which $\tan\beta > 1$. This result is obtained from renormalization group analyses where the low-energy supersymmetric parameters are obtained from an initial set of Planck scale parameters by RG evolution. The fact that $\tan\beta > 1$ appears to be a rather general result in models where the top-quark mass is heavy (as indicated by the present experimental bounds).

5.4 Higgs Boson Phenomenology in the MSSM

In section 4.5, I treated the phenomenology of two-Higgs-doublet models. Although much of the discussion there is also applicable to the MSSM, the phenomenology of the supersymmetric model is more constrained, since the MSSM Higgs sector depends on fewer free parameters. We begin by reviewing the present experimental limits from LEP. From eq. (5.18), the decay $Z \to H^+H^-$ is not kinematically allowed. Thus, there are no limits from LEP on the charged Higgs boson of the MSSM. The four LEP detector collaborations have made an extensive search for h^0 and A^0 with a simultaneous search for $Z \to h^0 f\bar{f}$ and $Z \to h^0 A^0$. As discussed in Lecture 4, these decay rates generally depend on three parameters: the two Higgs masses and $\sin(\beta - \alpha)$. In the MSSM, only two of these parameters are independent. Thus, it is possible to display the experimental limits in the m_{h^0}–m_{A^0} plane. The LEP results which have been presented are based on a tree-level analysis; thus the region $m_{A^0} < m_{h^0}$ is not allowed [see eq. (5.13)]. For example, the L3 collaboration claims to rule out the region of h^0 and A^0 masses up to 41.5 GeV (at 95% confidence level). The results of the other LEP experiments are similar.

We now turn to issues of MSSM Higgs phenomenology at future colliders. Some of the salient points are summarized below:

1. Perhaps the most important prediction is that $m_h \leq m_Z|\cos 2\beta| \leq m_Z$. Moreover, according to eq. (4.23), $g_{ZZh^0} = g_{ZZ\phi^0}\sin(\beta-\alpha)$ (where ϕ^0 is the minimal Higgs boson), and $\sin(\beta-\alpha)$ is near 1 over a very large region of supersymmetric parameter space (see fig. 10). These are tree-level predictions of the model which if satisfied imply that h^0 could be discovered (or completely ruled out) at LEP or LEP-II in $Z \to Z^* h^0$ or $e^+e^- \to Z^* \to Zh^0$. [See the discussion below eq. (3.55).] However, in section 5.2, I showed that large radiative corrections were possible (particularly if m_t is substantially larger than the present experimental bound) which raise the mass of h^0 significantly above its tree-level value. Thus, the maximum value of m_{h^0} is above m_Z as indicated in fig. 9, and one must conclude that LEP-II will not be able to explore the entire range of the MSSM Higgs sector parameter space. (Radiative corrections to g_{ZZh^0} are less important and do not appreciably affect this discussion.) If $m_{h^0} > m_Z$, then h^0 is certainly an intermediate mass Higgs with couplings similar to the minimal Higgs boson of the Standard Model. One would then need to make use of the hadron (or e^+e^-) supercolliders to discover the h^0 or definitively rule out the MSSM.
2. On the other hand, the other physical Higgs particles, H^0, A^0 and $H^\pm$, may be difficult to detect. In the same limit where the properties of h^0 approach those of the minimal Higgs boson, the other physical Higgs particles tend to decouple. [For example, the coupling of H^0, A^0 and $H^\pm$ to VV or Vh^0 final states are suppressed by $\cos(\beta-\alpha)$, as indicated above eq. (5.33).] The pseudoscalar A^0 can also be light (although there is no particularly favored value for its mass). However, there is no tree-level coupling of A^0 to vector bosons, which makes it impossible to observe using methods which depend on the W^+W^- and ZZ couplings to the Higgs boson. As mentioned in Lecture 4, the most promising method for A^0 detection is via $e^+e^- \to Z^* \to A^0 h$ ($h = h^0$ or H^0) at LEP-II. The charged Higgs boson $H^\pm$ is predicted to be heavier than the W. In Lecture 4, I noted that it is very difficult to detect the $H^\pm$ at a hadron supercollider unless $m_{H^\pm} > m_t$, while H^+H^- pair production is straightforward to detect at e^+e^- colliders, as long as $m_{H^+} \lesssim 0.4\sqrt{s}$. Thus, to discover the charged Higgs boson of the MSSM will require the services of an e^+e^- supercollider.
3. As emphasized above, the coupling of the heavier scalar Higgs, H^0, to vector boson pairs is suppressed. Explicitly, $g_{W^+W^-H^0} = g_{W^+W^-\phi^0}\cos(\beta-\alpha)$

which, according to fig. 10, is very small over a large range of parameter space. In particular, for $m_{H^0} > 2m_W$, the $W^+W^-H^0$ coupling is reduced in amplitude by at (at least) a factor 10. This rules out the standard techniques for Higgs detection at LHC or SSC, which depend on the decay of the Higgs boson to vector boson pairs.

The upshot of the above observations is twofold. In supersymmetric models, it should be rather straightforward to discover the lightest Higgs scalar of the model. However, it will be far more difficult to prove the existence of an extended Higgs sector, which is a necessary consequence of low-energy supersymmetry. It could be argued that the above observations were very specific to the assumption of a minimal supersymmetric structure beyond the Standard Model. There has been some investigation in the literature of Higgs boson phenomenology in more complicated (non-minimal) supersymmetric models. Results in these models tend to confirm the general observations described above. Some interesting counterexamples are known to exist (*e.g.*, models with a charged Higgs boson which is lighter than the W). However these models tend to predict a rich structure of new physics at rather "low" energies (around 100 GeV) which should be rather easy to expose at colliders in the near future. Thus, the verification of the supersymmetric scenario will require direct evidence of the supersymmetric particles. The decays of Higgs bosons into supersymmetric final states may also make up a major percentage of the Higgs branching ratio. However, in such a scenario, Higgs boson phenomenology would be simply one part of the general experimental exploration of the supersymmetric spectrum.

I will end this lecture with a few additional remarks about going beyond the minimal supersymmetric model. Many non-minimal supersymmetric models have been examined in the literature, including superstring inspired models, models with $SU(2) \times U(1)$ singlet superfields, and models with extended gauge groups. All such models have rather complicated Higgs sectors. In many cases, the models have one light Higgs boson whose mass is not much larger than $\mathcal{O}(m_Z)$. However, these non-minimal models can often include new terms in the superpotential which result in new quartic Higgs self-couplings, λ, which are not constrained. The constraints on λ are rather similar to those of the minimal Higgs model. Thus, one might expect Higgs mass upper bounds in such models to be similar to the ones described in Lecture 2.

