1. (2 pts) Connect each of Maxwell's equations with a corresponding statement summarizing the equation.

a) \[ \oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{1}{\epsilon_0} \int_V \mathbf{v} d\mathbf{v} \]  
Closed electric field lines are produced by changing magnetic flux.

b) \[ \oint_S \mathbf{B} \cdot d\mathbf{A} = 0 \]  
Electric field lines start on positive charges and end on negative charges.

c) \[ \oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 \oint_S \mathbf{J} \cdot d\mathbf{A} + \mu_0 \sigma_0 \frac{d}{dt} \oint_S \mathbf{E} \cdot d\mathbf{A} \]  
Magnetic field lines form continuous loops only, as there are no magnetic monopoles.

d) \[ \oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \oint_S \mathbf{B} \cdot d\mathbf{A} \]  
Closed magnetic field lines are produced by currents and by changing electric flux.

2. (3 pts) A rectangular wire loop moves through a region of uniform magnetic field, as indicated in the figure, with B pointing out of the page. What is the direction of the induced current in the loop at the positions (i), (ii), and (iii), looking down on the loop as in the figure?

a) Position a:
   i) Clockwise
   ii) Counterclockwise
   iii) No current

b) Position b:
   i) Clockwise
   ii) Counterclockwise
   iii) No current

c) Position c:
   i) Clockwise
   ii) Counterclockwise
   iii) No current

3. (2 pts) A loop of wire is moving away from another wire carrying a constant current, as indicated in the figure. What is the direction of the induced current in the moving loop?

a) No current.
   b) Clockwise.
   c) Counterclockwise.
   d) \( 45^\circ \) from the x, y plane, between the positive x and positive y axes.

4. (2 pts) In an electromagnetic plane wave, the amplitude of the electric field vector is 100 V/m. What is the corresponding amplitude of the magnetic field vector, in Tesla?

a) \( 3.3 \times 10^{-7} \) T
b) 0.010 T
c) 100 T
d) \( 3.0 \times 10^{10} \) T

5. (2 pts) The magnet in the figure is moving toward the loop of conducting wire. What is the direction of the induced current?

a) Direction of the arrow a.
   b) Direction of the arrow b.
   c) No current.
6. (2 pts) Two series LRC circuits have the same resonance frequency but differ in the size of the series resistance. The two graphs show the power dissipation versus frequency. Which graph represents the circuit with the larger resistance? Note that the vertical scales are not the same in the two graphs.

7. (2 pts) The graph shows the power dissipation versus angular frequency in a series LRC circuit. If the capacitance is 50 μF, what is the value of the inductance?
   a) 0.13 mH
   b) 5.0 mH
   c) 10 mH
   d) 0.20 H
   e) 10 H

8. (2 pts) Shown below are 4 separate situations with 2 conducting wires each. The wires all carry the same current of 1.0 A, but the direction is into the page or out of the page, as indicated. Rank the magnitude of the magnetic field at the point P from least to greatest, assigning the number 1 to the weakest and 4 to the strongest, and so forth.

9. (3 pts) Consider this drawing of two coils whose axes lie along a common line.
   a) Just after the switch on the left coil is closed, what direction does current flow through the meter of the right coil?
      i) Left to right (positive current).
      ii) Right to left (negative current).
      iii) Zero current.
   b) A very long time after the switch is closed in the left coil, what direction does current flow through the meter of the right coil?
      i) Left to right (positive current).
      ii) Right to left (negative current).
      iii) Zero current.
   c) Just after the switch is reopened, what direction does current flow through the meter of the right coil?
      i) Left to right.
      ii) Right to left.
      iii) Zero current.
10. (2 pts) A conducting loop around a magnetic field contains three identical light bulbs, A, B, and C. The magnetic field is increasing rapidly. How do the light bulbs compare in brightness?
   a) A > B = C
   b) A < B = C
   c) A > B > C
   d) A = B = C

11. (4 pts) A metal bar is resting on a U-shaped conducting rail. The rail is fixed in position, but the bar is free to slide on it. A uniform magnetic field is perpendicular to the plane of the rail and points out of the page.
   a) If the magnetic field is decreasing in strength, what is the direction of the induced current that flows around the circuit made up of the rail and bar?
      i) Clockwise.
      ii) Counterclockwise.
      iii) No current.
   b) Does the bar
      i) remain in place?
      ii) move left?
      iii) move right?
      iv) move up (toward the top of the page)?
      v) move down (toward the bottom of the page)?
      vi) rotate clockwise?
      vii) rotate counterclockwise?

