What's so special about the MSSM Higgs sector?

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1. P.M. Ferreira, H.E. Haber and J.P. Silva, "Basis-independent conditions for supersymmetry in the two-Higgs-doublet model," *Phys. Rev.* **D82**, 016001 (2010).
2. P.M. Ferreira, H.E. Haber, M. Maniatis, O. Nachtmann and J.P. Silva, "The geometric picture of generalized CP and Higgs-family transformations in the two-Higgs-doublet model," *Int. J. Mod. Phys.* **A26**, 769 (2011).
3. P.M. Ferreira, H.E. Haber and J.P. Silva, "Generalized CP symmetries and special regions of parameter space in the two-Higgs-doublet model," *Phys. Rev.* **D79**, 116004 (2009).
4. S. Davidson and H.E. Haber, "Basis-independent methods for the two-Higgs-doublet model," Phys. Rev. **D72**, 035004 (2005).
5. H.E. Haber and D. O'Neil, "Basis-independent methods for the two-Higgs-doublet model. II: The significance of $\tan\beta$," *Phys. Rev.* **D74**, 015018 (2006).
6. H.E. Haber and D. O'Neil, "Basis-independent methods for the two-Higgs-doublet model. III: The CP-conserving limit, custodial symmetry, and the oblique parameters S, T, U," *Phys. Rev.* **D83**, 055017 (2011).

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The general scalar potential of the two-Higgs-doublet model (2HDM)

The 2HDM consists of two identical weak SU$(2)_L$ doublet, hypercharge 1 scalar fields. The most general tree-level scalar potential is:

$$\begin{aligned}
\mathcal{V} = {} & m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}] + \tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 \\
& + \tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
& + \left\{\tfrac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right]\Phi_1^\dagger\Phi_2 + \text{h.c.}\right\} ,
\end{aligned}$$

where m_{11}^2, m_{22}^2 and $\lambda_{1,2,3,4}$ are real and m_{12}^2 and $\lambda_{5,6,7}$ are potentially complex. In the most general model, Φ_1 and Φ_2 couple to quarks and leptons via dimension-four Yukawa couplings. In terms of the Higgs mass-eigenstates, the most general Higgs-quark interactions are:

$$
\begin{aligned}
-\mathscr{L}_Y = &\frac{1}{v}\overline{D}\left\{M_D(q_{k1}P_R + q_{k1}^* P_L) + \frac{v}{\sqrt{2}}\left[q_{k2}\,[e^{i\theta_{23}}\rho^D]^\dagger P_R + q_{k2}^*\,e^{i\theta_{23}}\rho^D P_L\right]\right\}Dh_k \\
&+\frac{1}{v}\overline{U}\left\{M_U(q_{k1}P_L + q_{k1}^* P_R) + \frac{v}{\sqrt{2}}\left[q_{k2}^*\,e^{i\theta_{23}}\rho^U P_R + q_{k2}\,[e^{i\theta_{23}}\rho^U]^\dagger P_L\right]\right\}Uh_k \\
&+\left\{\overline{U}\left[K[\rho^D]^\dagger P_R - [\rho^U]^\dagger K P_L\right]DH^+ + \frac{\sqrt{2}}{v}\overline{U}\,[KM_D P_R - M_U K P_L]\,DG^+ + \text{h.c.}\right\},
\end{aligned}
$$

where M_U and M_D are 3×3 diagonal quark mass matrices, K is the CKM matrix, $e^{i\theta_{23}}\rho^{U,D}$ are arbitrary complex 3×3 coupling matrices, and the q_{ki} encode mixing angles that arise from the diagonalization of the 3×3 Higgs scalar squared-mass matrix.

That is, the most general 2HDM exhibits

- CP-violating Higgs interactions
- Tree-level Higgs-mediated flavor changing neutral currents (FCNCs)

which for generic values of the 2HDM parameters are inconsistent with observed particle physics phenomena.