The MSSM is clearly the simplest of the "low-energy" supersymmetric models. The Higgs sector of the MSSM is more constrained than an arbitrary two-Higgs doublet model, and thus presents an opportunity for experimental confirmation or rejection in the not too distant future. If the MSSM can be ruled out, one should not rule out the possibility of a somewhat more complicated non-minimal supersymmetric model. But perhaps we should keep an open mind and look elsewhere for the resolution of the hierarchy and naturalness problems. This is the subject of the next lecture, where we explore the major competitor of the supersymmetric approach.

Suggestions for Further Reading and a Brief Guide to the Literature

A review of the minimal supersymmetric extension of the Standard Model (MSSM) is given in

1. H.P. Nilles, *Phys. Rep.* **110**, 1 (1984); H.E. Haber and G.L. Kane, *Phys. Rep.* **117**, 75 (1985).

A detailed discussion of the Higgs sector of the MSSM and a complete compilation of the Feynman rules for Higgs bosons of this model can be found in

2. *The Higgs Hunter's Guide*, §4.2 and Appendices A–C.

Radiative corrections to the neutral Higgs mass sum rule [eq. (5.10)] are given in

3. M. Berger, *Phys. Rev.* **D41**, 225 (1990).

The material of section 5.2 is based on

4. H.E. Haber and R. Hempfling, SCIPP-90/42 (1990), *Phys. Rev. Lett.* **66** (1991), in press. [For complementary work, see also: J. Ellis, G. Ridolfi and F. Zwirner, CERN-TH.5946 (1990) and Y. Okada, M. Yamaguchi, and T. Yanagida, Tohoku University preprint TU-360 (1990).]

The most recent published limits on the h^0 and A^0 masses from LEP are

5. D. Decamp *et al.* [ALEPH Collaboration], *Phys. Lett.* **B237**, 291 (1990); P. Abreu *et al.* [DELPHI Collaboration], *Phys. Lett.* **B245**, 276 (1990); B. Adeva *et al.* [L3 Collaboration] *Phys. Lett.* **B251**, 311 (1990); M.Z. Akrawy *et al.* [OPAL Collaboration], CERN-EP-90-100 (1990), to be published in *Z. Phys. C*.

A thorough discussion of non-minimal low-energy supersymmetric models and their Higgs sectors is given in

6. *The Higgs Hunter's Guide*, Chapter 5.

6. Is Electroweak Symmetry Breaking Generated Without Elementary Scalars?

In the last four lectures, I have focused my attention primarily on the exploration of the origin of electroweak symmetry breaking via the search for fundamental Higgs bosons. An alternative view puts the origin of the symmetry breaking in a different sector of the theory, one with new fundamental fermions that have gauge theory interactions. In this latter approach, elementary scalar bosons are completely absent. However, composite bound state of the new fundamental fermions can arise (in analogy with the way mesons arise in the QCD theory of quarks), and may provide an experimental signature of the electroweak symmetry breaking sector.

Why throw out a simple picture—the Higgs mechanism via the dynamics of weakly interacting elementary scalar fields—in which calculations are perturbative and straightforward, and replace it with unknown strong interaction dynamics which require solving a strongly coupled theory? First, nature might actually be that way! For example, superconductivity is the most well known example of the Higgs mechanism in solid state physics in which $U(1)_{EM}$ is spontaneously broken. (The Meissner effect in which the magnetic field dies away exponentially as it penetrates the superconductor can be interpreted as arising from a nonzero photon mass generated by the Higgs mechanism.) In this case, the mechanism for spontaneous symmetry breaking does not involve an elementary scalar field, but rather a condensation of e^-e^- Cooper pairs in the vacuum. Second, elementary fields are "unnatural", as discussed at the beginning of Lecture 5. In contrast, large gauge hierarchies are "natural" in asymptotically free gauge theories. For example, in grand unified theories, a typical value for the grand unification mass [where the SU(3), SU(2) and U(1) running coupling constants meet] is $M_X \simeq 10^{15}$ GeV. Nevertheless, the ratio of the proton mass to M_X, $m_p/M_X \simeq 10^{-15}$ is perfectly reasonable. Solving the RGE for the evolution of the QCD strong coupling constant

$$\mu\frac{dg_s}{d\mu} = -bg_s^3 \qquad (b \equiv 11 - \tfrac{2}{3}n_f > 0) \tag{6.1}$$

(where $n_f = 6$ quark flavors) yields

$$\frac{1}{g_s^2(\mu)} = \frac{1}{g_s^2(M_X)} + b\ln\frac{\mu^2}{M_X^2}\,. \tag{6.2}$$

Let us define $\Lambda_{\rm QCD}$ such that $g_s^2(\Lambda_{\rm QCD}) = \infty$. Then it follows that*

$$\Lambda_{\rm QCD} = M_X \exp\left(\frac{-1}{2bg_s^2(M_X)}\right) \ll M_X \,. \tag{6.3}$$

On the other hand, the mass of the proton is proportional to $\Lambda_{\rm QCD}$ [ignoring quark mass effects; see eq. (3.27)]. Writing $m_p = c\Lambda_{\rm QCD}$ with $c \sim \mathcal{O}(1)$, it follows that $m_p/M_X \ll 1$. Such a large hierarchy is regarded as natural, since this conclusion does not change appreciably with small changes in the initial conditions [*i.e.*, the value of $g_s(M_X)$].

Let us examine some of the basic concepts associated with dynamical electroweak symmetry breaking. We have seen in Lecture 1 that in the absence of elementary Higgs scalars, the W and Z would still gain mass by virtue of the spontaneous symmetry breaking of chiral symmetry by QCD. Although the resulting gauge boson masses were phenomenologically unacceptable, this example serves as a prototype for more general (and hopefully more realistic) models of electroweak symmetry breaking by dynamical means. In this lecture, I would like to explore some of the buzz-words associated with dynamical electroweak symmetry breaking: technicolor, extended technicolor, walking technicolor, composite Higgs bosons, and $t\bar{t}$-condensates, and briefly describe the potential for discovery of these phenomena at future colliders. Some of the ideas we are about to examine are quite elegant, but my guess is that there are some important missing pieces. For example, gauge bosons are given mass in a rather attractive way, while fermions get mass by a brute force approach that is unconvincing and often difficult to reconcile with experimental constraints. By examining these alternative approaches carefully (and with a little help from experiment), perhaps we will learn how to construct a compelling model of dynamical electroweak symmetry breaking.

