Illustration of Rutherford scattering (e.g. for an attractive Coulomb potential):

\[ \varepsilon_{\infty} := 8.85 \times 10^{-12} \quad q_1 := 2 \times 10^{-4} \quad q_2 := 3 \times 10^{-4} \quad \text{Particle charges} \]

\[ k := \frac{1}{4\pi\varepsilon_0} q_1 q_2 \quad k = 539.508 \quad \text{Coulomb force constant} \]

\[ v := 3 \times 10^4 \quad \text{Initial velocity, at infinite distance} \quad \mu := 0.0001 \quad \text{Reduced mass (all coordinates are in the center-of-mass system)} \]

\[ E := \frac{1}{2} \mu \cdot v^2 \quad L(b) := b \cdot \mu \cdot v \quad \text{Conserved energy and angular momentum} \]

\[ \alpha(b) := \frac{L(b)^2}{\mu \cdot k} \quad \varepsilon(b) := \sqrt{1 + \frac{2E \cdot L(b)^2}{\mu \cdot k^2}} \quad \psi(b) := \cos \left( \frac{1}{\varepsilon(b)} \right) \]

\[ \tau(\theta, b) := \text{if} \left( \theta < \pi + 2 \cdot \psi(b) \land \theta > \pi, 10^6, \frac{\alpha(b)}{1 + \varepsilon(b) \cdot \cos(\theta - \psi(b))} \right) \quad \text{Trajectory or orbit} \]

\[ \Theta(b) := \pi - 2 \cdot \psi(b) \quad \text{Scattering angle as a function of impact parameter} \]
Monte Carlo illustration of Rutherford scattering:

\[ N := 1000000 \quad b_{\text{max}} := 1.5 \quad \text{Number of trials and the maximum impact parameter} \]

\[ y := \text{runif}(N, 0, 1) \quad \text{Generate a list of } N \text{ random numbers uniform from 0 to 1.} \]

\[ b := b_{\text{max}} \sqrt{y} \quad \text{Taking the square root of the uniform random numbers give us a list of random numbers distributed linearly in } b \text{ (i.e. corresponding to equal areas).} \]

\[ N_b := 1000 \quad \text{Number of histogram bins.} \]

\[ H_b := \text{histogram}(N_b, b) \quad \text{Make a histogram (frequency distribution) of impact parameters} \]

\[ \varphi := \text{runif}(N, 0, 2 \pi) \quad \text{Distribute the initial phi angle uniformly over } 2 \pi \]

\[ i := 0 \ldots \frac{N}{100} \quad x_{p_i} := b \cdot \cos(\varphi_i) \quad y_{p_i} := b \cdot \sin(\varphi_i) \quad \text{Cartesian coordinates} \]

Distribution of starting points, which should be distributed uniformly over a disc of radius \( b_{\text{max}} \). For efficiency, only 1% of the points are plotted.
\[ \theta_s := \Theta(b) \] Calculate the scattering angle for each impact parameter

\[ H := \text{histogram}(N_b, \theta_s) \] Make a histogram (frequency distribution) of scattering angles

Notes that the minimum scattering angle in this plot corresponds to the maximum impact parameter chosen above. The large scattering angles correspond to very small impact parameters, which are rare.
Compare the Monte Carlo simulation with the Rutherford scattering cross section

\[ \Theta(b_{\text{max}}) = 7.993 \times 10^{-3} \]

Minimum scattering angle, corresponding to the maximum impact parameter.

\[ \sigma(\theta) := \frac{k^2}{(4 \cdot E)^2} \cdot \frac{1}{\sin\left(\frac{\theta}{2}\right)^4} \]

Rutherford differential scattering cross section.

\[ I := 2\pi \int_{\Theta(b_{\text{max}})}^{\pi} \sigma(\theta) \cdot \sin(\theta) \, d\theta \]

Integral of the cross section from minimum to maximum angle in order to normalize it.

\[ dN(\theta) := 2\pi \frac{N}{I} \cdot \sigma(\theta) \cdot \sin(\theta) \]

Normalized theoretical distribution of the scattering angles.

\[ n := 0 \ldots N_b - 1 \quad \text{bw} := \left(\frac{H(\phi)}{n} - \left(\frac{H(\phi)}{n}\right)_0\right) = 2.564 \times 10^{-3} \]

Bin width in the histogram.

\[ \sum_n y_n = 9.757 \times 10^5 \]

Check that the total number of predicted scatters adds up close to \(N\). (This will come out systematically a bit low, as the distribution is evaluated at the bin center, where the value is a bit below the average for the bin.)