To avoid these unseemly features within the 2HDM, one has two options:

- fine-tune the relevant 2HDM parameters (and/or arrange for the decoupling limit, in which only a Standard Model-like Higgs boson remains in the low-energy effective theory) to reduce the CP-violating and FCNC effects to an acceptable level.
- introduce a new symmetry governing the Higgs interactions to eliminate both CP-violating and FCNC effects at tree-level.

The MSSM Higgs sector is one example of the second approach. It possesses a very special 2HDM scalar potential. In the SUSY limit,

$$m_{11}^2 = m_{22}^2 = |\mu|^2 \,, \qquad m_{12}^2 = 0 \,, \qquad \lambda_5 = \lambda_6 = \lambda_7 = 0 \,,$$

$$\lambda_1 = \lambda_2 = -(\lambda_3 + \lambda_4) = \tfrac{1}{4}(g^2 + g'^2) \,, \qquad \lambda_4 = -\lambda_1 - \lambda_3 = -\tfrac{1}{2}g^2 \,.$$

When soft-SUSY-breaking is added, m_{11}^2, m_{22}^2 and m_{12}^2 become independent (non-zero) parameters, while the tree-level relations among the λ_i continue to hold.

What is special about the tree-level relations among the MSSM 2HDM scalar potential parameters?

Basis-independent formalism for the 2HDM

The parameters of the 2HDM scalar potential introduced above are not physical, since they can changed by redefining the scalar fields. A basis change consists of a U(2) Higgs family transformation $\Phi_a \to U_{a\bar{b}}\Phi_b$.

$$\mathcal{V} = Y_{a\bar{b}}\Phi^\dagger_{\bar{a}}\Phi_b + \tfrac{1}{2}Z_{a\bar{b}c\bar{d}}(\Phi^\dagger_{\bar{a}}\Phi_b)(\Phi^\dagger_{\bar{c}}\Phi_d)$$

where $Z_{a\bar{b}c\bar{d}} = Z_{c\bar{d}a\bar{b}}$ and hermiticity implies $Y_{a\bar{b}} = (Y_{b\bar{a}})^*$ and $Z_{a\bar{b}c\bar{d}} = (Z_{b\bar{a}d\bar{c}})^*$. The barred indices help keep track of which indices transform with U and which transform with $U^\dagger$. The most general U(1)$_{\rm EM}$-conserving vacuum expectation value (vev) is:

$$\langle\Phi_a\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix} 0 \\ \widehat{v}_a \end{pmatrix}, \qquad \text{with} \qquad \widehat{v}_a \equiv e^{i\eta}\left(c_\beta\,,\, s_\beta\, e^{i\xi}\right) \quad \text{for } a=1,2\,,$$

where $v \equiv 2m_W/g = 246$ GeV. The overall phase η is arbitrary.

If we define the hermitian matrix $V_{a\bar{b}} \equiv \hat{v}_a \hat{v}^*_{\bar{b}}$, then the scalar potential minimum condition is given by the invariant condition:

$$\mathrm{Tr}\,(VY) + \tfrac{1}{2} v^2 Z_{a\bar{b}c\bar{d}} V_{b\bar{a}} V_{d\bar{c}} = 0\,.$$

The orthonormal eigenvectors of $V_{a\bar{b}}$ are $\hat{v}_b$ and $\widehat{w}_b \equiv \widehat{v}^*_{\bar{c}}\, \epsilon_{cb}$ (with $\epsilon_{12} = -\epsilon_{21} = 1$, $\epsilon_{11} = \epsilon_{22} = 0$). Note that $\hat{v}^*_{\bar{b}} \hat{w}_b = 0$. Under a U(2) transformation, $\hat{v}_a \to U_{a\bar{b}} \hat{v}_b$, but:

$$\widehat{w}_a \to (\det\, U)^{-1}\, U_{a\bar{b}}\, \widehat{w}_b\,,$$

where $\det\, U \equiv e^{i\chi}$ is a pure phase. That is, $\widehat{w}_a$ is a pseudo-vector with respect to U(2). One can use $\widehat{w}_a$ to construct a proper second-rank tensor: $W_{a\bar{b}} \equiv \hat{w}_a \hat{w}^*_{\bar{b}} \equiv \delta_{a\bar{b}} - V_{a\bar{b}}$.