6.1 Dynamical Breaking of Chiral Symmetry

The dynamical breaking of chiral symmetry is assumed to take place in the theory of QCD; the resulting Goldstone bosons are the pions. By exploiting this mechanism in a new setting (*e.g.*, technicolor) we can construct a theory in which the electroweak symmetry is dynamically broken. So, let us take a brief look at the theory of chiral symmetry and its breaking.

⋆ I could just as well define $g_s^2(\Lambda_{\rm QCD}) = 4\pi$ or any other finite number that would indicate the onset of strong coupling. The conclusion that $\Lambda_{\rm QCD} \ll M_X$ would remain unchanged.

Consider a QCD-like theory with n-flavors of massless fermions. The Lagrangian is

$$\begin{aligned}\mathcal{L} &= \overline{\psi} i\gamma^{\mu}(\partial_{\mu} + igA_{\mu}^{a}T^{a})\psi \\ &= \overline{\psi}_{L} i\gamma^{\mu}(\partial_{\mu} + igA_{\mu}^{a}T^{a})\psi_{L} + \overline{\psi}_{R} i\gamma^{\mu}(\partial_{\mu} + igA_{\mu}^{a}T^{a})\psi_{R} \,,\end{aligned} \tag{6.4}$$

where $\psi_{R,L} = \frac{1}{2}(1 \pm \gamma_5)\psi$. Note that there is no connection between L and R, so $\mathcal{L}$ exhibits a global $\mathrm{U(n)}_L \times \mathrm{U(n)}_R$ flavor symmetry. Actually, there is an overall $\mathrm{U(1)_{axial}}$ symmetry which is anomalous, so I shall simply discard it. There is also an overall $\mathrm{U(1)_{vector}}$ symmetry which corresponds to fermion number conservation. The remaining global symmetry is the chiral $\mathrm{SU(n)}_L \times \mathrm{SU(n)}_R$ symmetry. If the following mass term were added to the Lagrangian above

$$\delta\mathcal{L} = m\overline{\psi}\psi = m(\overline{\psi}_R\psi_L + \overline{\psi}_L\psi_R) \tag{6.5}$$

with a common mass for the N-flavors, then the $\mathrm{SU(n)}_L \times \mathrm{SU(n)}_R$ chiral symmetry would be explicitly broken down to the diagonal $\mathrm{SU(n)}_{L+R}$ symmetry (the analog of isospin symmetry in two-flavor QCD). In QCD, the origin of $\delta\mathcal{L}$ lies in the electroweak sector, so let us neglect it for now. (Since electroweak forces are much weaker than the strong QCD forces, such an approximation is a reasonable one.) However, even in the absence of explicit mass breaking, the strong QCD forces can spontaneously break $\mathrm{SU(2)}_L \times \mathrm{SU(2)}_R \rightarrow \mathrm{SU(2)}_{L+R}$. This would occur if the following non-zero expectation values develop

$$\langle\overline{\psi}_{iL}\psi_{jR}\rangle = \langle\overline{\psi}_{iR}\psi_{jL}\rangle = \delta_{ij}\Delta^3 \,, \tag{6.6}$$

where Δ has dimensions of mass. Note that the first equality above must be satisfied if QCD is to conserve parity.

To determine whether such expectation values develop requires a nonperturbative QCD computation. Nevertheless, it is easy to understand how such a result could come about. First, note that one can construct a $\overline{\psi}\psi$ state with the quantum numbers of the vacuum if the total angular momentum of the state is zero. One possible state can be pictured as follows

$$\overset{\leftarrow}{\longleftarrow} \;\bullet\; \overset{\rightarrow}{\longrightarrow}$$

The helicities (the small arrows in the above picture) must both point either parallel or antiparallel to the corresponding momenta (the larger arrows). For a massless fermion pair, I can only do this with $\overline{\psi}_L\psi_R$ (or $\overline{\psi}_R\psi_L$). Thus, it is possible to fill up the vacuum with zero momentum, color singlet pairs, *i.e.*, condensation! In this case, $\langle \overline{\psi}_L \psi_R \rangle \neq 0$. Whether this happens when strong QCD forces are turned on is a dynamical question. An intuitive argument of Nambu and Jona-Lasinio observed that the Hamiltonian consists of a piece which preserves the number of $\overline{\psi}\psi$ pairs and another piece which creates (or destroys) $\overline{\psi}\psi$ pairs. In such a case, strong attractive interactions can reduce the former at the expense of the latter. Thus, above some critical coupling, the vacuum would condense into an indefinite number of $\overline{\psi}\psi$ pairs, and dynamical symmetry breaking occurs.

Phenomenological observation strongly suggests that QCD [with two nearly massless flavors of quarks: u and d] breaks $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \rightarrow \mathrm{SU}(2)_{L+R}$.* Coleman and Witten presented a very clever argument that proved that an SU(N) gauge theory in the large N limit with quarks in the fundamental representation (and a few other reasonable assumptions) will break an n-flavor chiral $\mathrm{SU(n)}_L \times \mathrm{SU(n)}_R \rightarrow \mathrm{SU(n)}_{L+R}$. There are many other possible groups, representations and chiral symmetry breaking patterns. However, in this lecture, I will be content to appeal to the behavior of QCD in determining the pattern of chiral symmetry breaking.

6.2 Technicolor in its Simplest Manifestation

All the necessary formalism needed for the technicolor approach has been set up in Lecture 1. Let us invent some new fermions (called technifermions).

$$\begin{pmatrix} U \\ D \end{pmatrix}_L, \quad U_R\ , D_R \tag{6.7}$$

which have nontrivial quantum numbers under $\mathrm{SU}(2)_L \times \mathrm{U}(1)$. For simplicity, I will assume that U and D are color singlets. The following choice then will avoid a gauged U(1) anomaly:

	T_3	Y	$Q = T_3 + Y/2$
U_L	1/2	0	1/2
D_L	-1/2	0	-1/2
U_R	0	1	1/2
D_R	0	-1	-1/2

* Since the strange quark can also be considered light (compared to Λ_{QCD}), one can also discuss the breaking of $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \rightarrow \mathrm{SU}(3)_{L+R}$. The latter symmetry is the well-known SU(3) flavor symmetry of the quark model.

In addition, one invents a new non-abelian gauge theory called "technicolor". U and D are assumed to be technicolor non-singlets. In order to be able to apply our experience with QCD to a theory of the strong technicolor force, I shall assume that the technicolor gauge group is SU(N), with U and D transforming under the fundamental representation.

