Physics 160
Lecture 2

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Input Impedance vs Output Impedance

- *The concepts of **input and output impedance** are two of the most fundamental and important that you must come to understand in this course!*

$$Z_{in} = \frac{\Delta V_{in}}{\Delta I_{in}}$$

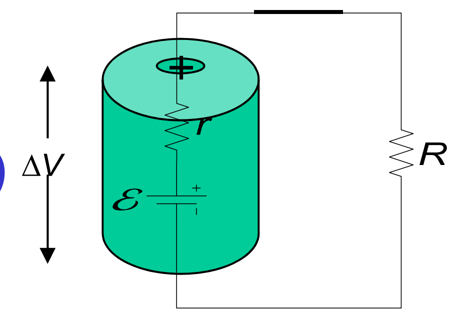
If Z_{in} is very large, then we can change V_{in} easily while supplying very little current (necessary if the signal source, for example, is a microphone).

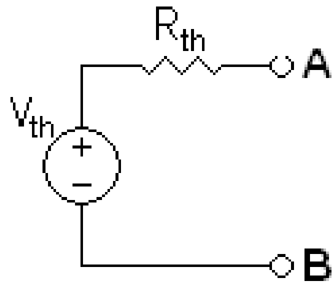
$$Z_{out} = \frac{\Delta V_{out}}{\Delta I_{out}}$$

If Z_{out} is very small, then changes in the "load" R will have little effect on V_{out}.

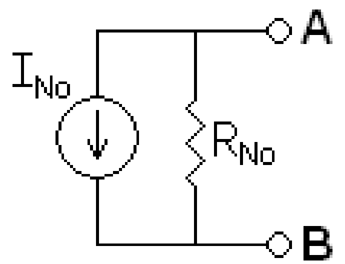
Voltage and Current Sources (DC examples)

- *Voltage source*
  - *Ideal: zero internal resistance (output impedance)*
  - *Real: EMF in series with an internal resistance*
  - *Desire: minimum output impedance R_{th}*
    - *Change in current (due to load change) results in little change in voltage*
  - *Non-ideal example: a good, well-charged battery*





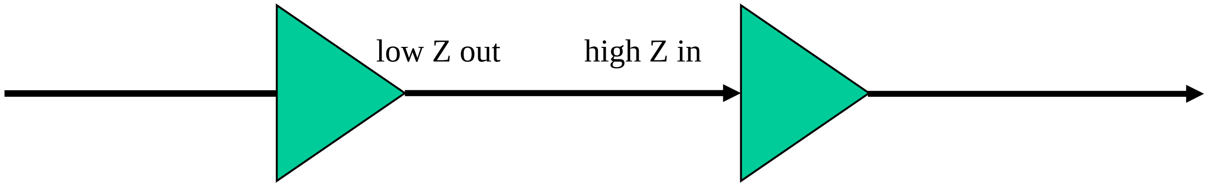
- *Current source*
  - *Ideal: constant current, independent of load*
  - *Real: limited output voltage will limit current into a high impedance load*
  - *Desire: maximum output impedance R_{th}*
    - *Change in voltage (due to load change) results in little change in current*
  - *Poor example: battery in series with a large resistor*



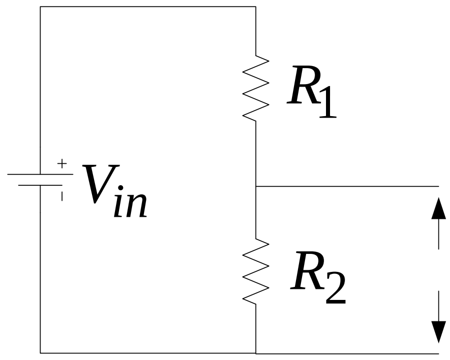
Power vs Signal Transfer

- *The power delivered to a load is maximum if the (input) impedance of the load matches the (output) impedance of the source*
  - *This is useful if power transfer is the goal. For example, you want the impedance of your stereo speakers to match the output impedance of the amplifier (typically 8 ohms).*
  - *This also eliminates reflections in a transmission line.*