Note that $\tan\beta \equiv v_2/v_1$ is a basis-dependent quantity, and hence it is *not* a physical parameter in a general 2HDM.

A list of invariant and pseudo-invariant quantities

$$Y_1 \equiv \text{Tr}\,(YV)\,, \qquad Y_2 \equiv \text{Tr}\,(YW)\,,$$

$$Z_1 \equiv Z_{a\bar{b}c\bar{d}}\,V_{b\bar{a}}V_{d\bar{c}}\,, \qquad Z_2 \equiv Z_{a\bar{b}c\bar{d}}\,W_{b\bar{a}}W_{d\bar{c}}\,,$$

$$Z_3 \equiv Z_{a\bar{b}c\bar{d}}\,V_{b\bar{a}}W_{d\bar{c}}\,, \qquad Z_4 \equiv Z_{a\bar{b}c\bar{d}}\,V_{b\bar{c}}W_{d\bar{a}}$$

are invariants, whereas the following (potentially complex) pseudo-invariants

$$Y_3 \equiv Y_{a\bar{b}}\,\widehat{v}_{\bar{a}}^{\,*}\,\widehat{w}_b\,, \qquad Z_5 \equiv Z_{a\bar{b}c\bar{d}}\,\widehat{v}_{\bar{a}}^{\,*}\,\widehat{w}_b\,\widehat{v}_{\bar{c}}^{\,*}\,\widehat{w}_d\,,$$

$$Z_6 \equiv Z_{a\bar{b}c\bar{d}}\,\widehat{v}_{\bar{a}}^{\,*}\,\widehat{v}_b\,\widehat{v}_{\bar{c}}^{\,*}\,\widehat{w}_d\,, \qquad Z_7 \equiv Z_{a\bar{b}c\bar{d}}\,\widehat{v}_{\bar{a}}^{\,*}\,\widehat{w}_b\,\widehat{w}_{\bar{c}}^{\,*}\,\widehat{w}_d\,.$$

transform as

$$[Y_3, Z_6, Z_7] \to (\det\,U)^{-1}[Y_3, Z_6, Z_7] \quad \text{and} \quad Z_5 \to (\det\,U)^{-2} Z_5\,.$$

Physical quantities must be invariants. For example, the charged Higgs boson mass is $m_{H^\pm}^2 = Y_2 + \frac{1}{2}Z_3 v^2$. Pseudo-invariants are useful because one can always combine two such quantities to create an invariant.

The MSSM Higgs scalar potential revisited

Perhaps the significance of the relations among the MSSM Higgs scalar potential parameters will become clearer when expressed in terms of invariant quantities.

$$Z_1 = Z_2 = \tfrac{1}{4}(g^2 + g'^2)\cos^2 2\beta\,, \qquad Z_3 = e^{-2i\chi} Z_5 + \tfrac{1}{4}(g^2 - g'^2)\,,$$

$$Z_4 = e^{-2i\chi} Z_5 - \tfrac{1}{2}g^2\,, \qquad e^{-2i\chi} Z_5 = \tfrac{1}{4}(g^2 + g'^2)\sin^2 2\beta\,,$$

$$e^{-i\chi} Z_7 = -e^{-i\chi} Z_6 = \tfrac{1}{4}(g^2 + g'^2)\sin 2\beta\cos 2\beta\,,$$

where the phase χ is chosen such $e^{-2i\chi} Z_5$ is real and positive. (Since Z_5, Z_6 and Z_7 are pseudo-invariants, these quantities can be appropriately rephased without affecting any physical observable.)

