Homework Assignment #1
Due Friday, October 2 by 4:00 pm.

Submitted solutions must be done neatly, following the guidelines on the back of this sheet.
Nontrivial indefinite integrals needed for these problems can be found in Appendix E of the course textbook! In your solutions, you must note clearly at each step which integration formula was used.

For projectile problems, before starting your calculation consider carefully whether you are being asked to solve for a time, a distance, or a velocity, so that you know what variables need to be eliminated versus solved for. In some (but by no means all!) cases it is useful to eliminate time from Newton’s second law by using the chain rule:
\[
\frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = v \frac{dv}{dx},
\]

1. A projectile is fired with initial speed \( v_0 \) and angle \( \theta \) (from the horizontal) from the top of a cliff of height \( h \) above a level valley. Assuming zero air resistance and a constant gravitational field, calculate the following:
   a) The time at which the projectile strikes the ground in the valley below the cliff.
   b) The horizontal range of the projectile.
   c) The time and range for the specific case \( v_0 = 300 \) m/s, \( \theta = 40^\circ \), \( h = 50 \) m.

2. A boat with initial speed \( v_0 \) is slowed by a frictional force \( F = -bv^a \). Calculate \( v(t) \) and \( x(t) \). How much time will it take for the boat to stop, and what distance will it travel before stopping?

3. Consider a projectile fired vertically in a constant gravitational field. Find as a function of the initial velocity the time required for the projectile to reach its maximum height, for two cases: (a) zero resistance and (b) resistance proportional to the instantaneous velocity. Use the Taylor-series expansion for \( \ln(1+z) \) to show that the result for Case (b) reduces to the result for Case (a) when the resistance goes to zero. What is the lowest order correction to the time? Is the time larger or smaller than the zero-resistance case, for a given starting velocity?

4. A particle of mass \( m \) slides down an inclined plane under the influence of gravity. If the motion is resisted by a force of magnitude \( kmv^2 \), show that the time required to move a distance \( d \) after starting from rest is
   \[
t = \frac{\cosh^{-1}(e^{kd})}{\sqrt{kg \sin \theta}},
\]
   where \( \theta \) is the angle of inclination of the plane.

5. Show directly that the time rate of change of the angular momentum about the origin for a projectile fired from the origin, with initial speed \( v_0 \) and elevation angle \( \alpha \), is equal to the torque about the origin. Note that there is no air resistance in this problem, and the motion takes place in a constant gravitational field. Note that I am not asking you to prove the general relationship 2.83 (which is already done in the textbook). Instead, you are to calculate both the left and right-hand sides of that equation for this specific example, to show that they are equal at every instant in time.

6. Find which of the following forces are conservative by calculating in each case the curl. If conservative, find a suitable potential energy function \( U(x, y, z) \). Note that the potential energy function can always be found rigorously by calculating a line integral from an arbitrary constant point (such as the origin or a point at infinity) to the point \( (x, y, z) \). The path, remember, is arbitrary, so choose one that is simple to integrate. It is a good idea to check that the negative gradient of your solution gives back the original force.
   a) \( F_x = -ze^{-x} \quad F_y = \ln z \quad F_z = e^{-x} + \frac{y}{z} \)
   b) \( F_x = ayz + bx + c \quad F_y = axz + bz \quad F_z = axy + by \)
   c) \( F_x = 2xyz \quad F_y = z(x^2 - 3y^2) - 6y^2z \quad F_z = y(x^2 - 3y^2) \)
   d) \( F_x = -\sin(yz) \quad F_y = -xz \cos(yz) \quad F_z = -z \cos(yz) \)
Guidelines for Presentation of Homework Solutions

Grading homework is never an easy task, so please help our graders by making your solutions neat and clear. It is to your advantage to do so, as well, because the amount of feedback and credit that the grader can give to you will depend on whether she or he can follow your work. Here are some points to keep in mind while writing your solutions. The graders will be instructed that they should take off points, or even not grade your work at all, if you do not follow reasonably well these guidelines. Full credit can be assured only if your solution is correct and your presentation follows reasonably well these guidelines.

1. Use standard unfolded 8½ by 11 inch paper, preferably not torn out of a spiral-bound notebook. If you do use spiral-bound paper, then please remove all of the little torn pieces along the edge after you rip it out. Recycle paper printed on only one side is okay.
2. Please staple together all of your pages. If not possible, then be sure to write your name on each page.
3. Print your name and the number of the assignment at the top of the first page.
4. Do not use red ink or red pencil!
5. Present your solutions in the order that the problems are assigned. Number them as in the assignment. The grader should not be expected to hunt through your pages for randomly ordered or unlabeled problems.
6. Each solution to a problem or answer to a question should begin at the left margin of the paper. In other words, do not work in multiple columns. Your work should flow neatly from left to right and top to bottom.
7. We do not want to see your scratch work, doodles, false starts, etc.
8. However, your presentation of the solution must show each step of your work, except those that you can do entirely in your head.
9. Some notes explaining what you are doing, when not obvious, are always appreciated and often necessary in order to make sense of your work.
10. Dimensional results must be presented along with the correct units and correct number of significant figures (if in doubt, one extra digit is fine).
11. If a graph or plot is requested, then it should be done to proper scale on graph paper (computer generated graphs, such as those plotted by Mathematica, are even better). The graph should be drawn such that it fills most of the page. Each axis should be clearly labeled with the values along the axis, the name of the variable, and the units. Points should be plotted to the full accuracy possible with the graph paper, but curves may be interpolated freehand, or with a French curve, between points. If only a sketch is requested, then graph paper is not required.
12. If you use a computer program, such as Mathematica, to check your work or help you with the algebra, you still have to show each step of the algebra in your solution. Mathematica printout will not be accepted as a replacement for your algebra.
13. Indefinite integrals can be done by referring to a standard table of integrals (e.g. Appendix E of the textbook) or by a symbolic computer program. But if you do so, then you must state clearly from where you obtained the integral.