❖ *More commonly, however, we want to transfer a **voltage signal**, not power, in which case you desire*
  - *Very low output impedance source driving a*
  - *Very high input impedance load.*
  - *Then the load does not alter significantly the properties of the source and you can analyze them separately.*



Voltage Dividers



- *Open output (no load)*
  - *Current*

$$I = \frac{V_{\text{in}}}{R_1 + R_2}$$

  - *Output voltage vs. input*

$$V_{\text{out}} = IR_2 = \frac{R_2}{R_1 + R_2} \cdot V_{\text{in}}$$

  - *"Transfer function" H*

$$H = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{R_2}{R_1 + R_2}$$

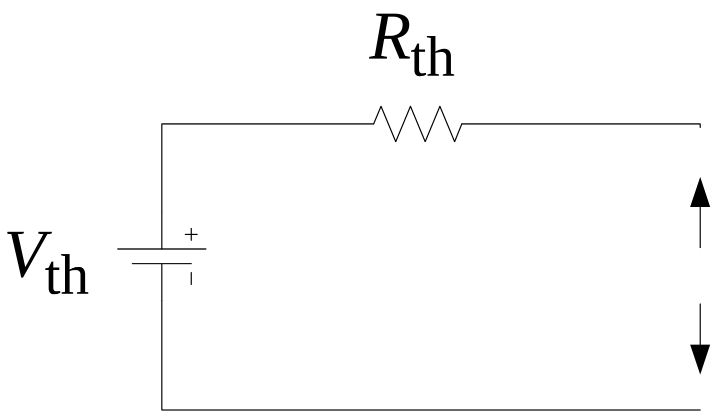
- *Input impedance*
  - *How much does the input current change if V_{in} changes?*
- *Output impedance*
  - *How much does V_{out} change if I_{out} changes?*
  - *Low output impedance requires high current!*

$$Z_{\text{in}} = R_1 + R_2$$

$$Z_{\text{out}} = R_{\text{Th}}$$

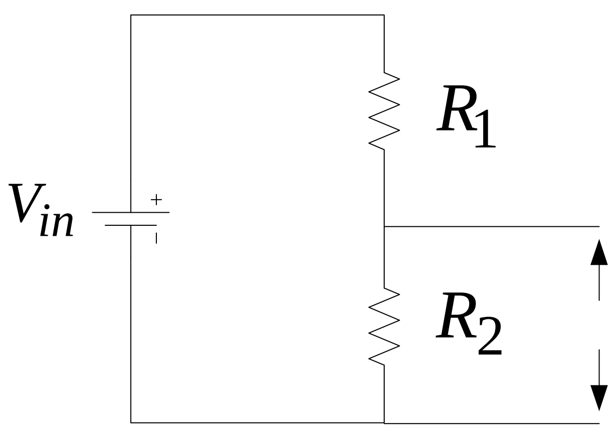
- *Effect of a load: V_{out} will decrease with increasing load ($Z_{\text{out}} \neq 0$)*
- ***This simple device is extremely common and important!***

Thévenin Equivalent for Voltage Divider



$$V(\text{open circuit}) = V_{\text{th}}$$

$$I(\text{short circuit}) = \frac{V_{\text{th}}}{R_{\text{th}}}$$



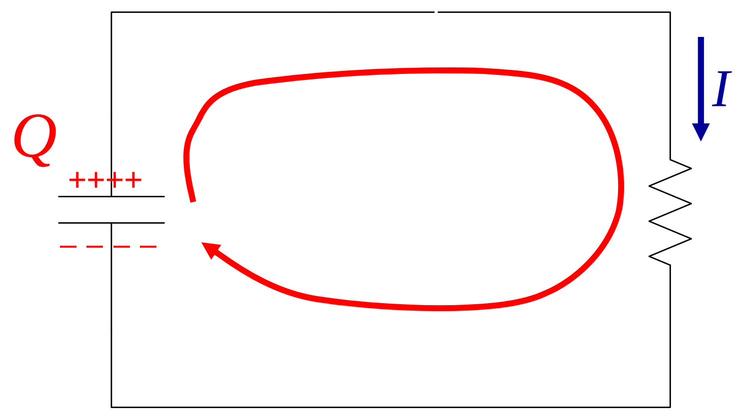
$$V(\text{open circuit}) = V_{\text{in}} \frac{R_2}{R_1 + R_2} = V_{\text{th}}$$

$$I(\text{short circuit}) = \frac{V_{\text{in}}}{R_1}$$

$$R_{\text{th}} = \frac{R_1 R_2}{R_1 + R_2} = Z_{\text{out}}$$

Looking back into the output, R_1 and R_2 appear to be in parallel!