However, as $\tan\beta$ is basis-dependent, the above cannot be regarded as a basis-independent characterization of the MSSM Higgs sector.

How to discover a symmetry of the 2HDM

Suppose we impose a $\mathbb{Z}_2$ symmetry, $\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$, on the scalar potential. Then it follows that $m_{12}^2 = \lambda_6 = \lambda_7 = 0$. In a different basis for the scalar fields, the symmetry defined above is no longer manifest. Nevertheless, the symmetry still persists. One can formulate a basis-independent test for the existence of the $\mathbb{Z}_2$ symmetry. Defining

$$Z^{(1)}_{a\bar{d}} \equiv \delta_{b\bar{c}} Z_{a\bar{b}c\bar{d}} \,, \qquad Z^{(11)}_{c\bar{d}} \equiv Z^{(1)}_{b\bar{a}} Z_{a\bar{b}c\bar{d}} \,, \qquad Y^{(1)}_{c\bar{d}} \equiv Y_{b\bar{a}} Z_{a\bar{b}c\bar{d}} \,,$$

then we require the following conditions to be satisfied:

$$[Z^{(1)}, Y] = [Z^{(1)}, Z^{(11)}] = [Y^{(1)}, Y] = 0 \,.$$

As an example, suppose that the 2HDM scalar potential satisfies the following conditions:

$$m_{11}^2 = m_{22}^2 \,, \quad \lambda_1 = \lambda_2 \,, \quad \lambda_7 = \lambda_6^* \,, \quad m_{12}^2 \text{ and } \lambda_5 \text{ are real} \,.$$

The three commutators above vanish, which implies that there exists some basis where the discrete $\mathbb{Z}_2$ symmetry $\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$ is manifest.*

*In fact, in the original basis, the symmetry just exhibited corresponds to a permutation symmetry $\Phi_1 \leftrightarrow \Phi_2$.

Classification of 2HDM symmetries

The 2HDM scalar potential is invariant with respect to the electroweak SU(2)×U(1)$_Y$ gauge symmetry. However, additional global symmetries (either discrete or continuous) may be present.

- Higgs family symmetries

Consider the U(2) basis transformations, $\Phi_a \to U_{a\bar{b}}\Phi_b$. The 2HDM scalar potential is invariant under global U(1)$_Y$ transformations, which is a subgroup of U(2). The Higgs family symmetry consists of the largest subgroup of U(2) orthogonal to U(1)$_Y$ that is a global symmetry of the 2HDM scalar potential.

- Generalized CP (GCP) symmetries

The standard CP symmetry transformation is $\Phi_a \to \Phi^*_{\bar{a}}$. But, one can also consider generalized CP transformations,

$$\Phi_a \to X_{\bar{a}b}\Phi^*_{\bar{b}}\,,$$

where X is a unitary matrix.

Three distinct classes of GCP symmetries are possible.

- CP1: X is also symmetric $\Longrightarrow$ $(\text{CP1})^2 = \mathbb{1}$
- CP2: X is also antisymmetric $\Longrightarrow$ $(\text{CP2})^2 = -\mathbb{1}$
- CP3: X is neither symmetric nor antisymmetric $\Longrightarrow$ $(\text{CP3})^2 \neq \pm\mathbb{1}$

Note: In the CP1 case (and only in this case), one can always find a basis such that the CP1 transformation is simply $\Phi_a \to \Phi^*_{\bar{a}}$.

