Quiz #2
January 19, 2007

1. A circular loop of radius $a$ is charged uniformly with linear charge density $\lambda$. The loop lies in the $x,y$ plane centered on the origin (such that the $z$ axis is the symmetry axis). Starting from the general expression for the electric potential due to a linear charge density

$$ V(\vec{r}) = \frac{1}{4\pi \varepsilon_0} \int \frac{\lambda(\vec{r}') d\ell'}{C |\vec{r} - \vec{r}'|} $$

write an expression for the potential in spherical polar coordinates, $V(r,\theta)$, as an integral over an angle $\phi$:

$$ V(r,\theta) = \frac{\lambda}{4\pi \varepsilon_0} \frac{2\pi}{0} (\text{something}) d\phi $$

2. Use Gauss’ law to find the electric field as a function of radius inside of a uniform sphere of charge with radius $R$ and total charge $Q$.

3. Demonstrate explicitly that

$$ V(\vec{r}) = \frac{1}{4\pi \varepsilon_0} \int \frac{\rho(\vec{r}') d\tau'}{|\vec{r} - \vec{r}'|} $$

satisfies Poisson’s equation $\nabla^2 V = -\frac{1}{\varepsilon_0} \rho$. 