Physics 6C
Introduction to Physics III
Electricity and Magnetism

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Midterm 1

- Grading is still in progress.
- Probably the exams will be returned Monday.
- See the solutions posted on the web site.
  - Two slightly different versions: A & B
Capacitance Calculations

Parallel Plates

\[ C = \varepsilon_0 \frac{A}{d} \]

Area of plate = A

Concentric Spheres

\[ C = 4\pi\varepsilon_0 \frac{r_a r_b}{r_b - r_a} \]

Good approximation if \( d \) is small compared with lateral size.

Coaxial Cylinders

\[ C = \frac{2\pi\varepsilon_0}{\ln\left(\frac{r_b}{r_a}\right)} \cdot L \]

Good approximation if \( L \) is big compared with \( r_b \).
Electroscope Demo

UCSC Demo Electroscope

\[ Q_1 + Q_2 = \text{constant} \]

If \( d \) increases, then \( C \) decreases, and \( Q_1 \) decreases.

\[ \Rightarrow Q_1 \text{ increases.} \]
Capacitors in Parallel

(a) Parallel capacitors have the same $\Delta V_C$.

\[ Q_1 = C_1 \Delta V_C \quad Q_2 = C_2 \Delta V_C \]

(b) Same $\Delta V_C$ as $C_1$ and $C_2$

\[ Q = Q_1 + Q_2 \]

Same total charge as $C_1$ and $C_2$

\[ C_{eq} = C_1 + C_2 \]
Capacitors in Series

(a) Series capacitors have the same \( Q \).

\[
\Delta V_1 = \frac{Q}{C_1}
\]

\[
\Delta V_2 = \frac{Q}{C_2}
\]

No net charge on this isolated segment

(b) Same \( Q \) as \( C_1 \) and \( C_2 \)

\[
\Delta V_C = \Delta V_1 + \Delta V_2
\]

Same total potential difference as \( C_1 \) and \( C_2 \)

\[
\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}
\]
What are the charge and potential difference across each capacitor?
Stored Energy

• Work is required to move charge from Object 1 to Object 2, pushing against the repulsive force of the electric field.
• Suppose charge $q$ has already been moved, and we want to move a little bit more: $dq$.
• That requires work $dW = \Delta V \cdot dq$.

$$W = \int dW = \int_0^Q \frac{q}{C} dq = \frac{1}{2} \frac{Q^2}{C}$$
Capacitor Relations

\[ Q = C \cdot \Delta V \]  
Definition of capacitance

\[ U_C = \frac{1}{2} \frac{Q^2}{C} \]  
Energy stored in a capacitor

or, equivalently,

\[ U_C = \frac{1}{2} C (\Delta V)^2 \]  
Energy stored in a capacitor
Example: Parallel Plates

The instantaneously charge on the plates is \( \pm q \).

\[ U_C = \frac{1}{2} C (\Delta V)^2 \]

\[ C = \varepsilon_0 \frac{A}{d} \]

\[ \Delta V = Ed \]

Volume = \( Ad \)

\[ U_C = \frac{1}{2} \varepsilon_0 \frac{A}{d} (Ed)^2 = \frac{1}{2} \varepsilon_0 E^2 \cdot Ad \]

Energy density of the electric field:

\[ u_E = \frac{U_C}{\text{Volume}} = \frac{1}{2} \varepsilon_0 E^2 \]

The charge escalator does work \( dq \Delta V \) to move charge \( dq \) from the negative plate to the positive plate.
Potential Energy of an Electric Field

• The work that we do in moving charge from conductor 1 to conductor 2 goes into potential energy.

• That means that this energy is stored up and can be recovered (e.g. in a flash lamp).
  – Short-term storage of electrical energy is the principal use of capacitors in practical applications.

• We can think of this energy as being stored in the electric field itself, with an energy density:

\[
\mathcal{u}_E \equiv \frac{dU}{dV} = \frac{1}{2} \varepsilon_0 E^2
\]

Valid for any geometry, not just parallel plates!
Circuit Analysis: Simplest Circuit

I = \frac{\Delta V}{R}

Resistance of short copper or aluminum wires is usually negligible compared with the “resistors” in a circuit.
Kirchhoff’s Rules

• Two simple but essential rules for analyzing circuits.
• These apply strictly speaking to steady-state DC circuits only.
• We will generalize the applications later.
Kirchhoff’s loop rule

- The sum of potential changes around a closed loop is zero.
- The is simply a special case of the rule for \textit{electrostatic} fields:

\[ \oint \vec{E} \cdot d\vec{\ell} = 0 \quad \text{Around any closed loop in space.} \]

\[ V - IR_1 - IR_2 = 0 \]

\[ I = \frac{V}{R_1 + R_2} \]
Equivalent Resistance

This gives us a simple rule for adding resistors that are in series.

\[ V - I(R_1 + R_2) = 0 \]
\[ V - IR_{eq} = 0 \]
Current Conservation

Kirchhoff’s junction law: the total current flowing out of a junction must equal the total current flowing into the junction.

\[ \sum I_{\text{in}} = \sum I_{\text{out}} \]

The current in a wire is the same at all points.

\[ I = \text{constant} \]
Realistic Battery

- The source of emf, such as a battery, always has some resistance of its own.

With the switch open, the current is zero and the potential drop across the internal resistance $r$ is zero ($I \cdot r = 0$).

In that case, $\Delta V = \mathcal{E} - I \cdot r = \mathcal{E}$.

With the switch closed the current is

$$I = \frac{\mathcal{E}}{R_{\text{equiv}}} = \frac{\mathcal{E}}{r + R}$$

In that case, the battery voltage looks like

$$\Delta V = I \cdot R = \mathcal{E} \cdot \frac{R}{r + R}$$
Measuring Current & Voltage

Must have very low resistance, so it doesn't disturb the circuit.

Must have very high resistance, so it doesn't disturb the circuit.

Must have very low resistance, so it doesn't disturb the circuit.