12. (3 pts) The graph shows the current passing through the inductor versus time. Positive current is in the direction of the arrow. Graph the emf of the inductor versus time. Positive emf means that the lower end is at a higher potential than the upper end. You do not need to label the vertical scale.
13. (3 pts) An upward magnetic field passes through a horizontal loop of wire, as illustrated. The magnetic field is changing with time as shown in the graph. Graph the induced current in the loop versus time. A positive current is one that is clockwise, looking from above (i.e., a current into the page on the left and out of the page on the right). You do not need to label the vertical scale.

14. (4 pts) The figure shows a cross-section of a solenoid on the left and capacitor on the right. The current in the solenoid and the voltage on the capacitor are decreasing with time.
   a) Sketch the induced \( E \) field lines both inside and outside the solenoid, indicating clearly the field direction.
   b) Sketch the induced \( B \) field lines both inside and outside the capacitor.

Solenoid. The current is rapidly decreasing with time. \( B \) out of page

Capacitor Plate. The \( E \) field points out of the page and is rapidly decreasing with time.
15. (8 pts) The graph to the right shows a phaser representing the voltage across the resistor in a series RLC circuit. The grid spacing in the graph is 5 V.
   a) What is the magnitude of the potential difference across the resistor at this instant in time?
      \[ V = 20 \text{ V} \]
   b) What is the rms voltage across the resistor?
      \[ V_{\text{rms}} = \frac{1}{\sqrt{2}} \sqrt{20^2 + 10^2} = 16 \text{ V} \]
   c) At this instant, is the resistor voltage increasing with time?
      i) increasing with time?
      ii) decreasing with time?
      iii) or momentarily not changing with time?
   d) Sketch two more phasors on the plot, one for each of the capacitor and inductor voltages. Don’t worry about the magnitude—just get the directions of the phasors correct.

16. (2 pts) Rank the four identical light bulbs from brightest to dimmest.
   a) A = B = C = D
   b) A > B > C > D
   c) A > C > B > D
   d) A > C = D > B
   e) C = D > B > A

17. (2 pts) An electron is traveling with velocity \( \vec{v} \) through a magnetic field as indicated. What is the direction of the force on the electron? Either draw the force vector or state clearly its direction.

18. (2 pts) Which electric field configuration could be responsible for the parabolic trajectory of the proton that is shown?

   (a) \hspace{1cm} (b) \hspace{1cm} (c) \hspace{1cm} (d) \hspace{1cm} (e)

   constant acceleration \( \Rightarrow \) parabola
19. (10 pts) A current \( I_1 = 31.8 \, \text{A} \) flows in the 
xy plane around a ring of radius \( a = 5 \, \text{cm} \) in 
the clockwise sense as indicated in the 
drawing. A second current \( I_2 = 100 \, \text{A} \) flows 
on a long straight wire in the 
\(-z\) direction 
(into the page) at a distance 10 cm from the 
center of the ring, as indicated.

a) Draw and label vectors \( \vec{B}_1 \) and \( \vec{B}_2 \) on 
the picture indicating the direction of the 
magnetic field contributions at the origin 
from currents \( I_1 \) and \( I_2 \) respectively. 
If necessary, use a \( \bigcirc \) to indicate a vector 
into the page or a circle with a dot in it to 
indicate a vector out of the page.

b) What is the magnitude of the magnetic 
field at the origin?