RC Circuits: Discharging



Kirchhoff loop rule: $$\frac{Q}{C} - IR = 0 \qquad I = -\frac{dQ}{dt}$$

$$\frac{dQ}{dt} + \frac{1}{RC}Q = 0$$

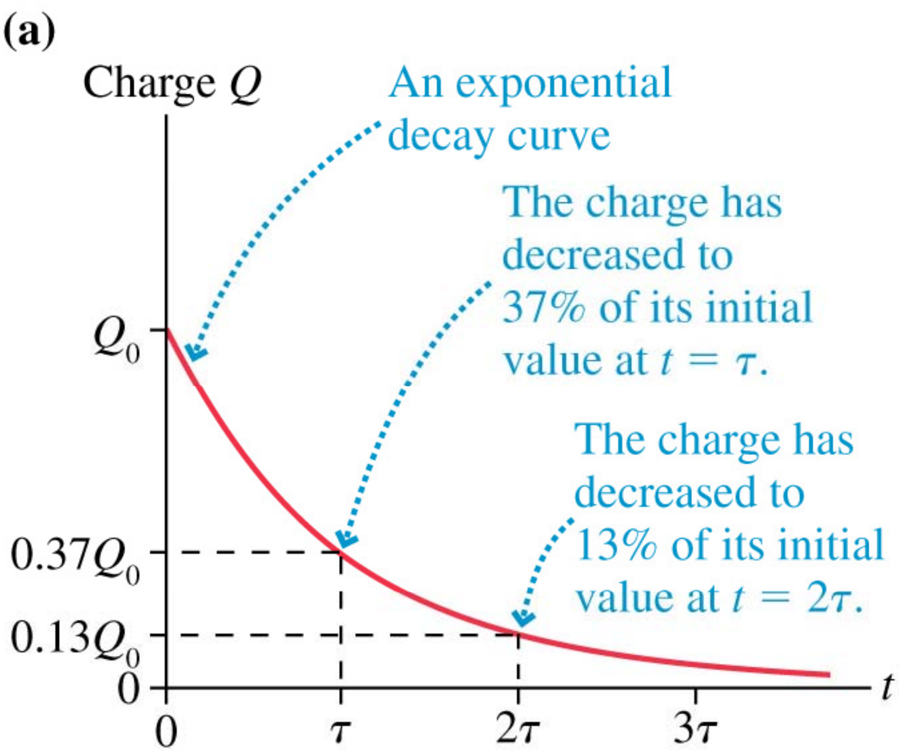
$$Q(t) = Q_0 e^{-t/RC} \qquad I(t) = -\frac{dQ}{dt} = \frac{V_0}{R} e^{-t/RC}$$