Recall that in the two-flavor QCD example of Lecture 1, I argued that the $SU(2)_L \times SU(2)_R$ chiral symmetry was broken down to $SU(2)_{L+R}$ by quark condensates: $\langle \bar{q}_L q_R \rangle = \langle \bar{q}_R q_L \rangle$, since QCD conserves parity, and $\langle \bar{u}_L u_R \rangle = \langle \bar{d}_L d_R \rangle$, in order that a vector SU(2) isospin symmetry remain unbroken. For the technicolor model above, we simply make the same conjecture. Proceeding as in Lecture 1, the techniquarks couple to the gauge bosons via vector and axial currents

$$\mathcal{L} = \overline{Q}\gamma^\mu \left[g W_\mu^a \frac{\tau^a}{2} \left(\frac{1-\gamma_5}{2} \right) + g' B_\mu \frac{\tau^3}{2} \left(\frac{1+\gamma_5}{2} \right) \right] Q \tag{6.8}$$

with $Q = \binom{U}{D}$ and Pauli matrices τ^a. The coupling of the gauge bosons to the axial current is obtained by writing

$$\mathcal{L} = -\frac{g}{2} W_\mu^a J_5^{\mu a} + \frac{g'}{2} B_\mu J_5^{\mu 3} + \text{vector current couplings}\,, \tag{6.9}$$

where $J_{\mu 5}^a \equiv \overline{Q}\gamma_\mu \gamma_5 \frac{1}{2}\tau^a Q$. When the technicolor breaks the $SU(2)_L \times SU(2)_R$ chiral symmetry down to $SU(2)_{L+R}$, three Goldstone bosons appear which transform as a triplet under isospin (in complete analogy with the way the pions arise when QCD breaks ordinary chiral symmetry). Thus,

$$\langle 0 | J_{\mu 5}^a(0) | \Pi^b \rangle = i F_\pi p_\mu \delta^{ab}\,. \tag{6.10}$$

Using the methods outlined in Lecture 1, it is now straightforward to compute the vector boson mass matrix. The end result is

$$m_W^2 = m_Z^2 \cos^2\theta_W = \tfrac{1}{4} g^2 F_\pi^2\,. \tag{6.11}$$

Thus $\rho_0 = 1$ has been automatically implemented due to a custodial SU(2) symmetry of the model, which in this case is $SU(2)_{L+R}$. Moreover, if we identify

$F_\pi = v = 246$ GeV, we would get the correct masses for the W and Z.† Continuing in analogy with QCD, I would like to identify the "scale" of technicolor, Λ_{TC} (analogous to Λ_{QCD}). Now, F_π/Λ_{TC} should be a pure number which is predicted by the gauge theory. Since the main difference between QCD and QTCD‡ is the gauge group, one can use "large N" arguments to estimate

$$\frac{F_\pi}{f_\pi} \sim \sqrt{\frac{N}{3}}\,\frac{\Lambda_{TC}}{\Lambda_{QCD}} \tag{6.12}$$

for an SU(N) technicolor (TC) group. The proof that $F_\pi \sim \sqrt{N}$ goes as follows. Start with eq. (6.10), and note that $J^a_{\mu 5} = \sum_{i=1}^N \overline{Q}_i \gamma_\mu \gamma_5 \, \frac{1}{2}\tau^a Q_i$, where i is a TC-index, so $J^a_{\mu 5} \sim N$. On the other hand, Π^b is a TC-singlet. When properly normalized, the wave function of $|\Pi^b\rangle$ scales like $1/\sqrt{N}$. Comparing the large N behavior of both sides of eq. (6.10) yields $F_\pi \sim \sqrt{N}$.

Therefore, eq. (6.12) leads to the following estimate (for $\Lambda_{QCD} \simeq 200$ MeV)

$$\Lambda_{TC} \sim \sqrt{\frac{3}{N}}\,\frac{F_\pi}{f_\pi}\Lambda_{QCD} \sim \sqrt{\frac{3}{N}}\,(500 \text{ GeV})\,. \tag{6.13}$$

Presumably, Λ_{TC} is the appropriate scale for the masses of technihadrons which are the analogs of the mesons and baryons of QCD. For example, the mass and width of the techni-rho (ρ_T) would be

$$\begin{aligned} m_{\rho_T} &\sim \sqrt{\frac{3}{N}}\,\frac{F_\pi}{f_\pi}\,m_\rho \sim \sqrt{\frac{3}{N}}\,(2 \text{ TeV})\,, \\ \Gamma_{\rho_T} &\sim \left(\frac{3}{N}\right)\frac{\Gamma_\rho}{m_\rho} m_{\rho_T} \sim \left(\frac{3}{N}\right)^{3/2} (500 \text{ GeV})\,, \end{aligned} \tag{6.14}$$

where I have made use of 't Hooft's large N scaling. The ρ_T would dramatically affect $W_L^+ W_L^-$ scattering in the same way that the ρ dominates $\pi^+\pi^-$ scattering at $\sqrt{s} \simeq m_\rho$.

---

† To be complete, I should include QCD effects as well. The calculation is nearly identical to the one presented in Lecture 2, and I find

$$|G\rangle = \frac{1}{\sqrt{f_\pi^2 + F_\pi^2}}\Big(F_\pi\,|\Pi\rangle + f_\pi\,|\pi\rangle\Big)$$

$$|\pi\rangle_{\text{phys}} = \frac{1}{\sqrt{f_\pi^2 + F_\pi^2}}\Big(F_\pi\,|\pi\rangle - f_\pi\,|\Pi\rangle\Big)$$

where $|G\rangle$ is the Goldstone bosons which are eaten by the W and Z, and $|\pi\rangle_{\text{phys}}$ is the physical pion of QCD. Since $f_\pi \ll F_\pi$, I can normally neglect this mixing.

‡ Quantum technicolor dynamics.

This is all very elegant. Unfortunately, this theory possesses zero mass quarks and leptons. Unlike the case of the elementary Higgs boson, we must introduce additional structure to generate fermion masses. This is where the model begins to lose its elegance.

6.3 The Problem of Fermion Mass in Technicolor Models

In a model where electroweak symmetry breaking occurs through the dynamics of elementary Higgs fields, it is easy to generate masses for the quarks and leptons. One introduces an (arbitrary) set of Yukawa interactions between the Higgs bosons and the fermions. When the Higgs bosons acquire their VEVs, fermion masses are generated. In contrast, consider the simple technicolor model that was just introduced. We can attempt a similar construction if the technicolor model possesses states analogous to the Higgs bosons. Naively, such states would correspond to 0^{++} resonant bound states of technifermions analogous to the infamous ϵ of QCD.* Suppose we try to mimic the Higgs-fermion Yukawa interaction of the minimal Higgs model. This would require us to connect two fermions with two technifermions.