Because the terms of the tree-level scalar potential are at most dimension-4, the possible symmetries of the 2HDM scalar potential are quite limited. Here is the complete list.

symmetry class	maximal symmetry	m^2_{22}	m^2_{12}	λ_2	λ_4	λ_5	λ_6	λ_7
$\mathbb{Z}_2$	$(\mathbb{Z}_2)^2$		0			0	0	
U(1)	O(2)		0			0	0	0
SO(3)	O(3)	m^2_{11}	0	λ_1	$\lambda_1 - \lambda_3$	0	0	0
CP1	$\mathbb{Z}_2$		real			real	real	real
CP2	$(\mathbb{Z}_2)^3$	m^2_{11}	0	λ_1				$-\lambda_6$
CP3	O(2)$\otimes\mathbb{Z}_2$	m^2_{11}	0	λ_1		$\lambda_1 - \lambda_3 - \lambda_4$ (real)	0	0

The hierarchy of possible symmetries has the following structure:

$$\text{CP1} < \mathbb{Z}_2 < \left\{ \begin{array}{c} \text{U}(1) \\ \text{CP2} \end{array} \right\} < \text{CP3} < \text{SO}(3)\,.$$

Basis-independent conditions for each symmetry class have also been obtained.

- Custodial symmetry

Consider the 2HDM scalar potential that is invariant with respect to *global* $\text{SU}(2)_L \times \text{U}(1)_Y$ transformations. The largest symmetry allowed global group (corresponding to the symmetry group of the kinetic energy terms) is:

$$\Phi'_a = U_{ab}\Phi_b + V^*_{ab}\widetilde{\Phi}_b\,,$$

where $\widetilde{\Phi} \equiv i\sigma^2\Phi^*_a$, and the matrices U and V satisfy

$$UU^\dagger + VV^\dagger = \mathbb{1}\,, \qquad U^{\mathsf{T}}V - V^{\mathsf{T}}U = 0\,.$$

We can then construct the 4×4 matrix

$$\mathcal{Q} = \begin{pmatrix} U & V^* \\ -V & U^* \end{pmatrix}\,.$$

The matrix $\mathcal{Q}$ satisfies:

$$\mathcal{Q}^{\mathsf{T}} P \mathcal{Q} = P\,, \qquad \mathcal{Q}^{\dagger}\mathcal{Q} = \mathbb{1}_{4\times 4}\,, \qquad \text{where} \quad P = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}\,.$$

That is $\mathcal{Q} \in$ Sp(2). This conclusion continues to hold if we gauge the SU(2)$_L$ symmetry. Thus in the limit of $g' = 0$, the largest symmetry group of the 2HDM scalar potential is SU(2)$_L\times$ Sp(2). The custodial symmetry group corresponds to SU(2)$_L\times$ SU(2)$_R$, which is a subgroup of the maximal symmetry group. We can explicitly identify SU(2)$_R$ by choosing,

$$U = \cos\theta\, e^{i\alpha}\,\mathbb{1}\,, \qquad V = \sin\theta\, W\,,$$

where W is an arbitrary symmetric unitary matrix and α and θ are arbitrary angles. Different choices of W correspond to different basis choices. The global U(1)$_Y$ transformations provide the diagonal generator for SU(2)$_R$ and corresponds to $\cos\theta = 1$.

After electroweak symmetry breaking, the SU(2)$_L\times$ SU(2)$_R$ symmetry is spontaneously broken down to the diagonal SU(2)$_V$ subgroup, which is commonly identified as the custodial symmetry. Of course, the custodial symmetry is broken once $g' \neq 0$, corresponding to the gauging of the U(1)$_Y$.

Basis-independent conditions for a custodial-symmetric 2HDM potential

- The 2HDM scalar potential is CP-conserving. That is, one can choose a phase convention such that the pseudo-invariants Z_5, Z_6 and Z_7 are simultaneously real.

- One additional condition must be satisfied:

$$Z_4 = \begin{cases} \varepsilon_{56}|Z_5|\,, & \text{for } \; Z_6 \neq 0\,, \\ \varepsilon_{57}|Z_5|\,, & \text{for } \; Z_7 \neq 0\,, \\ \pm|Z_5|\,, & \text{for } \; Z_6 = Z_7 = 0\,, \end{cases}$$

where ϵ_{56} and ϵ_{57} correspond to the sign of Z_5 in the phase convention where the pseudo-invariants are real.