\[
\vec{B}_1 = \frac{\mu_0}{2\pi} \frac{I_1}{r} = 2\pi \cdot 10^{-7} \cdot \frac{31.8}{0.05} = 4.0 \cdot 10^{-4} \, \text{T}
\]

\[
\vec{B}_2 = \frac{\mu_0}{2\pi} \frac{I_2}{a} = 2\pi \cdot 10^{-7} \cdot \frac{100}{0.1} = 2.0 \cdot 10^{-4} \, \text{T}
\]

\[
\vec{B} = \sqrt{\vec{B}_1^2 + \vec{B}_2^2} = 4.5 \cdot 10^{-4} \, \text{T}
\]

c) What is the direction of the magnetic field at the origin? Give the direction as an angle, and 
specify clearly in words or in a drawing the definition of your angle. For example, a sketch 
showing the addition of the magnetic field vectors, along with the angle, would be good.

\[
\theta = \tan^{-1} \frac{\vec{B}_2}{\vec{B}_1} = \tan^{-1} 0.5
\]

\[
\theta = 27^\circ
\]

d) An electron happens to be at the origin at this instant in time, traveling in the \( +z \) direction at high 
speed. What is the direction of the magnetic force on the electron?

\[
+\hat{x} \, \text{direction}
\]
20. (10 pts) A loop of radius \( b \) and resistance \( R \) surrounds and is co-axial with a solenoid of radius \( a \). The solenoid has \( n \) turns per unit length and carries the current \( I_{\text{solenoid}}(t) = I_0 \sin(2\pi ft) \), where the positive direction of \( I_{\text{solenoid}}(t) \) is indicated on the figure and \( I_0 \) is a positive constant. Assume an ideal, long solenoid, with uniform field inside and zero field outside.

a) Write an expression for the time-dependent magnetic field inside the solenoid, and indicate on the figure its direction at \( t = t_0 \).

\[
\vec{B} = \mu_0 n I_0 \sin 2\pi ft
\]

b) Write an expression for the time-dependent magnetic flux \( \Phi_B \) through the large loop.

\[
\Phi_B = \oint \vec{B} \cdot d\vec{A} \quad \text{\( \vec{B} \) is zero outside \( r = a \)}
\]

\[
\Phi_B = (\mu_0 n I_0 \sin 2\pi ft) (\pi a^2)
\]

\[
\Phi_B = \mu_0 n \pi a^2 I_0 \sin 2\pi ft
\]

c) Write an expression for the time-dependent induced current in the large loop.

\[
I_{\text{ind}} = \frac{\mathcal{E}}{R} = \frac{1}{R} \frac{d\Phi_B}{dt}
\]

\[
I_{\text{ind}} = \frac{1}{R} \mu_0 n \pi a^2 I_0 \sin 2\pi ft
\]

d) Show on the diagram the direction of the induced current in the large loop at time zero.

e) If the frequency is \( f = 1.0 \, \text{kHz} \), how long after \( t = 0 \) does the induced current become zero for the first time?

\[
\frac{1}{\mathcal{f}} \text{ period } = \frac{1}{\mathcal{f}} 0.001 = 250 \mu s
\]
21. (10 pnts) Switch S1 has been closed for a very time when, at t=0, it is opened. The capacitance is C=12.7 μF and the inductor and wires are idealized (zero ohmic resistance).

![Circuit Diagram]

a) What is the current through the inductor at t=0?

\[
I_0 = \frac{6 \text{ V}}{12 \Omega} = 0.5 \text{ A}
\]

b) How much charge is on the capacitor at t=0?

\[
Q = 0,000_0 \text{ C}
\]

c) Which of the following correctly describes the capacitor charge after the switch is opened, and what is the numerical value of \( \omega \)?