To accomplish this, let us introduce an effective four-fermion coupling

$$\begin{aligned}\mathcal{L}_{\text{eff}} =& G(\overline{Q}\gamma_\mu q)(\overline{q}\gamma^\mu Q) \\ =& -G\left[(\overline{Q}Q)(\overline{q}q) + (\overline{Q}i\gamma_5 Q)(\overline{q}i\gamma_5 q)\right. \\ &\left. -\tfrac{1}{2}(\overline{Q}\gamma_\mu Q)(\overline{q}\gamma^\mu q) - \tfrac{1}{2}(\overline{Q}\gamma_\mu\gamma_5 Q)(\overline{q}\gamma^\mu\gamma_5 q)\right]\end{aligned} \tag{6.15}$$

where the last equality was obtained by a Fierz transformation. In eq. (6.15), Q denotes a techniquark and q denotes a quark. When the techniquark pairs condense (breaking the techni-chiral symmetry which breaks the electroweak symmetry group in the desired way),

$$\langle \overline{Q}_i Q_j \rangle = \Delta^3 \delta_{ij}\,. \tag{6.16}$$

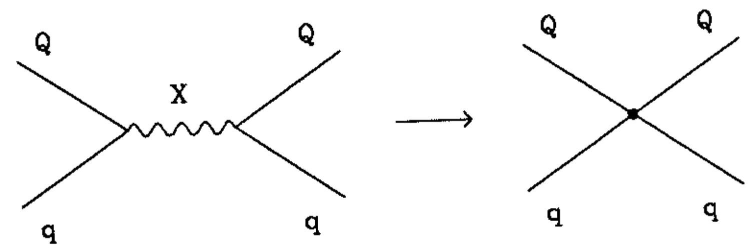
Therefore, when we "shift" to the correct vacuum, we generate a fermion mass term. That is, in eq. (6.15), $-G(\overline{Q}Q)(\overline{q}q) \to -G\Delta^3\overline{q}q$, and we identify $m_q = G\Delta^3$.

Where did this four-fermi coupling come from? Let us introduce a new gauge theory whose matter multiplets contain both fermions and technifermions. Suppose the gauge group is broken in the direction corresponding to generators which mix fermions and technifermions. The corresponding gauge bosons, (generically called

---

⋆ The s-wave $\pi\pi$ phase shift does go through 90° somewhere near 1 GeV, corresponding to a resonance called the ϵ, but the ϵ "width" is nearly as wide as its "mass". The ϵ is still a questionable particle in the Particle Data Group compilations.

X_{ETC}) couple quarks to techniquarks. Thus, if I consider the scattering $\bar{q}Q \to \bar{q}Q$ via s-channel X_{ETC}-exchange, then in the limit of large gauge boson mass, one obtains the effective four-fermion operator



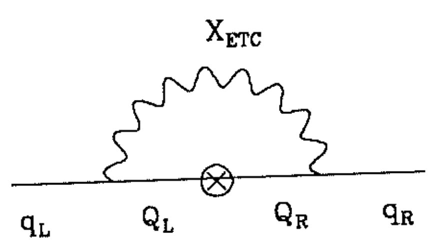
given in eq. (6.15). This new gauge theory just described is called extended technicolor (ETC), and the X_{ETC} (there can be more than one depending on the model) are the extended technicolor gauge bosons. If g_{ETC} is the $\bar{q}QX_{\mathrm{ETC}}$ coupling and M_{ETC} is the mass of X_{ETC}, then the strength of the effective four-fermion interaction is

$$G = \frac{g_{\mathrm{ETC}}^2}{M_{\mathrm{ETC}}^2}\,. \tag{6.17}$$

Therefore, one obtains the following estimate for the quark mass

$$m_q \simeq \frac{g_{\mathrm{ETC}}^2 \Delta^3}{M_{\mathrm{ETC}}^2}\,. \tag{6.18}$$

Diagrammatically, the quark mass arises from



Because the Q_L-Q_R mixing results from the condensation $\langle \overline{Q}Q \rangle \neq 0$, the Q has effectively acquired a dynamical mass, which must vanish like Δ^3/k^2 as $k \to \infty$ inside the loop. As a result, the loop is convergent and one reproduces the result of eq. (6.18).

Thus, in order to generate fermion masses, we are forced to introduce a new ETC gauge interaction. In order to see whether such a picture is phenomenologically viable, we must estimate the parameters M_{ETC} and Δ. By analogy with QCD, we first evaluate $\langle \bar{q}q \rangle$. Using the well known relation from current algebra [*cf.* eq. (3.39)]

$$f_\pi^2 m_\pi^2 = \tfrac{1}{2}(m_u + m_d)\langle \bar{u}u + \bar{d}d\rangle \tag{6.19}$$

with $\langle \bar{u}\bar{u}\rangle = \langle \bar{d}d\rangle$ (by isospin invariance), one obtains the numerical estimate: $\langle \bar{q}q\rangle \simeq 17 f_\pi^3$. Using large N arguments to scale this result to the technicolor theory, and identifying $F_\pi = v = 246$ GeV as before, one obtains

$$\langle \overline{Q}Q\rangle \equiv \Delta^3 \simeq \sqrt{\frac{3}{N}}\, 17 v^3 \,. \tag{6.20}$$

Thus, roughly speaking, [up to factors of $\mathcal{O}(1)$],

$$m_q \sim \frac{17 v^3}{M_{\rm ETC}^2}\,. \tag{6.21}$$

We conclude that a 1 GeV quark mass would require $M_{\rm ETC} \simeq 16$ TeV, while a 100 GeV top quark mass would require $M_{\rm ETC} \simeq 1.6$ TeV.

So far, we have discussed the basic building blocks that are required in order to generate electroweak symmetry breaking and fermion masses. Unfortunately, it is not an easy matter to construct a specific ETC model which does not possess some serious phenomenological problems. Perhaps the most serious problem of such models is the existence of flavor changing neutral currents (FCNCs) which are larger than what is allowed by current data. The main problem is that we need to make use of the ETC generated four-fermion $q\bar{q}Q\overline{Q}$ interactions in order to generate the full (non-diagonal) quark mass matrix. However, having done so, one finds that the ETC interactions also generate four-fermion $q\bar{q}q\bar{q}$ interactions which are not flavor diagonal. Unless one can construct a techni-GIM mechanism (in analogy with the GIM mechanism of the Standard Model) to avoid the undesirable FCNC's, one would be forced to raise $M_{\rm ETC}$ to a large enough value to avoid current experimental bounds. This requires rather large values of $M_{\rm ETC}$, perhaps above 1000 TeV, to avoid the strong FCNC constraints in the K–$\overline{K}$ system. But, with such large ETC gauge boson masses, it is not possible (if the above estimates are valid) to generate heavy quark masses. Perhaps one can arrange for a spectrum of ETC masses corresponding to the spectrum of observed quark masses. One has some freedom here since the FCNC constraints involving heavy quark systems are not as severe as those related to the kaons. However, technicolor models that have attempted to surmount these problems are extremely baroque. Furthermore, the top quark mass poses a particular problem for model builders. The top quark is now known to be much heavier than what was anticipated in the heyday of technicolor model building. In addition to requiring a rather low ETC scale, a 100 GeV top quark mass generated by the mechanism described above almost certainly produces too large a radiative correction to the electroweak ρ parameter.