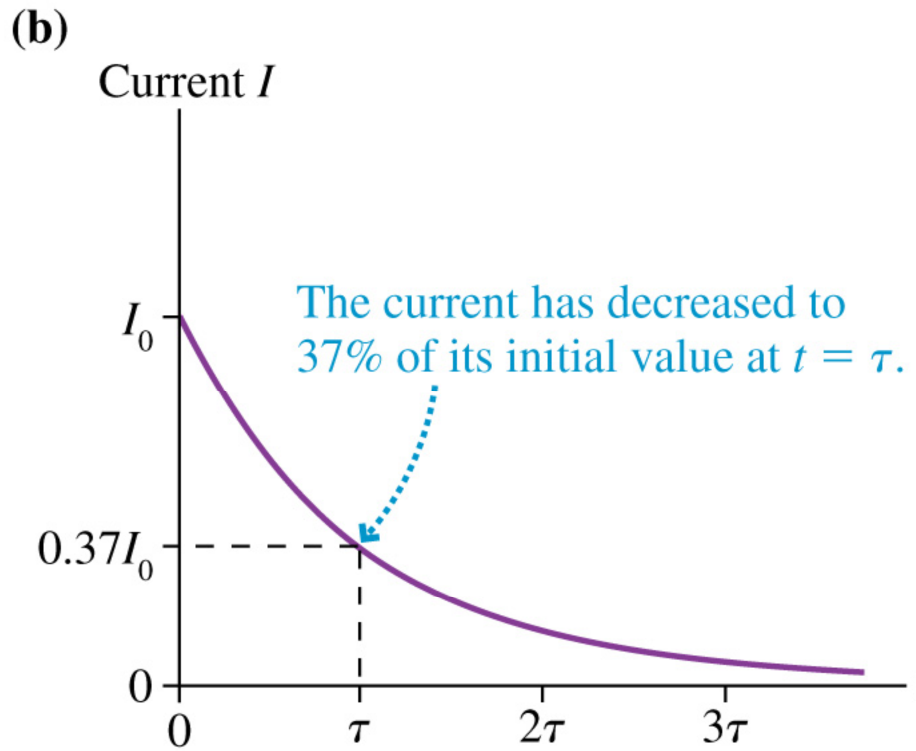
$Q = CV$ *so* $$V(t) = V_0 e^{-t/RC}$$

RC Circuits: Discharging

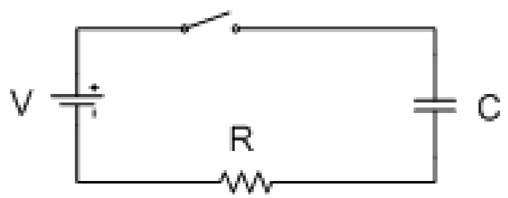
$$Q(t) = Q_0 e^{-t/RC}$$

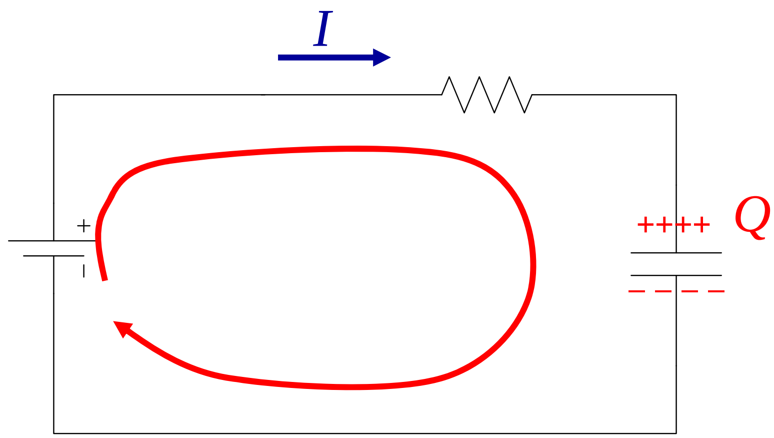
$$I(t) = \frac{V_0}{R} e^{-t/RC}$$





RC Circuit: Charging (transient solution)





Assume Q=0 at time t=0.

