Department of Physics & Astronomy
Laboratory III

Course 3C20

Experiment CP15

The charge on an oil drop: **Millikan's experiment**

Experiment Objectives :

- To search for evidence of charge quantisation.
- To estimate the value of the quantum of charge.

Relevant Lecture Courses :

1B23 Modern Physics, astronomy and Cosmology
1B26, Electricity and Magnetism
2B24 Atomic and Molecular Physics

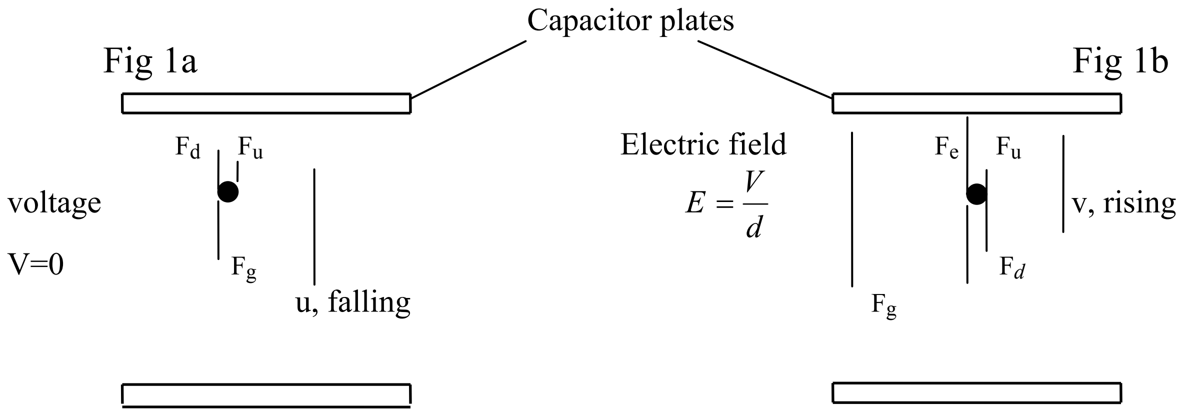
Measurement of the Charge carried by an Oil Drop: Millikan's experiment for the determination of the quantum of charge

1. Introduction

Millikan's experiment is one of the classic experiments of Physics. It provided evidence for the quantised nature of electric charge and was the first to measure convincingly a value for its magnitude. It is discussed in most introductory courses on Modern Physics. The present experiment uses Millikan's method but rather different equipment to that which he employed.

2. Principles of the Experiment

The method is based on measuring the speed of a charged oil drop, first at its terminal velocity reached while falling under gravity and second its terminal velocity reached rising under the net force on it due to gravity and an opposing uniform electric field. The field is supplied by the plates of an air gap capacitor. The determination proceeds in the following stages.



Drop falling under gravity

Drop rising in the electric field E

2.1 An oil drop is timed falling freely under gravity between the capacitor plates (Fig la).

This effectively measure the radius of the drop as follows. As it falls it quickly reaches the terminal velocity *u* when the drag force due to viscosity, F_d, equals the force due to gravity on the drop, F_g, less the Archimedean upthrust, F_u, equal to the weight of air displaced by the drop.

$$F_d = 6\pi\eta a u \qquad \text{Drag due to viscosity (Stoke'sLaw)}$$

$F_g = mg = \frac{4}{3}\pi a^3 \rho g$ Force due to gravity

$F_u = \frac{4}{3} a^3 \sigma g$ Archimedean upthrust

where a = radius of the drop, u =velocity of fall, η = viscosity of air, σ = density of air, and ρ = density of the oil drop.

Thus $F_d = F_g - F_u$

or
$$6\pi\eta a u = \frac{4}{3}\pi a^3 (\rho - \sigma) g$$

If u is measured and ρ, σ, η, and g are known, the radius of the drop is given by;

$$\text{a} = \left[\frac{9\eta u}{2(\rho - \sigma) g}\right]^{\frac{1}{2}} \qquad \textbf{[1]}$$