There is a second problem which plagues the technicolor models described above. In order to successfully overcome the FCNC problem, one must certainly go beyond the minimal technicolor model described above. However, one then typically finds that such models will possess techni-chiral symmetries much larger than $SU(2)_L \times SU(2)_R$ which are spontaneously broken by the technicolor forces. This leads to additional Goldstone bosons (sometimes called technipions) which are not be eaten by the W, Z. Like the charged pions and kaons of QCD, these Goldstone bosons are in fact pseudo-Goldstone bosons (PGB's), since when SU(3) $\times$ SU(2) $\times$ U(1) gauge forces are taken into account, the unbroken "isospin" global symmetry is perturbed, thereby giving mass to the Goldstone bosons. (By the same mechanism, even in the absence of explicit quark masses, the charged pions in QCD would not be massless due to the electromagnetic interactions.) Technipion masses can also be generated from ETC interactions. In general, a catalog of technipions would include: (i) a colorless, SU(2) $\times$ U(1) singlet called P'^0, (ii) a colorless, electrically neutral, SU(2) $\times$ U(1) nonsinglet called P^0, (iii) colorless, electrically charged states, $P^\pm$, (iv) color triplets (P_3), and (v) color octets (P_8). Both P_3 and P_8 types of technipions can have a variety of electroweak quantum numbers.

The lightest technipions are colorless; they can only get mass from electroweak (and ETC) interactions. Thus, P'^0, P^0 and $P^\pm$ are expected to be the lightest technipions of the model and of immediate interest for phenomenology. Since the technipions are associated with axial vector global symmetries which are spontaneously broken when the technicolor forces become strong, it follows that the technipions are CP-odd scalars.* Thus, the phenomenology of the $P^\pm$, P^0 and P'^0 is *indistinguishable* from that of the elementary charged and CP-odd neutral Higgs bosons of some elementary multi-Higgs model. More precisely, the interactions of the colorless technipions can always be reproduced as the low-energy limit of some elementary multi-Higgs model, in which the masses of the CP-even Higgs scalars are taken to infinity. This means that much of the phenomenology of $P^\pm$, P^0 and P'^0 is the same as the phenomenology of $H^\pm$ and A^0 studied in Lecture 4.† Since it is desirable to have $\rho \equiv m_W^2/(m_Z^2 \cos^2\theta_W) = 1$, the technicolor models of interest have technipions which resemble the charged and CP-odd neutral Higgs bosons of

---

⋆ It is possible to construct technicolor models in which some of the pseudo-Goldstone bosons are CP-even. Consider a model in which technikaons exists. These would be analogous to technipions in precisely the same way that the K and π mesons are related in ordinary QCD. Then, if CP is conserved in the technicolor and extended technicolor interactions, the actual neutral technikaon mass eigenstates would consist of a CP-even techni-K_S and a CP-odd techni-K_L. Technipions which couple to CP-even final states could also arise in extended technicolor models with CP-violation.

† Some differences do arise in the technipion couplings to the fermions, since these depend in detail on the structure of the ETC sector.

multi-doublet Higgs models.

The major problem with the above picture is the experimental non-observation of any colorless technipions at e^+e^- colliders. The electroweak contribution to the technipion masses is theoretically well understood and can be reliably computed (with some dependence on the technicolor model). One finds that

$$m_{P^\pm}|_{EW} \approx 5 - 14 \text{ GeV}\,, \qquad m_{P^0}|_{EW} = m_{P'^0}|_{EW} = 0\,. \tag{6.22}$$

Unfortunately, the contributions from the ETC interactions are very model-dependent, so that it is difficult to make precise predictions for the light technipion masses. In addition, P^0 and P'^0 are not necessarily mass eigenstates; in general, one would expect them to mix. However, in the above picture, it is difficult to imagine a mechanism which would yield light (colorless) technipion masses above about 40 GeV (Various model calculations invariably yield masses somewhat smaller than this generous upper limit.) However, the LEP limits on the charged Higgs boson ($m_{H^\pm} \lesssim 40$ GeV) also apply to the charged technipions, $P^\pm$. Thus, if the above estimates of technipion masses are correct, then models with colorless (charged) technipions are already ruled out.

In summary, the (standard) technicolor models, with their associated ETC sectors, appear not to be viable models of electroweak symmetry breaking. Nevertheless, many of the ideas behind the technicolor approach are quite appealing. As a result, there have been a number of attempts to construct variations of the technicolor idea which might be able to overcome the phenomenological problems discussed above.

6.4 New Directions in Dynamical Symmetry Breaking

Composite Higgs bosons

An alternative approach to the theory of elementary Higgs bosons is one in which the Higgs bosons are retained but are taken to be composite states. Such a theory has been constructed by Kaplan, Georgi and collaborators. Their models possess a number of features in common with technicolor. Again, one introduces a new gauge force ("ultracolor"), and a set of new massless ultrafermions. The resulting global symmetry of the model is broken dynamically to a smaller subgroup by ultrafermion condensates at an energy scale where the ultracolor force becomes strong. Once again, there are pseudo-Goldstone bosons corresponding to the generators of the broken global symmetry. However, the new feature here is that the electroweak symmetry remains unbroken at this stage. This is in contrast to technicolor models in which the electroweak gauge symmetry is broken when condensates of technifermions form. The pseudo-Goldstone bosons include both

CP-even and CP-odd scalars and are composites of the ultrafermions. In this approach, the electroweak symmetry is broken when a pseudo-Goldstone boson with the same quantum numbers as those of the Standard Model Higgs boson acquires a vacuum expectation value. This will occur because a Higgs potential can develop when electroweak gauge interactions are taken into account. [To build a successful model, one must expand the low-energy electroweak gauge group beyond SU(2) × U(1).] Because the Higgs bosons of the model are pseudo-Goldstone bosons, their masses are calculable, at least in part. In models which have been constructed, one typically finds $m_H \sim m_W$.