Stability under renormalization group (RG) running

Any true symmetry properties of the dimension-4 terms of the scalar potential must be stable with respect to RG-running. The MSSM Higgs potential fails this test when naively applied. Consider

symmetry class	m_{22}^2	m_{12}^2	λ_2	λ_4	λ_5	λ_6	λ_7
SO(3)	m_{11}^2	0	λ_1	$\lambda_1 - \lambda_3$	0	0	0
MSSM	m_{11}^2	0	λ_1	$-\lambda_1 - \lambda_3$	0	0	0

Focusing on the scalar and vector boson contributions to RG-running, given the constraints $\lambda_1 = \lambda_2$ and $\lambda_5 = \lambda_6 = \lambda_7 = 0$, compare

$$\mathcal{D}(\lambda_4 + \lambda_3 - \lambda_1) = \tfrac{1}{2}(\lambda_4 + \lambda_3 - \lambda_1)(12\lambda_1 + 4\lambda_4 - 9g^2 - 3g'^2)$$

$$\begin{aligned}\mathcal{D}(\lambda_4 + \lambda_3 + \lambda_1) = &-\tfrac{1}{2}\left(9g^2 + 3g'^2\right)(\lambda_4 + \lambda_3 + \lambda_1) \\ &2\left(3\lambda_1^2 + (3\lambda_3 + 2\lambda_4)\lambda_1 + 2\lambda_3^2 + 2\lambda_4^2 + 3\lambda_3\lambda_4\right) \\ &+\tfrac{1}{4}\left(9g^4 + 6g^2g'^2 + 3g'^4\right).\end{aligned}$$

The SO(3) relation $\lambda_4 = \lambda_1 - \lambda_3$ is stable under RG-running. The MSSM relation $\lambda_4 = -\lambda_1 - \lambda_3$ is not. Perhaps you are concerned that I have not yet made use of the fact that the parameters of the scalar potential are related to gauge couplings in the MSSM. Imposing those conditions yields:

$$\mathcal{D}(\lambda_4 + \lambda_3 + \lambda_1) = 3g^4 + 2g^2 g'^2 + g'^4 \,,$$

i.e., $\lambda_1 = -\lambda_3 - \lambda_4$ is still not stable under RG-running.

In order to achieve stability under RG-running for the MSSM Higgs scalar potential, one must also include the effects of the superpartners. The gaugino and higgsino interactions generate additional terms, which in the SUSY limit yield

$$\delta_{\rm SUSY}(\mathcal{D}\lambda_1) = -\tfrac{5}{2}g^4 - g^2 g'^2 - \tfrac{1}{2}g'^4 \,,$$

$$\delta_{\rm SUSY}(\mathcal{D}\lambda_3) = -\tfrac{5}{2}g^4 + g^2 g'^2 - \tfrac{1}{2}g'^4 \,,$$

$$\delta_{\rm SUSY}(\mathcal{D}\lambda_4) = 2g^4 - 2g^2 g'^2 \,.$$

Hence,

$$\delta_{\rm SUSY}\{\mathcal{D}(\lambda_4 + \lambda_3 + \lambda_1)\} = -(3g^4 + 2g^2 g'^2 + g'^4) \,,$$

which is precisely what is required to obtain: $\mathcal{D}(\lambda_4 + \lambda_3 + \lambda_1) = 0$.

Reconciling the MSSM scalar potential

Do basis-invariant conditions exist that guarantee the existence of a basis in which the MSSM conditions of the Higgs scalar potential parameters are satisfied? Ferreira, Silva and I found a necessary but not sufficient condition:

The Higgs sector of the MSSM is a CP3-symmetric 2HDM.