2.2 A voltage V is switched on across the plates establishing a field $E = \frac{V}{d}$ (Fig lb).

If the drop has a charge q, it experiences an electrostatic force $F_e = \frac{qV}{d}$. If V has the right polarity this force will overcome gravity and the drop will rise upwards reaching a terminal velocity v when,

$$F_d = F_e - F_g + F_u$$

or
$$6\pi\eta a v = q\frac{V}{d} - \frac{4}{3}\pi a^3 (\rho - \sigma) g$$

but from 1.
$$F_d = 6\pi\eta a u = \frac{4}{3}\pi a^3 (\rho - \sigma) g$$

so
$$q = 6\pi\eta a (u + v)\frac{d}{V}$$

substituting for the radius of the drop, a, from **[1]** yields

$$q = \frac{4}{3}\pi\left(\frac{9\eta}{2}\right)^{\frac{3}{2}} \{g(\rho - \sigma)\}^{-1/2} (u + v) u^{1/2} \left(\frac{d}{V}\right) \qquad \textbf{[2]}$$

2.3 Correction to Stokes Law

Unfortunately the above formulae has a flaw. Stokes' Law is not quite correct for droplets of the size used in the experiment. It is true for spheres falling in an infinite extent of a continuous fluid. In the experiment the droplets fall in a fluid confined between the capacitor plates and the fluid is air which is composed of molecules rather than continuous. On the scale of size of the droplets (order 10^{-6}m), Millikan showed experimentally that Stokes Law is modified to;

$$F_d = 6\pi\eta au(1 + b/pa)^{-1}$$

where p is the atmospheric pressure in m of mercury (note this may vary during the course of your experiment, so keep track of it) and *b* is a constant having the value 6.18 x 10^{-8}m^2

Carrying this correction through the theory shows that the value of the charge on the droplet, *q*, is corrected to q_c, where

$$q_c^{2/3} = q^{2/3}(1 + b/pa)^{-1}$$

You may assume the following values for the various physical constants
used in the formulae;

Viscosity of air at 23C, $\eta = 1.81 \times 10^{-5}\ Nsm^{-2}$
density of air σ= *1.29kgm* $^{-3}$
density of the oil ρ = 873.3kgm^{-3}
The spacing of the plates, *d*, is determined as part of the experiment