As in technicolor, one must introduce additional (extended ultracolor) interactions in order to generate fermion masses. However, these models are more flexible than the technicolor models discussed in the previous section. In particular, one can generate quark and lepton masses of an appropriate size, and at the same time avoid large flavor-changing neutral currents. In addition, colorless pseudo-Goldstone bosons can also be heavier than in traditional technicolor models, with masses of order m_W or less. The phenomenology of these scalars has yet to be explored in detail.

Walking Technicolor

Recently, the dynamics of technicolor has been reexamined in an approach known as "walking technicolor". This name derives from the idea that models can be constructed in which the technicolor gauge coupling runs much more slowly (than is typical for an asymptotically free coupling) above the scale of electroweak interactions. This allows the extended technicolor scale to be much larger than in standard extended technicolor models, thereby suppressing flavor-changing neutral currents while not affecting the size of fermion masses which can be generated. At the same time, it will be possible to generate phenomenologically viable fermion masses. Said another way, such theories can raise the value of m_f [computed in eq. (6.18)] without lowering the value of $M_{\rm ETC}$. To do this, one must raise the value of Δ in eq. (6.18) without altering the technicolor scale and changing the value of $F_\pi = v = 246$ GeV. It is possible to do this because F_π is generated when TC becomes strong and thus is sensitive to momentum scales at 1 TeV and below. On the other hand, Δ in eq. (6.18) is being evaluated at the ETC scale. If one could change the nature of the dynamics between the TC and ETC scale, then one could boost Δ without affecting F_π.

I shall briefly sketch how these ideas are implemented. Consider the technifermion propagator

$$S(p) = \frac{1}{A(p^2)\not{p} - \Sigma(p^2)} \,. \tag{6.23}$$

The generation of a dynamical mass simply means that $\Sigma(p^2) \neq 0$. Roughly speaking, $\Sigma(\Lambda_{TC}) \simeq \Lambda_{TC}$. To obtain an expression for $\Sigma(p^2)$ requires a dynamical calculation. One can attempt to solve the Schwinger–Dyson equation in some approximation and extract the behavior of $\Sigma(p)$ at large p^2. This can be done in the ladder approximation by solving an integral equation which is represented diagrammatically as follows:

The results of such a calculation simplify considerably in the Landau gauge since the wave function renormalization vanishes at one-loop. Hence we can take $A(p^2) = 1$. One then finds

$$\Sigma(p) \underset{p\to\infty}{\longrightarrow} \Sigma_0 \left(\frac{p}{\Lambda_{TC}}\right)^{-1-\sqrt{1-\alpha(p)/\alpha_c}}, \tag{6.24}$$

where α_c is the critical coupling above which techni-chiral symmetry is spontaneously broken and $\Sigma_0 \simeq \mathcal{O}(\Lambda_{TC})$. Finally, Δ evaluated at the ETC scale can be estimated very roughly as follows [in the Landau gauge with $A(k^2) = 1$]:

$$\begin{aligned}\Delta^3 \equiv \langle\overline{\psi}\psi\rangle &= -\lim_{x\to 0} \operatorname{Tr} \langle 0|\, T\psi(x)\overline{\psi}(0)\,|0\rangle \\ &= -i \int \frac{d^4k}{(2\pi)^4} \operatorname{Tr} \frac{1}{\not{k} - \Sigma(k^2)} \\ &= \frac{N}{4\pi^2} \int dk^2 k^2 \frac{\Sigma(k^2)}{k^2 + \Sigma^2(k^2)},\end{aligned} \tag{6.25}$$

where N is the number of techni-colors. The last step is obtained after a Wick rotation. In the case of interest, Δ is dominated by momenta at the ETC scale. Since $\Sigma(\Lambda_{TC}) \simeq \Lambda_{TC}$ and falls with higher momentum scales, we can approximate

$$\Delta^3 \simeq \frac{N}{2\pi^2} \int^{M_{\rm ETC}} k dk \Sigma(k^2). \tag{6.26}$$

In typical asymptotically free theories, $\alpha(p)$ runs, so we expect $\alpha(p) \ll \alpha_c$ for most of the momentum range from Λ_{TC} to $M_{\rm ETC}$. Then, $\Sigma(p) \sim 1/p^2$ and $\Delta^3 \simeq \mathcal{O}(\Lambda^3_{TC})$ up to logarithmic corrections. However, in "walking" technicolor, $\alpha(p)$ runs so slowly that $\alpha(p)$ is not very different from α_c all the way out to $p \simeq M_{\rm ETC}$. In this case $\Sigma(p) \sim 1/p$ which enhances Δ^3 by a factor of $M_{\rm ETC}/\Lambda_{TC}$, without affecting the value of F_π.

The dynamics of walking technicolor models are clearly very different from the original technicolor models which were based on an analogy with QCD. The walking technicolor approach and various related variations thereof present the possibility of solving many of the phenomenological problems which plagued the original technicolor models. By altering the dynamics of the strong technicolor forces between the TC and ETC scales, it is possible to maintain the good features of TC (mass generation for the $W^{\pm}$ and Z), while avoiding problems formerly attributed to the ETC sector. It is still not clear whether realistic models can be constructed which can produce a heavy enough top-quark, and is consistent with all other phenomenological constraints.

Models of $t\bar{t}$ Condensates

The large value of the top-quark mass has led to other speculations. Suppose there exists a four-fermion interaction $t\bar{t}q\bar{q}$ generated at some very high mass scale (call it M_X). With the t-quark so heavy, one can imagine that there is a condensation of $t\bar{t}$ pairs in the vacuum. The low-energy effective theory (at scales far below M_X) of such a picture looks identical to the Standard Model. This approach makes one prediction: the masses of the Higgs boson and the top-quark are determined (as a function of M_X). The predicted values of m_t and m_H are given (roughly) by the maximum allowed values shown in fig. 2 (for the appropriate value of M_X). The reason is that this point in the m_t–m_H plane corresponds to the point at which the Landau poles of the Higgs self-coupling and $H^0t\bar{t}$ Yukawa couplings are both at M_X. Thus the predicted value for m_t and m_H is almost completely insensitive to the details of the physics at the M_X scale. However, implementing such a picture requires extreme fine-tuning of the parameters which govern the physics at M_X, since the natural value of the electroweak symmetry breaking scale would be at M_X. Thus, in these approaches, the naturalness and hierarchy problems are not addressed.