We also provided an explicit basis-invariant condition for CP3 symmetry (you don't want to see it!). But suffice it to say that for a CP3-symmetric scalar potential, one can always find a basis in which

$$m_{11}^2 = m_{22}^2 \,, \quad \lambda_1 = \lambda_2 \,, \quad \text{and} \quad m_{12}^2 = \lambda_5 = \lambda_6 = \lambda_7 = 0 \,.$$

These conditions are satisfied by the MSSM Higgs sector, but the additional condition $\lambda_4 = -\lambda_1 - \lambda_3$ and the relation of the λ_i to gauge couplings are not imposed.

To achieve a complete basis-invariant formulation of the MSSM Higgs sector, one must also incorporate the gaugino-Higgs-higgsino interaction. In the MSSM, the latter is given by:

$$\mathscr{L}^{\rm MSSM}_{\rm gaugino\text{-}Higgs} = \mu\epsilon_{ij}\psi^i_{H_D}\psi^j_{H_U} + \frac{ig}{\sqrt{2}}\lambda^\alpha\tau^\alpha_{ij}\left(\psi^j_{H_U}\Phi_2^{i\,\dagger} + \epsilon^{ik}\psi^j_{H_D}\Phi_1^k\right) + \frac{ig'}{\sqrt{2}}\lambda'\left(\psi^j_{H_U}\Phi_2^{i\,\dagger} - \epsilon^{ik}\psi^i_{H_D}\Phi_1^k\right) + \text{h.c.}\,.$$

However, if we relax the constraints imposed by SUSY, we should consider more general dimension-4 interaction terms,

$$\mathscr{L}_{\rm gaugino\text{-}Higgs} = \frac{i}{\sqrt{2}}\lambda^\alpha\tau^\alpha_{ij}\left(\psi^j_{H_U}f^a_U\Phi^{i\,\dagger}_{\bar{a}} + \epsilon^{ik}\psi^j_{H_D}f^{\bar{a}\,*}_D\Phi^k_a\right) + \frac{i}{\sqrt{2}}\lambda'\left(\psi^j_{H_U}f'^{\,a}_U\Phi^{i\,\dagger}_{\bar{a}} - \epsilon^{ik}\psi^i_{H_D}f'^{\,\bar{a}\,*}_D\Phi^k_a\right) + \text{h.c.}\,,$$

where f^a_U, f^a_D, $f'^{\,a}_U$ and $f'^{\,a}_D$ transform as vectors under a Higgs basis U(2) transformation.

Basis-invariant relations that enforce SUSY gaugino-Higgsino-Higgs couplings are:

$$f_U^a f_D^{\bar{a}*} = 0, \qquad f_U^a f_D^{\prime\bar{a}*} = 0,$$

$$f_U^{\prime a} f_D^{\bar{a}*} = 0, \qquad f_U^{\prime a} f_D^{\prime\bar{a}*} = 0,$$

$$f_U^a f_U^{\bar{a}*} = f_D^a f_D^{\bar{a}*} = g^2, \qquad f_U^{\prime a} f_U^{\prime\bar{a}*} = f_D^{\prime a} f_D^{\prime\bar{a}*} = g^{\prime\,2},$$

$$f_U^a f_U^{\prime\,\bar{a}*} = gg^\prime, \qquad f_D^{\bar{a}*} f_D^{\prime a} = gg^\prime\,.$$

Basis-invariant relations that enforce a SUSY scalar Higgs potential involve the basis-invariant quantities constructed from *both* the Higgs scalar potential and the gaugino-Higgs interactions,

$$\mathcal{Y}_{DD} = \hat{f}_D^{\bar{a}*}\,\hat{f}_D^b\,Y_{a\bar{b}}, \qquad \mathcal{Y}_{UU} = \hat{f}_U^{\bar{a}*}\,\hat{f}_U^b\,Y_{a\bar{b}},$$

$$\mathcal{Y}_{DU} = \hat{f}_D^{\bar{a}*}\,\hat{f}_U^b\,Y_{a\bar{b}}, \qquad \mathcal{Y}_{UD} = \hat{f}_U^{\bar{a}*}\,\hat{f}_D^b\,Y_{a\bar{b}},$$

and

$$\mathcal{Z}_{\alpha\beta\gamma\delta} = \hat{f}_\alpha^{\bar{a}*}\,\hat{f}_\beta^b\,\hat{f}_\gamma^{\bar{c}*}\,\hat{f}_\delta^d\,Z_{a\bar{b}c\bar{d}}\,,$$

where the indices α, β, γ, and δ can take the values D or U.