3. Experimental Details

Millikan Apparatus: schematic diagrams

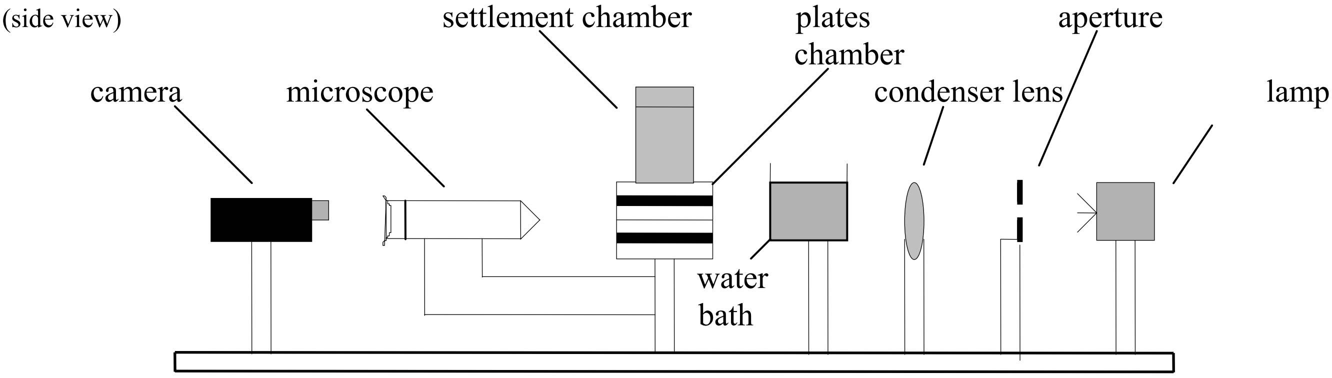


Fig 2

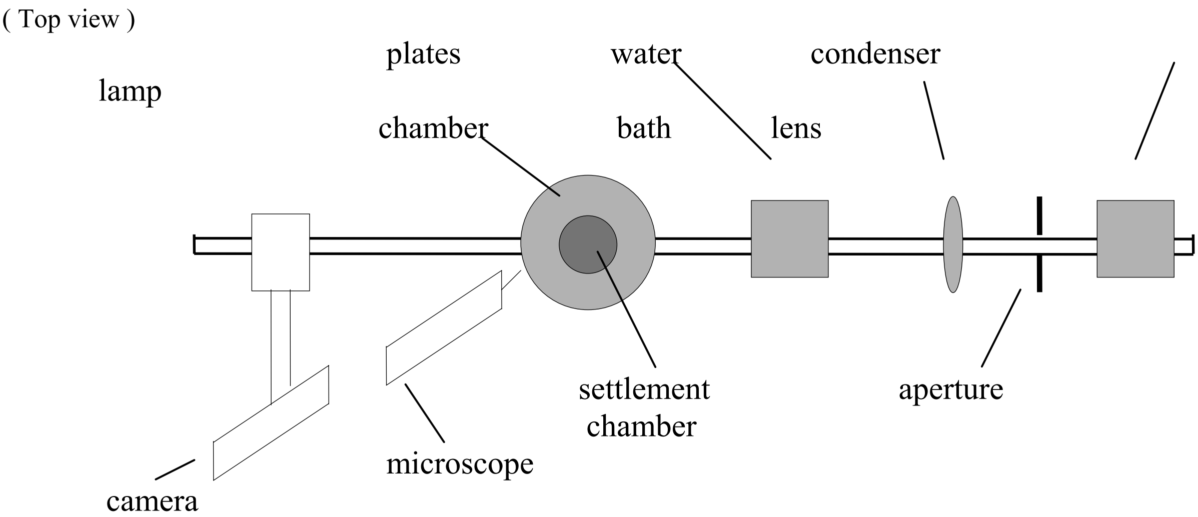


Fig 3

Levelling the capacitor plates: It is important that when the drops are falling vertically they are moving perpendicular to the plates. The equipment should be set up so that this is already so. You can check that it has been set correctly by using the small spirit provided. Correction may be made using the screw feet on the optical bench. There are also levelling screws under the body of the chamber for fine adjustment.

Setting the High Voltage (HV): The high voltage is already connected up to the equipment via a monitor box to which a digital voltmeter is connected. The latter reads the voltage accurately scaled so that 1000V reads as 1V. There is no need for you to do anything to the HV power unit other than switch it on at the begining of the session, it has already been adjusted to deliver a nominal 2500V. Read the precise voltage from the voltmeter.

On no account turn up the high voltage to exceed 2500V. You must not tamper with the equipment in any way which will lead to the high voltage being exposed.

If you suspect something is wrong with the HV supply, then inform the lab technician or a demonstrator.

Choosing suitable drops: Using the atomiser provided, droplets are sprayed into the settlement chamber the top of which can be rotated off to give access. The droplets drift down, through the hole on the top plate, into the space between the plates. **WARNING:** direct only a very short burst from the atomiser into the settlement chamber. If you produce too many they will swamp the field of view and make it difficult to pick out one to observe. In the spray of droplets produced by the atomiser are a few which are charged by a friction process and which respond when the voltage is switched on. You will find that there are few of these in the first droplets which come into view, indeed most of the initial burst are so large that they fall far too quickly to measure. Wait for these to clear. Obviously charged droplets are more numerous amongst the small droplets which arrive later so be patient, you could wait up to 10 minutes before finding charged drops. The short burst from the atomiser will produce a continuous stream of falling droplets which lasts for a couple of hours. As soon as you have injected the droplets with the atomiser, make sure you rotate the top of the settlement chamber to the closed position before observing the droplets. Failure to do so will mean that their smooth motion will be upset by draughts.

> ***Warning:* The falling droplets are particularly sensitive to vibrations conducted via the optical bench. Make sure that you do not knock or even tap the optical bench or components on it. If you do so the drops will be dispersed and lost from view.**

