Homework Assignment #5
Quiz in class on Friday, February 9

Submit only problems 3, 4, 7, and 9 for grading.

1. I worked out part (a) of this problem in lecture. Derive the formula for the expansion in Legendre polynomials of the potential $V(r, \theta)$ of a point charge $q$ located at $\vec{r}' = r'\hat{z}$. The potential must take the form given in Eqn. 3.65. Find the values of $A_i$ and $B_i$ by expanding the Coulomb potential $\theta \pi \varepsilon_0 \cos^2 \frac{1}{4} \left( \frac{r}{r} - 2rr' \cos \theta \right)$ into a power series for the special case $\theta = 0$ (i.e. along the $z$ axis) and matching term by term with the general form of Eqn. 3.65.
   a. Do the derivation first for the case $r > r'$. The result should agree with Eqn. 3.95 for the case where the charge density is a delta function of amplitude $q$ at location $\vec{r}'$.
   b. Then do the derivation for the case $r < r'$.


4. Problem 3.45 on Page 158, parts (a), (b), and (c) only. I did part (a) in lecture. In part (b), also write out $V_{\text{quad}}$ with the double sum fully expanded.

5. Problems 4.7 and 4.8, Page 165. Dipole interaction energy. Problem 4.7 is done in any introductory textbook.

6. Write out in detail the derivation starting from Equation 4.8 and concluding with Equation 4.13. Be sure to understand what “integration by parts” here means in the context of vectors in a 3-dimensional space.

7. Problem 4.10 on Page 169. Radial polarized sphere. Make use of the spherical symmetry and use Gauss’ law in integral form where appropriate (i.e. don’t do this simple problem the hard way).

8. Problem 4.13 on Page 173. A cylinder with a uniform polarization perpendicular to the axis. The solution should be very similar to Example 4.2, the analogous problem in spherical coordinates. Use the form of solution to Laplace’s equation that you derived in Problem 3.23. You don’t need to go through a lot of formalism with “Fourier’s” trick and so forth, however, because you should find that the form of the bound charge matches one of the eigenfunctions in the general expansion, resulting in a very simple form for the potential.

9. Problem 4.15 on Page 177. A spherical shell with a frozen polarization. Again, the spherical symmetry should make this calculation easy.