Then, the basis-invariant relations that enforce a SUSY scalar Higgs potential are given by:

$$\mathcal{Y}_{DD} = \mathcal{Y}_{UU}, \qquad \mathcal{Y}_{DU} = \mathcal{Y}_{UD} = 0,$$

$$\mathcal{Z}_{DDDD} = \mathcal{Z}_{UUUU} = \tfrac{1}{4}\left[f_U^{\bar{a}\,*} f_U^{a} + f_D^{\bar{a}\,*} f_D^{a}\right],$$

$$\mathcal{Z}_{DDUU} = \mathcal{Z}_{UUDD} = \tfrac{1}{4}\left[f_U^{\bar{a}\,*} f_U^{a} - f_U^{\prime\,\bar{a}\,*} f_U^{\prime\,a}\right],$$

$$\mathcal{Z}_{DUUD} = \mathcal{Z}_{UDDU} = -\mathcal{Z}_{DDDD} - \mathcal{Z}_{DDUU},$$

$$\begin{aligned}\mathcal{Z}_{DUDU} = \mathcal{Z}_{UDUD} = \mathcal{Z}_{UDDD} = \mathcal{Z}_{DUDD} = \mathcal{Z}_{DDUD} = \mathcal{Z}_{DDDU} \\ = \mathcal{Z}_{DUUU} = \mathcal{Z}_{UDUU} = \mathcal{Z}_{UUDU} = \mathcal{Z}_{UUUD} = 0.\end{aligned}$$

In the basis-independent formalism, $\tan\beta$ is not a physical parameter. However, basis-invariant $\tan\beta$-like parameters can be defined. To give two examples,

$$|\tan\beta_U| \equiv \left|\frac{\hat{f}_U^{\bar{a}*}\hat{v}_a}{\epsilon^{\bar{a}\bar{b}}\hat{v}_a\hat{f}_U^{b}}\right|, \qquad |\tan\beta_D| \equiv \left|\frac{\epsilon^{\bar{a}\bar{b}}\hat{v}_a\hat{f}_D^{b}}{\hat{f}_D^{\bar{a}*}\hat{v}_a}\right|,$$

where the hats indicate unit vectors. In the SUSY-limit, $\tan\beta_U = (\tan\beta_D)^* = \tan\beta$. Since $\tan\beta_U$ and $\tan\beta_D$ are defined directly in terms of basis-invariant quantities (which are thus physical), one can in principle test experimentally for supersymmetry by verifying that the $\tan\beta$-like parameters coincide (at leading order).

Conclusions and future directions

• Basis-independent methods provide a powerful technique for studying the theoretical structure of the two-Higgs doublet model.

• All physical observables can be expressed in terms of basis-invariant quantities. This allows for model-independent experimental analysis of 2HDM phenomena. Then, experiment can determine the nature of any additional global discrete or continuous symmetries (or supersymmetry) that govern the 2HDM interactions.

• The classification of 2HDM symmetries can be extended to include other sectors of the theory. In particular, it is important to include the Higgs-fermion interactions in classifying all possible symmetries of the Higgs interactions. Some work in this direction has already been carried out.

• A set of collider tools needs to be more fully developed to carry out this program in its full generality. I expect that there is still many improvements that can be made to the LHC analysis of 2HDM phenomena., Eventually, the full power of this program can only be realized at a precision Higgs factory such as the ILC.