Kirchhoff loop rule: $$V_0 - IR - \frac{Q}{C} = 0$$

$$I = \frac{dQ}{dt}$$

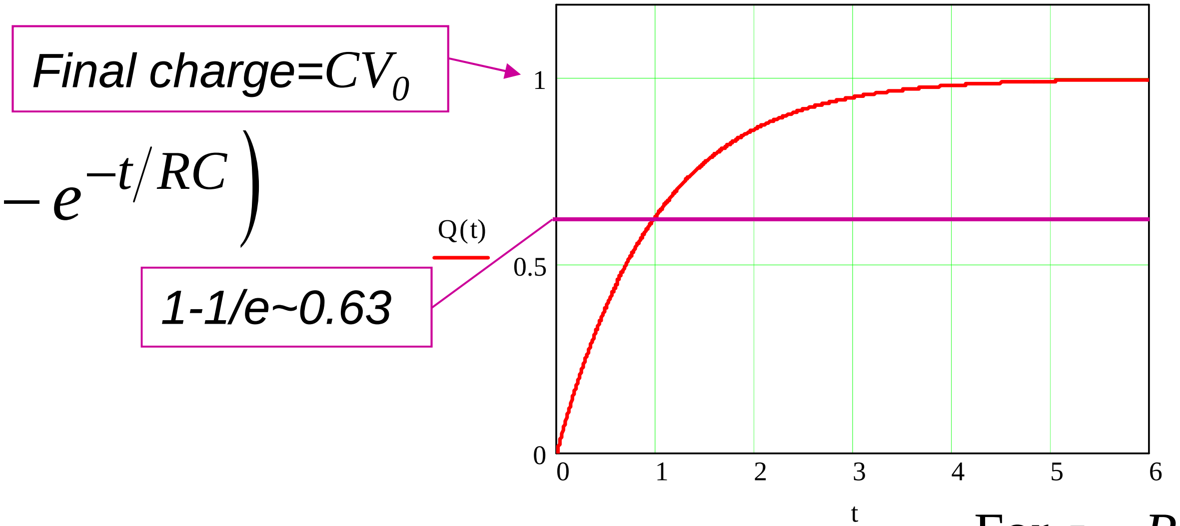
First order linear differential equation: $$\frac{dQ}{dt} + \frac{1}{RC}Q = \frac{V_0}{R}$$

$$\left(\text{Note that } \frac{V_0}{R} = I_0 \right)$$

$$\Rightarrow \quad Q(t) = CV_0 \cdot \left(1 - e^{-t/RC}\right) \quad \Rightarrow \quad I(t) = I_0 \cdot e^{-t/RC}$$

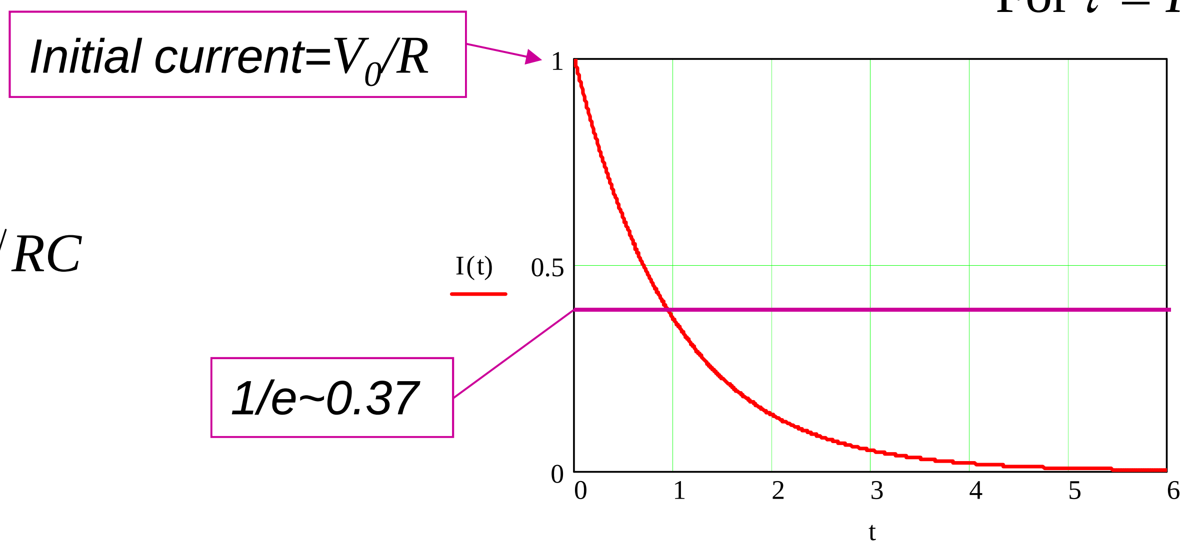
RC Circuit: Charging

$$Q(t) = CV_0 \cdot \left(1 - e^{-t/RC}\right)$$

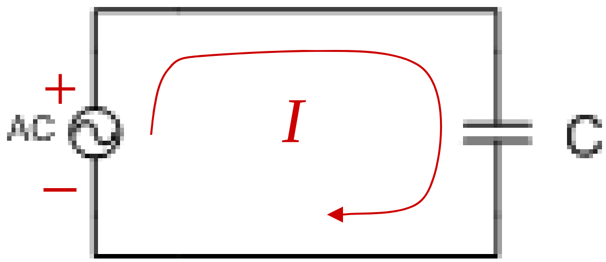


For $\tau = RC = 1$

$$I(t) = I_0 \cdot e^{-t/RC}$$



Capacitor (AC Steady State)

