Damped Oscillator with Square Wave Forcing Function

\[ n := 1, 3 \ldots 9 \]  \quad T := 2 \cdot \pi \]  \quad T is the period of the driving function

\[ b_n := \frac{4}{n \cdot \pi} \]  \quad \omega_n := n \cdot \frac{2 \cdot \pi}{T} \]  \quad Frequency harmonics 1, 3, 5, etc

\[ F_a(t) := \begin{cases} -1 & \text{if } t \geq -\pi \land t \leq 0 \\ \text{otherwise} & \\ 1 & \text{if } t > 0 \land t \leq \pi \\ 0 & \text{otherwise} \end{cases} \]  \quad Exact square wave driving function, defined here over a single period of T=2\pi.

\[ F(t) := \sum_{n} \left( b_n \cdot \sin\left(\omega_n \cdot t\right) \right) \]  \quad Square wave driving function (truncated Fourier series). Note that the constant term and cosine terms are zero, as F is an odd function.

\[ \omega_0 := 1.5 \]  \quad \beta := .5 \]  \quad \text{m} := 1 \quad \text{Oscillator parameters}

\[ \delta_n := \text{atan} \left[ \omega_0^2 - \left( \omega_n \right)^2, 2 \cdot \omega_n \cdot \beta \right] \]  \quad Phase of the response for each harmonic

\[ D_n := \frac{b_n}{\sqrt{\left[ \omega_0^2 - \left( \omega_n \right)^2 \right]^2 + 4 \left( \omega_n \right)^2 \cdot \beta^2}} \]  \quad Amplitude of the response for each harmonic

\[ s(n, t) := D_n \cdot \sin\left(\omega_n \cdot t - \delta_n\right) \]  \quad Fourier components of the solution

\[ x(t) := \frac{1}{m} \sum_{n} (s(n, t)) \]  \quad Solution for the steady-state response

Input Square Wave
Input Square Wave Fourier Components

\[ b_1 \sin(\omega_1 t) \cdot b_3 \sin(\omega_3 t) \cdot b_5 \sin(\omega_5 t) \cdot b_7 \sin(\omega_7 t) \cdot b_9 \sin(\omega_9 t) \cdot F_a(t) \]

Fourier Amplitude of the Response

\[ D_n \]
Oscillator Steady-State Response

\[ x(t) \]

\[ \frac{t}{\pi} \]

Oscillator Response Fourier Components

\[ s_{1}(t), s_{3}(t), s_{5}(t), s_{7}(t) \]
Damped Oscillator with Saw-Tooth Wave Forcing Function

\[ n := 1, 2, \ldots, 9 \quad T := 2\pi \quad \text{T is the period of the driving function} \]

\[ b_n := \frac{(-1)^{n+1}}{n\cdot\pi} \quad \omega_n := \frac{n\cdot 2\cdot\pi}{T} \quad \text{Frequency harmonics 1, 3, 5, etc} \]

\[ F_a(t) := \begin{cases} \frac{t}{2\pi} & \text{if } t \geq -\pi \land t \leq \pi \\ 0 & \text{otherwise} \end{cases} \quad \text{Exact saw tooth driving function, defined here over a single period of T=2\pi.} \]

\[ F(t) := \sum_n \left( b_n \cdot \sin\left( \omega_n \cdot t \right) \right) \quad \text{Square wave driving function (truncated Fourier series). Note that the constant term and cosine terms are zero, as F is an odd function.} \]

\[ \omega_0 := 2.5 \quad \beta := .5 \quad m := 1 \quad \text{Oscillator parameters} \]

\[ \delta_n := \text{atan2}\left( \frac{\omega_0^2 - \left( \omega_n \right)^2 - 2\cdot\omega_n\cdot\beta}{\left( \omega_n \right)^2 - \beta^2} \right) \quad \text{Phase of the response for each harmonic} \]

\[ D_n := \frac{b_n}{\sqrt{\left[ \omega_0^2 - \left( \omega_n \right)^2 \right]^2 + 4\left( \omega_n \right)^2\cdot\beta^2}} \quad \text{Amplitude of the response for each harmonic} \]

\[ s(n, t) := D_n \cdot \sin\left( \omega_n \cdot t - \delta_n \right) \quad \text{Fourier components of the solution} \]

\[ x(t) := \frac{1}{m} \sum_n \left( s(n, t) \right) \quad \text{Solution for the steady-state response} \]

Input Saw Tooth
Input Saw Tooth Fourier Components

$\sin(\omega_1 t) b_1 \quad \sin(\omega_2 t) b_2 \quad \sin(\omega_3 t) b_3 \quad \sin(\omega_4 t) b_4 \quad \sin(\omega_5 t) b_5 \quad F_a(t)$

Fourier Amplitude of the Response

$D_n$
Oscillator Steady-State Response

Oscillator Response Fourier Components