Quiz #5 Solution — Physics 110A

1) $$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\vec{P}(\vec{r}\,')\cdot(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\,d\tau'$$

$$= \frac{1}{4\pi\varepsilon_0}\int \vec{P}\cdot\nabla'\frac{1}{|\vec{r}-\vec{r}\,'|}\,d\tau'$$

$$= \frac{1}{4\pi\varepsilon_0}\int \nabla'\cdot\left(\frac{\vec{P}}{|\vec{r}-\vec{r}\,'|}\right)d\tau' - \frac{1}{4\pi\varepsilon_0}\int(\nabla'\cdot\vec{P})\frac{1}{|\vec{r}-\vec{r}\,'|}\,d\tau'$$

$$= \frac{1}{4\pi\varepsilon_0}\oint_S \frac{\vec{P}\cdot\hat{n}}{|\vec{r}-\vec{r}\,'|}\,da' + \frac{1}{4\pi\varepsilon_0}\int_V \frac{-\nabla'\cdot\vec{P}}{|\vec{r}-\vec{r}\,'|}\,d\tau'$$

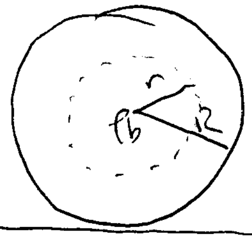
$$= \frac{1}{4\pi\varepsilon_0}\oint_S \frac{\sigma_b\,da'}{|\vec{r}-\vec{r}\,'|} + \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho_b}{|\vec{r}-\vec{r}\,'|}\,d\tau'$$

$$\sigma_b = \vec{P}\cdot\hat{n} \qquad \rho_b = -\nabla\cdot\vec{P}$$

2) a) $$\rho_b = -\nabla\cdot\vec{P} = -\nabla\cdot(k\vec{r}) = -k\left(\frac{\partial x}{\partial x}+\frac{\partial y}{\partial y}+\frac{\partial z}{\partial z}\right) = -3k$$

$$\sigma_b = \vec{P}\cdot\hat{n} = +kR$$

b)



$$\oint E\cdot da = \frac{1}{\varepsilon_0}\int \rho_b\,d\tau$$

$$E_r 4\pi r^2 = \frac{1}{\varepsilon_0}(-3k)\frac{4}{3}\pi r^3$$

$$E_r = \frac{1}{4\pi\varepsilon_0}\frac{4\pi}{3}(-3k)r$$

$$E_r = \frac{-k}{\varepsilon_0}r \qquad \boxed{\vec{E} = -\frac{k}{\varepsilon_0}\vec{r}}$$

Alternate: since $\nabla\times\vec{P}=0$

$\rho_f = 0 \Rightarrow \vec{D}=0=\varepsilon_0\vec{E}+\vec{P}$

$\Rightarrow \vec{E} = -\frac{1}{\varepsilon_0}\vec{P} = \frac{-k\vec{r}}{\varepsilon_0}$