Quiz #3
January 26, 2007

1. The electrostatic potential energy of a charge distribution can be calculated from

\[ U = \frac{1}{2} \int \rho V \, d\tau \]

a) Starting with this expression, show that an equivalent expression is

\[ U = \frac{\varepsilon_0}{2} \left[ \int_{V} \frac{|\vec{E}|^2}{2} \, d\tau + \int_{S} \vec{E} \cdot \, d\vec{a} \right] \]

where \( V \) is any volume that encloses all of the charge, and \( S \) is the surface of that volume. Be sure to use proper vector notation. You may make use of the vector identities inside the front cover of the textbook.

b) Explain why the surface integral becomes zero if the volume is taken to be all of space.

2. A sphere of radius \( R \) carries a charge density \( \rho(r) = kr \) (where \( k \) is a constant). Starting from Gauss’ law and using symmetry, find the electric field inside the sphere.

Gauss’ law in two forms:

\[ \nabla \cdot \vec{E} = \frac{1}{\varepsilon_0} \rho \]

and

\[ \oint_{S} \vec{E} \cdot \, d\vec{a} = \frac{1}{\varepsilon_0} \int_{V} \rho \, d\tau \]