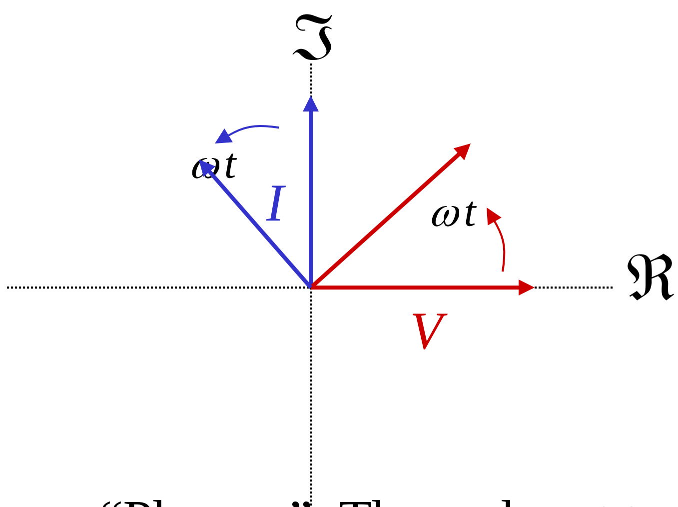


$$V = V_0 \cos \omega t$$

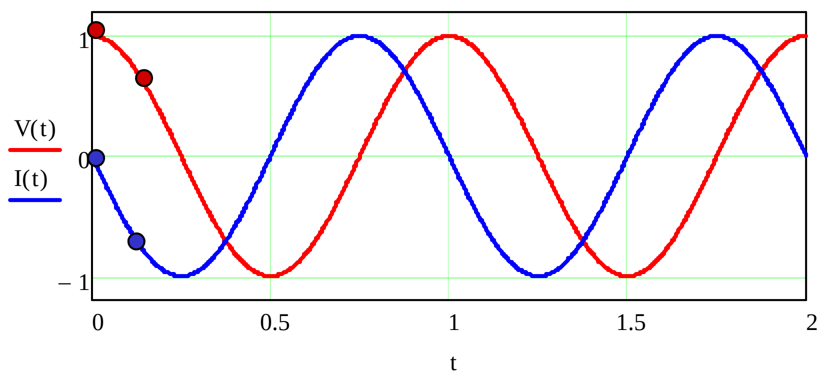
$$Q = CV$$

$$I = \frac{dQ}{dt} = -\omega C V_0 \sin \omega t$$

$$I = \omega C V_0 \cos(\omega t + \frac{\pi}{2})$$



Current leads voltage by ¼ period (90°)



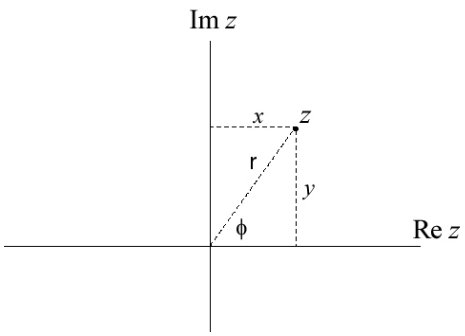
$\omega = 2\pi$

“Phasors”: The real part traces out the cosine curves as the vector rotates.

Phasors are Complex Numbers

Physical voltage and current are understood to be the real part of the complex quantity.

