1. Single-particle solutions for an initial-value problem can always be generated numerically in a very straightforward way, as long as a model for the forces exists. This problem illustrates using a spreadsheet (either Open-Office Calc or MS Excel) to solve a projectile problem by numerical integration. Assume a linear model for the air resistance: \( \vec{F}/m = -kv \). This particular problem can be integrated analytically, but the important point is that the numerical recipe given here will work just as easily for any model of the air resistance. I demonstrated this during lecture #2.

Use the first row of the spreadsheet to label columns for \( t \), \( x \), \( y \), \( v_x \), \( v_y \), \( a_x \), and \( a_y \). Further to the right somewhere, say in column K, arrange the constants needed in the calculation. For example, put \( v_0 \) in K2, the initial elevation angle \( \theta \) in degrees in K3, \( g = -9.8 \) in K4, \( k \) in K5, and the time step \( \delta t \) in K6. Then, enter the initial values into the second row. That is, fill in zero for the initial position in both coordinates, and type “\( =K2*cos(K3*PI()/180) \)” into D2 (\( x_v \) column) for the initial velocity in \( x \), and similarly into E2 (\( y_v \) column) for the initial velocity in \( y \). Enter the force model (i.e. acceleration \( \vec{a} = \vec{F}/m \)) into the acceleration columns (cells F2 and G2):

\[
\begin{align*}
F_x &= -K5*D2 \\
F_y &= -K5*E2+K4
\end{align*}
\]

The dollar signs are inserted so that the row number referenced for the constant (\( k \) or \( g \)) will not change when the formulas are dragged later on to copy them into the succeeding rows. Then, in row 3 enter formulas to calculate the corresponding values for \( x \), \( y \), \( v_x \), \( v_y \) after the first time step, assuming that the acceleration doesn’t change significantly over the short time step:

\[
\begin{align*}
A_x &= A2+K6 \\
A_y &= B2+D2*K6+0.5*F2*K6^2 \\
A_x &= D2+F2*K6
\end{align*}
\]

You can figure out cells C3 and E3. For cells F3 and G3, just select cells F2 and G2 with the cursor, and then drag them down one row by holding onto the little square at the lower right corner of the selected region. To complete the integration, select cells A3 through G3 and drag them down to fill in about 250 rows. Try it out by reproducing the curves in Fig. 2-8 of the text, using a time step of 0.5 seconds. Plot the trajectories by doing the following. Select the columns of \( x \) and \( y \). Go to the Insert menu and click on Chart. Then select the type “XY (Scatter)”. Locate the chart somewhat below the value for \( \delta t \). The plot will update whenever you change any of the input parameters. Print and submit for grading just the first page for each of the cases \( k = 0 \) and \( k = 0.01 \). You can find on the web site an example print-out, in case the above recipe isn’t sufficiently clear.

2. Derive Eqn. 3.37 from Eqn. 3.34 by making the substitution \( x(t) = e^{rt} \) and solving for the two roots \( r_1 \) and \( r_2 \). Then, derive Eqn. 3.40 for the case that the roots are complex. What are \( A_1 \) and \( A_2 \) (of Eqn. 3.39) in terms of \( A \) and \( \delta \)?
3. A mass of 0.1 kg is attached to a spring having a spring constant of 10 N/m. The mass is displaced from the equilibrium point by 0.030 m and released.
   a) Calculate the initial total energy.
   b) Calculate the natural frequency and period (assuming zero damping).
   c) Calculate the amplitude of the motion (maximum displacement) and the maximum velocity (assuming zero damping).
   d) Suppose that in 10 seconds the energy is reduced to one half of its initial value. Calculate the approximate damping constant $\beta$, the frequency (compare with the undamped frequency), and the decrement of the motion.
   e) Plot the displacement versus time for both the undamped and damped motion. Excel or Open-Office Calc can do the job. In that case, make a column of time $t$ and two columns for $x(t)$ (one undamped, the other damped). Enter the formulas into the $x(t)$ columns in such a way that they can be dragged downward to fill many rows. Then use “Insert Chart” to plot the columns versus time.

4. The potential energy for the force between two atoms in a diatomic molecule has the form
   \[ V(x) = -\frac{a}{x^6} + \frac{b}{x^{12}} \]
   where $x$ is the distance between atoms and $a, b$ are positive constants.
   a) Find the force.
   b) Assuming that one of the atoms is very heavy and remains at rest while the other moves along a straight line, describe the possible motions.
   c) Find the equilibrium distance and the period of small oscillations about the equilibrium position if the mass of the lighter atom is $m$.

5. Verify by substitution into the differential equation that Eqn. 3.43 is a solution to Eqn. 3.35 for the critically damped case, where $\omega_0^2 = \beta^2$.

6. Let the initial position and speed of an overdamped, nondriven oscillator be $x_0$ and $v_0$, respectively.
   a) Show that the amplitudes of the solution (Eqn. 3.44) are given by
      \[ A_1 = \frac{\beta_2 x_0 + v_0}{\beta_2 - \beta_1} \quad \text{and} \quad A_2 = -\frac{\beta_1 x_0 + v_0}{\beta_2 - \beta_1} \]
      where $\beta_1 = \beta - \omega_2$ and $\beta_2 = \beta + \omega_2$.
   b) Show that when $A_1 = 0$, the phase paths of Figure 3-11 must be along the dashed curve given by $\dot{x} = -\beta_2 x$, otherwise the asymptotic paths are along the other dashed curve given by $\dot{x} = -\beta_1 x$. *Hint:* Note that $\beta_2 > \beta_1$ and find the asymptotic paths when $t \to \infty$.

7. Equation 3.53 can be rewritten with a complex forcing function, $\dot{x} + 2\beta \dot{x} + \omega_0^2 x = Ae^{iot}$. This form is equivalent, because the real part is exactly Eqn. 3.53 and because the equation is linear, such that the real and imaginary parts never get mixed together. Substitute the form $x_p(t) = De^{(i\omega t - \delta)}$ (note that Eqn. 3.55 is the real part of this) and solve for $D$ and $\tan \delta$ to derive the results given in Eqns. 3.59 and 3.57. Hopefully it will be apparent that with some knowledge of the algebra of complex numbers (which you should know well by now) this calculation is easier than the one given in the book, which relies on trig identities. This is a general and common method for simplifying work with linear differential equations.