\begin{enumerate}
  \item \( Q(t) = -Q_0 \sin \omega t \)
  \item \( Q(t) = Q_0 \cos \omega t \)
  \item \( Q(t) = Q_0 \tan \omega t \)
\end{enumerate}

\[
\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{12.7 \times 10^{-6} \text{ F} 	imes 0.001 \text{ A}}} = 6270 \text{ rad/s}
\]

d) At what time after t=0 does the absolute value of the capacitor voltage first reach its maximum?

\[
\frac{1}{4} \text{ cycle} = \frac{1}{4} \times \frac{2\pi}{\omega} = \frac{1}{4} \times \frac{2\pi}{0.001} = 250 \mu s
\]

c) What is the maximum voltage on the capacitor?

\[
I(t) = +\omega Q_0 \cos \omega t = -\frac{dQ}{dt}
\]

\[
Q_0 = \frac{I_{\text{max}}}{\omega} = \frac{I_0}{\omega} = \frac{0.5 \text{ A}}{6270 \text{ rad/s}} = 80 \mu \text{ C}
\]
22. (8 pts) The switch in the circuit to the right, with $V_0 = 12 \text{ V}$, has been open for a very long time. It is closed at $t=0$. What is the current through the 24 $\Omega$ resistor?

a) immediately after closing the switch?

$$I = \frac{V_0}{24 \Omega + 24 \Omega} = \frac{12 \text{ V}}{48 \Omega}$$

$$I = 0.25 \text{ A}$$

b) after the switch has been closed for a very long time?

$$R_{eq} = 24 \Omega + \frac{24 \Omega \cdot 12 \text{ V}}{24 \Omega + 12 \text{ V}} = 24 \Omega + 8 \Omega = 32 \Omega$$

$$I = \frac{12 \text{ V}}{32 \Omega} = 0.375 \text{ A}$$

$$I_{LV} = \frac{12 - (0.375)(24)}{24} = \frac{3}{24} = 0.125 \text{ A}$$

c) immediately after the switch is reopened?

$$I_L = \frac{3}{12} = 0.25 \text{ A} \text{ before opening switch, and}$$

this equals $I_{LV}$ after opening.

$$I_{LV} = 0.25 \text{ A}$$

d) 100 $\mu$s after the switch is reopened?

$$\tau = (3.6 \text{ s})(3.6 \text{ mH}) = 100 \mu\text{s}$$

$$I_{LV} = 0.25 \text{ A} e^{-1} = 0.092 \text{ A}$$
23. (12 pts) The impedance of this series LRC circuit is $Z=40 \, \text{ohms}$, with the current lagging the source emf when driven at a frequency of 60 Hz.

a) What is the rms current in the circuit?
$$I_{\text{rms}} = \frac{120 \, \text{V}}{40 \, \Omega} = 3.0 \, \text{A rms}$$

b) What is the average power dissipation in the resistor?
$$P = I_{\text{rms}}^2 R = 3^2 \times 24 = 216 \, \text{W}$$

c) How much power is delivered by the source?
$$P_{\text{delivered}} = 216 \, \text{W}$$

d) What are the values of the power factor $\cos \phi$ and the phase angle $\phi$?
$$P = I_{\text{rms}} V_{\text{rms}} \cos \phi$$
$$= 216 \, \text{W}$$
$$\cos \phi = \frac{216 \, \text{W}}{3 \, \text{A} \times 120 \, \text{V}} = 0.6$$
$$\phi = \cos^{-1} 0.6 = 53^\circ$$

e) Sketch the current on the same graph above where the emf is shown. The horizontal axis is in degrees and the vertical axis on the left is in volts. Label the vertical axis on the right in amperes for the current.

f) Calculate the value of the instantaneous current when the emf is at its peak (170 V).
$$t = 0$$
$$i = \sqrt{2} \times 3 \, \text{A} \times (\sqrt{2} - 53)$$
$$= \sqrt{2} \times 3 \times 0.6 = 2.5 \, \text{A}$$

g) Calculate the value of the instantaneous current when the emf is zero and decreasing.
$$i = \sqrt{2} \times 3 \, \text{A} \times (\sqrt{2} - 53) = 3.4 \, \text{A}$$