6.5 Prospects for Phenomenology in the Models Without Elementary Scalars

The theoretical questions and problems of models of dynamical electroweak symmetry breaking are difficult and challenging. Presumably, data from experiments in the coming decade will provide some important clues as to whether nature has chosen this course. The most interesting initial information will be obtained as the precision electroweak measurements improve. Already, the precise measurements of Z decay properties are beginning to impose interesting constraints on the structure of technicolor models. At the end of this decade, experiments at the LHC and SSC will begin to make measurements that will test the TeV energy scale for the first time. Via the VV fusion process (where $V= W^{\pm}$ or Z), one will be able to test the Standard Model predictions for $W_L^+W_L^-$ (and Z_LZ_L) scattering. It is this channel which is most sensitive to the physics of the electroweak symmetry breaking sector. For example, many technicolor models contain a ρ_T and other resonant states which couple to the VV channel. Such phenomena would appear as resonant enhancements of the VV cross section and should be clearly visible at the SSC.

Unfortunately, not all new physics at the TeV scale would lead to such dramatic results. In order to parametrize the most general VV interaction which is theoretically consistent, one omits the Higgs boson and makes use of the low-energy theorems for VV scattering for $s < s_c$, where $\sqrt{s_c}$ is the energy at which unitarity is violated (as discussed at the end of Lecture 2). Unitarity is then repaired by introducing a model. For example, one can parametrize the unknown physics which repairs the unitarity with a chiral Lagrangian. Here, the longitudinal vector bosons play the role analogous to the pions in the chiral Lagrangian of strong interactions. These analyses suggest that there are parameter choices in the chiral Lagrangian for which the detection of new physics in VV scattering at the SSC may be very difficult. Clearly, further work in this direction is important in order to determine how to best utilize the future supercolliders in exploring the origin of electroweak symmetry breaking.

Suggestions for Further Reading and a Brief Guide to the Literature

Some of the classic reviews of technicolor models and dynamical symmetry breaking can be found in

1. E. Farhi and L. Susskind, "Technicolour," *Phys. Rep.* **74**, 277 (1981).
2. K. Lane and M.E. Peskin, "An Introduction to Weak Interaction Theories with Dynamical Symmetry Breaking," 15th Rencontre de Moriond (1980).
3. K. Lane, "The Scalar Sector of the Electroweak Interactions," Proceedings of the 1982 Snowmass Workshop, edited by R. Donaldson, R. Gustafson and F. Paige, p. 222.
4. R.K. Kaul, "Technicolor," *Rev. Mod. Phys.* **55**, 449 (1983).
5. S. Kaptanoglu and N.K. Pak, "Technicolor, Extended-Technicolor, and Tumbling," *Fortschr. der Phys.* **30**, 451 (1982).
6. M.E. Peskin, "Chiral Symmetry and Chiral Symmetry Breaking," Proceedings of the Les Houches Summer School, Session XXXIX, 1982, edited by J.-B. Zuber and R. Stora, (North-Holland, Amsterdam, 1984) p. 217.

A brief review of dynamical electroweak symmetry breaking, along with a more up-to-date bibliography can be found in

7. M.S. Chanowitz, "Electroweak Symmetry Breaking," *Ann. Rev. Nucl. Part. Phys.* **38,** 323 (1988).
8. *The Higgs Hunter's Guide*, Chapter 7.

A brief review of the walking technicolor ideas is given in

9. Tom Appelquist, "Technicolor and Mass Hierarchies," in *Proceedings of the Last Workshop on Grand Unification*, edited by Paul H. Frampton (World Scientific, Singapore, 1989), p. 123.

A review of the $t\bar{t}$ condensate models can be found in

10. Christopher T. Hill, "Minimal Dynamical Symmetry Breaking of the Electroweak Interactions and $m_{\rm top}$" in *Proceedings of the Rice Meeting*, edited by Billy Bonner and Hannu Miettinen (World Scientific, Singapore, 1990), p. 906.

The implications of precision electroweak measurements for technicolor models are discussed in

11. M.E. Peskin and T. Takeuchi, *Phys. Rev. Lett.* **65**, 964 (1990); B. Holdom and J. Terning, *Phys. Lett.* **B247**, 88 (1990); M. Golden and L. Randall, FERMILAB-PUB-90/83 (1990).

For a discussion of the chiral Lagrangian approach to $W_L W_L$ scattering and the implications for searches at LHC and SSC, see

12. A. Dobado, M.J. Herrero, and J. Terron, CERN-TH-5959-90 (1990), to appear in the Proceedings of the ECFA Large Hadron Collider Workshop, Aachen, Germany, October 4–9, 1990.
13. J. Bagger, S. Dawson, and G. Valencia, JHU-TIPAC-91-002 (1991), to appear in the Proceedings of the 1990 Summer Study on High Energy Physics, Snowmass, Colorado, June 25–July 13, 1990.

Afterword

Particle physics is only now beginning to explore the TeV energy scale. This energy scale holds the secrets to a number of crucial theoretical questions:

- Is electroweak symmetry breaking dynamical in origin or is the Higgs field elementary?

Of course, I cannot rule out that an elementary Higgs boson is in reality a composite state associated with physics at energy scales far above the scale of electroweak symmetry breaking. If this is the case, we may never be able to distinguish between these two alternatives. But if you believe in the "naturalness" principle, then you believe that the elementary Higgs boson, if present, will be accompanied by supersymmetry (or other new physics at the TeV scale), so that there will be no ambiguity in the Higgs boson identity.

- Are mass generation mechanisms for fermions and vector bosons distinct?

The large value of m_t provides very strong constraints on models and raises the hope that the relevant energy scale controlling m_t and m_W is roughly the same. Of course, if $m_t \sim m_W$ is "expected", then we have to understand why all other fermions are so light. It is not clear whether the solution to the fermion mass spectrum puzzle lies at the TeV scale.

- Does the solution to the gauge hierarchy problem lie at the electroweak scale?

If it turns out that the SSC discovers one elementary Higgs boson as predicted by the Standard Model and *no other new physics*, then there will have to be a major reassessment of our overall picture of particle physics at high energies. Personally, I find this possibility rather remote. However, this is one crucial question which must be addressed by the next generation of supercolliders.

- Is there a "low-energy" window to Planck scale physics?

Here, "low-energy" supersymmetry may be the key that allow us to unlock some of the secrets of the Planck scale. On the other hand, dynamical symmetry breaking approaches, if correct, could impose a fundamental limit on our ability to look beyond the next layer of compositeness.

The Standard Model has been a remarkable success story. Yet one crucial aspect of it—the electroweak symmetry breaking sector—remains to be tested. It is in this direction that most of the efforts of the particle physics community will be focused during the coming decade. These are exciting times. Unlike the discovery of the $W^\pm$ and Z, it is not clear what lies just ahead. Let us anticipate many surprises in store as we begin to explore the TeV scale.