Illumination and viewing of the droplets: The droplets between the plates are illuminated (Fig 2) by a light source/aperture/condenser lens system which you will find set up on the optical bench. The components have been placed to focus light on the droplets and should not need adjustment. If, however, you find the illumination to be poor, move the lens by small amounts until you see an improvement. The droplets are viewed via scattered light (Fig 3) by a microscope offset from the beam in angle to avoid the glare of the direct beam. Don't rotate it to look directly down the beam, this is uncomfortable and could damage your eye. For less stressful operation measurements are made by viewing the droplets through the microscope with a TV camera/monitor set up. When you start the experiment the optics should have been left so that on switching on the lamp, TV camera etc you should be able to see the droplets immediately. However, it is advisable to make the following adjustments to make sure the measurement graticule in the microscope and the droplets are both brightly illuminated and in optimal focus. Move the eyepiece of the microscope in or out to bring the graticule into focus on the TV screen. Focus the image of the droplets on the graticule scale by movement of the microscope forward or backward using the adjustment knob on the side. Optimise the illumination of both droplets and graticule scale by moving the microscope around in angle order to see the drops and scale clearly without undue glare from the direct beam. The body of the microscope can be rotated so that the droplets fall parallel to the scale. The line of fall of the droplets may be made to coincide with the scale by making small adjustments of the microscope angle by means of the fine adjustment knob on the side of the microscope mount.

Determining the speed of the droplets: To measure the speed of fall or rise of a droplet it is timed passing between two convenient marks on the microscope graticule scale using a stopwatch. Pick out a suitable droplet on which to make measurements. A good droplet will be one which takes at least 30 secs to fall over 3 or 4 of the large graticule spacings when no voltage is applied. Of these the ones which rise the most slowly when the voltage is applied will carry the smallest charge. Thus if you want to maximise your chances of picking up droplets carrying one, two or three elctron charges make most of your timings on the slowest rising droplets. However, until you are thoroughly familiar with choosing droplets, time any you come across apart from the fastest and very slowest. By switching the voltage on and off at the HV unit make the droplet rise and fall between the timing marks and make several measurements of the rise and fall times. Note that the precision on any individual timing is limited not only by the usual observer uncertainty in pressing the stopwatch but by the effects of Brownian motion on the droplet. The droplet is so small that its motion is affected by the constant bombardment of molecules of air. This has a bigger effect than the usual stopwatch uncertainty and for this reason several (at least 5 and preferably 10) measurements of the fall and rise are necessary to get sufficient precision. You will probably need to follow the rise and fall of one or two droplets to get the feel of the equipment before making serious measurements.

NOTE: Taking repeated timings using a stopwatch is actually quite difficult to do and quite tedious. You make this aspect easier the computer with the experiment has a program, Milikan4 installed. If you run it you can use the computer as a stopwatch and the timings will be recorded automatically in a file. The file can afterwards be read into Excel (or another analysis program) for calculations to be done.

Calibration of the distance of fall: To turn the timing into speeds of rise and fall, the distance over which the droplets have been timed must be determined. This is not simply a matter of reading off the number of graticule divisions over which the droplet falls. This measures distances in the image plane of the microscope and the droplets fall in the object space. Distances in the two spaces are related by the magnification. The graticule may be calibrated to measure distances in the object space by focusing on the O.5mm grid provided. To effect this the Millikan chamber is disconnected from the HV (**make sure it is off at the HV unit**!!) and gently lifted off its mount. The calibration grid holder locates directly on the mount so that the grid is in approximately the same plane as the droplets. A small adjustment of the microscope focus will bring it sharply into view. Note, the precision of all your determinations of the electronic charge depends on this calibration and so it is worth doing well. Make a number of independent determinations of it. To do this honestly, adjust the height of the grid between determinations so that it falls into different positions relative to the graticule, refocussing on the grid each time. Make measurements of the positions of both the timing marks relative to the grid interpolating between marks where necessary.

After the calibration replace the Millikan chamber and reconnect the HV and make sure the optics are sufficiently well ajusted that the microscope graticule and droplets can be seen.

Measuring the distance between the plates: This cannot be done directly. Instead a set of plate spacers manufactured from the same sample of PTFE to the same tolerance as the set used in the Millikan chamber is provided. Measurement of the thickness of the material of these in several places will yield an average which is a high precision estimate of the plate spacing.

Analysis: Measurements on at least 25 droplets is necessary for a convincing demonstration of the quantised nature of charge and thus the same calculation will have to be repeated many times. A spreadsheet programme is well suited to this type of analysis. Use Excel to perform your calculations of the charge carried by the droplets and to make a frequency histogram of the charges to look for evidence of quantisation. Estimate the size of the quantum of charge.

NOTE: You will find it advantageous to write the analysis spreadsheet at the outset since this will faciltate the analysis of preliminary data.

NTS Sept 04, JHB Sept 99, April 00, Aug 01