Topographic Map

An object will roll \( \perp \) to the lines

Flat terrain

Steep terrain

From one line to the next is a change in elevation of 20 feet.
Vector Field Example, for a conservative field

\[i := 1, 2 \ldots 20\]
\[x_i := \frac{i}{10}\]

\[j := 1, 2 \ldots 20\]
\[y_j := \frac{j}{10}\]

\[F_x(x, y) := y\]
\[F_y(x, y) := x\]

\[M_{i,j} := F_x(x_i, y_j)\]
\[N_{i,j} := F_y(x_i, y_j)\]
Scalar function for this field (F is the gradient of this function)

\[ g(x, y) := x \cdot y \]

\[ O_{i, j} := g(x_i, y_j) \]
Stationary or Critical Points of a 2D Function

\[ f(x, y) := x^4 - 3x^2 + \frac{x \cdot y}{2} - 2y^3 + 3y \]  

Example Function of 2 variables.

\[ N := 100 \]

\[ i := 0, 1 .. N \]

\[ x_i := \left(i - \frac{N}{2}\right) \frac{4}{N} \]

Array of points to plot the function from -2 to +2 in both x and y.

\[ j := 0, 1 .. N \]

\[ y_j := \left(j - \frac{N}{2}\right) \frac{4}{N} \]

\[ M_{i,j} := f(x_i, y_j) \]
Find the critical points by searching for \(x,y\) pairs that make BOTH components of the gradient zero.

Calculate the gradient and the 2nd-derivative matrix:

\[
\begin{align*}
\text{grad}_x(x,y) & := 4x^3 - 6x + \frac{y}{2} \\
\text{grad}_y(x,y) & := \frac{x}{2} - 6y^2 + 3
\end{align*}
\]

\[
J(x,y) := \begin{pmatrix} \frac{12}{2} & \frac{1}{2} \\
\frac{1}{2} & -12y \end{pmatrix}
\]

The Hessian determinant is the determinant of \(J\)

\[
x := -1 \quad y := 1 \quad \text{Initial guesses for } x \text{ and } y
\]

Given

\[
\begin{align*}
\text{grad}_x(x,y) & = 0 \\
\text{grad}_y(x,y) & = 0
\end{align*}
\]

\[
\text{vec} := \text{Find}(x,y) \quad \text{vec} = \begin{pmatrix} -1.25 \\
0.629 \end{pmatrix} \quad \text{Solution vector for } x,y
\]

\[
J(\text{vec}_0, \text{vec}_1)_{0,0} = 12.755 \quad x,x \text{ component of } J
\]

\[
|J(\text{vec}_0, \text{vec}_1)| = -96.546 \quad \text{Determinant of } J
\]

\[
f(\text{vec}_0, \text{vec}_1) = -1.25 \quad \text{Function value at this critical point}
\]

Mathcad labels the axes with the indices, not the values of \(x\) and \(y\)