The real and imaginary parts don't get mixed up by the circuit equations ***as long as the circuit is linear.***



$$V = V_0 \cos \omega t = \Re\left\{ V_0 e^{j\omega t} \right\}$$

$$I = \omega C V_0 \cos(\omega t + \tfrac{\pi}{2}) = \Re\left\{ j\omega C \cdot V_0 e^{j\omega t} \right\} \qquad \left(j = \sqrt{-1} = e^{j\frac{\pi}{2}} \right)$$

Considering *I* and *V* to be complex numbers:

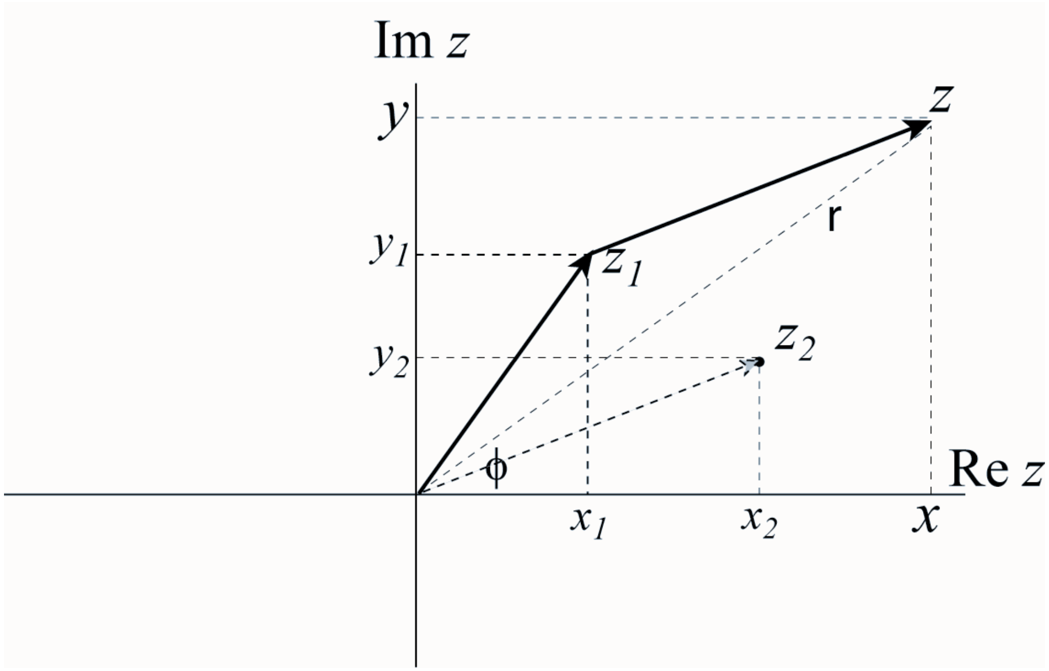
$$\Rightarrow \quad V = I \cdot Z \qquad \boxed{Z = \frac{1}{j\omega C}}$$

"Impedance"

Phasors Add as Vectors

But that exactly corresponds to addition of complex numbers:

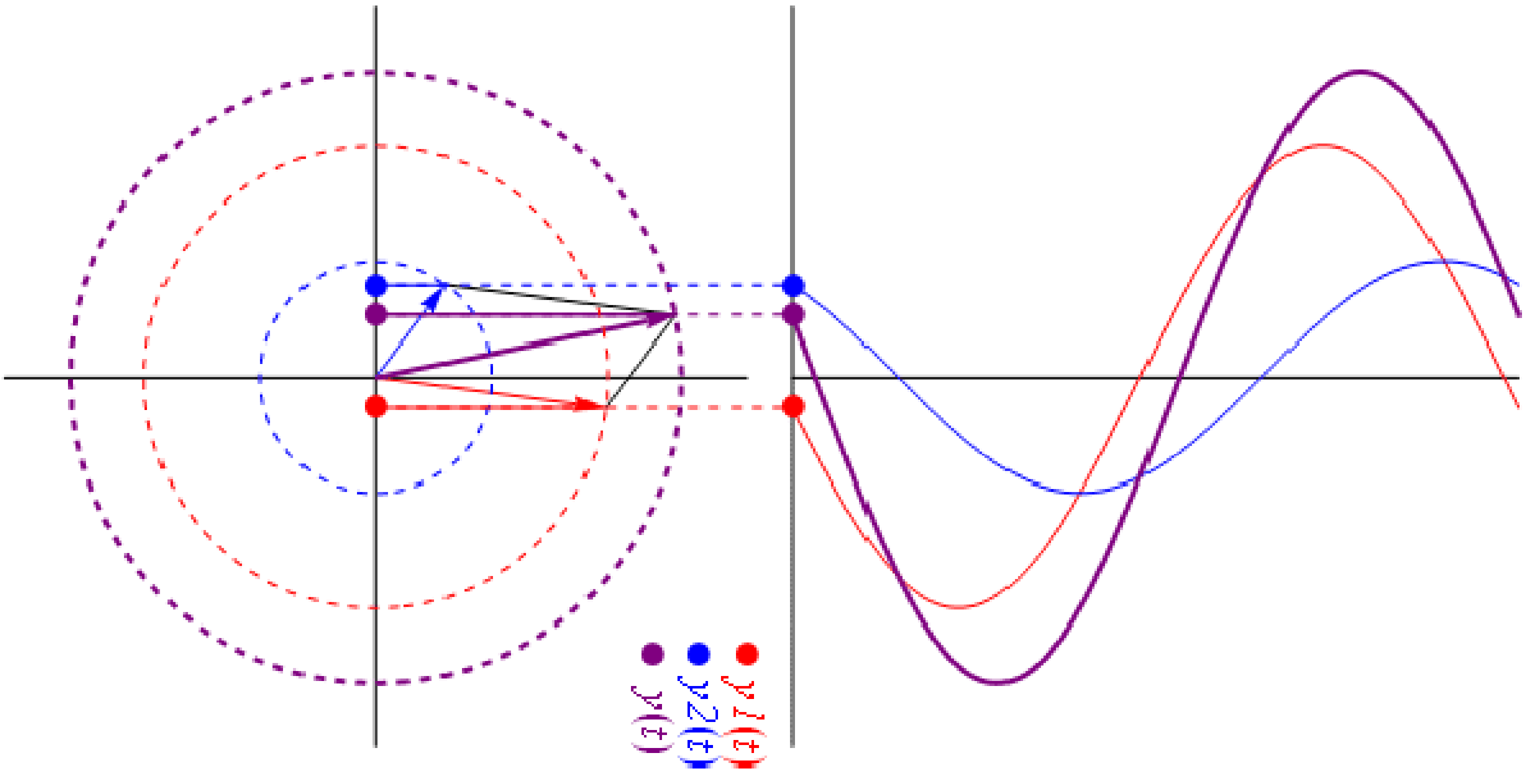
- Add real parts together
- And add the imaginary parts together



This is far easier than adding together cosine functions!

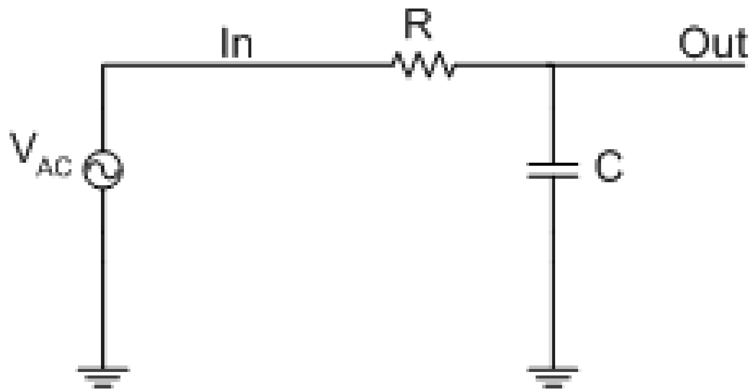
http://linus.highpoint.edu/~atitus/physlets/physlet.php?filename=cir_RC.html

Adding Two Sine Waves as Phasors



(From Wikipedia)

RC Circuit (Low-Pass Filter)



$$\tau \equiv RC$$

$$I = V_{\text{in}} / Z \qquad \text{where} \qquad Z = R + \frac{1}{j\omega C}$$

$$V_{\text{out}} = I \cdot X_C = V_{\text{in}} \cdot \frac{X_C}{Z} = V_{\text{in}} \times \frac{1/j\omega C}{R + 1/j\omega C}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega RC} = \frac{1}{\sqrt{1 + (\omega\tau)^2}} e^{-j\phi} \quad \text{where} \qquad \phi = \tan^{-1}(\omega\tau)$$