Physics 6C
Introduction to Physics III
Electricity and Magnetism

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Faraday’s Law

EMF around a closed loop: \[ \mathcal{E} = -\frac{d\Phi_B}{dt} \]

- You don’t need to worry about the minus sign! Use Lenz’s law to get the direction of the EMF.
- Use Faraday’s law to get the magnitude:

\[ |\mathcal{E}| = \left| \frac{d\Phi_B}{dt} \right| \]
AC Generator

- Uniform magnetic field $B$
- Coil rotating with uniform angular velocity $\omega = \frac{d\theta}{dt}$

Magnetic flux through the loop: $\Phi = BA \cos \theta = BA \cos \omega t$

EMF around loop: $\mathcal{E} = -\frac{d\Phi}{dt} = \omega BA \sin \omega t$
AC Generator

The EMF can be readily obtained from the magnetic force law, but it is much easier to apply Faraday’s law.

\[ \mathcal{E} = -\frac{d\Phi}{dt} = \omega BA \sin \omega t \]
According to the “principle of relativity,” the physical result (i.e. the current) caused by moving the loop to the right with the magnet at rest should be exactly the same as the result of moving the magnet to the left with the loop at rest!

This is indeed what is seen experimentally. (In fact, this was part of Einstein’s motivation to develop his 1905 “Theory of Relativity”, which led to $E=mc^2$ and new concepts on the relation between space and time!)

But in the figure at the bottom the EMF is not a “motional EMF” because the wire is not in motion.

We have to conclude that any change in flux in the loop produces an EMF.
ON THE ELECTRODYNAMICS OF MOVING BODIES

By A. Einstein
June 30, 1905

It is known that Maxwell's electrodynamics—as usually understood at the present time—when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. Take, for example, the reciprocal electrodynamic action of a magnet and a conductor. The observable phenomenon here depends only on the relative motion of the conductor and the magnet…
Faraday’s Law Example

\[ B(t) = 0.012t + 3.0 \times 10^{-5} t^4 \text{ T} \]
(with \( t \) expressed in seconds)

- Radius of coil = 0.040 m
- \( N=500 \) turns in the coil
- 600 ohm resistance

a) Magnitude of induced EMF
b) Current at \( t=5.0 \) seconds

This is not a “motional EMF,” so we cannot obtain the result from the magnetic force law. We must use Faraday’s law.
Lenz’s Law Example

• When the switch is closed, which way will current flow in the loop?
  a) In the direction of the red arrow.
  b) In the direction of the green arrow.

After closing the switch, the current and magnetic field in the solenoid will be increasing, so the flux through the loop will be increasing. Current will flow in the loop to oppose the increasing flux.
Induced Electric Field

When an EMF is induced in the loop of wire, there must be a force field inside the wire pushing the otherwise static charges. We call such a force field an electric field!

It should not come as a surprise, then, that the electric field is still there even if we remove the charges (i.e. remove the wire).

If \( E \) is constant around the loop of length \( \ell \) and is always parallel to the path of the loop, then we can write the EMF (work per unit charge) simply as

\[
\mathcal{E} = E \ell
\]

More generally:

\[
\mathcal{E} = \int \vec{E} \cdot d\vec{l}
\]
Calculate the EMF around the wire loop:

Inside the solenoid  \( B = \mu_0 n I \)  \( \Rightarrow \Phi_B = BA = \mu_0 n I A \)

\[ \mathcal{E} = -\frac{d\Phi_B}{dt} = -\mu_0 n A \frac{dI}{dt} \]
Now forget about the wire loop and consider just the field at a radius $r$.

From symmetry, the field is constant around a ring of radius $r$ and always tangent to the ring.

\[ \mathcal{E} = \oint E \cdot d\ell = E 2\pi r \]

\[ 2\pi r E = \mu_0 n A \frac{dI}{dt} \quad \Rightarrow \quad E = \frac{\mu_0 n A}{2\pi} \frac{dI}{dt} \cdot \frac{1}{r} \quad \text{Outside the solenoid} \]
Example: Induced Field Inside Solenoid

Inside the solenoid \( B = \mu_0 nI \)

Flux through a circle of radius \( r \)
\[
\Phi_B = BA = \mu_0 nIA
\]
\[
\Phi_B = \mu_0 nI \pi r^2
\]
\[
E = -\frac{d\Phi_B}{dt} = -\mu_0 n \pi r^2 \frac{dI}{dt}
\]
\[
\int \vec{E} \cdot d\vec{l} = E2\pi r = -\mu_0 n \pi r^2 \frac{dI}{dt}
\]

The electric field is zero at the center and increases linearly with radius out to the solenoid coil.
Inductance

• This is a concept used in connection with Faraday’s law when the circuits (coils of wire) are fixed in space and not changing in size or shape.
• What is changing, then, is the current, and therefore the magnetic field and flux.
• Since the flux is proportional to the magnetic field, and the magnetic field is proportional to the current, then

\[ \Phi_m \propto I \]

• We call the constant of proportionality “inductance”:

\[ \Phi_m = LI \]

Units of L: \( H = T m^2 / A \)
Inductance of a Solenoid

\[ B = \frac{\mu_0 NI}{\ell} \]

Flux through 1 coil: \( \Phi_1 = BA = \frac{\mu_0 NIA}{\ell} \)

Flux through N coils: \( \Phi_m = \frac{\mu_0 N^2 A}{\ell} I \)

Inductance: \( L = \frac{\mu_0 N^2 A}{\ell} \)

Note: we use here a fairly crude approximation that the field is uniform everywhere inside the solenoid and zero outside.
In accordance with Faraday’s law and Lenz’ law, an inductor always opposes a change in current.

\[ \Phi = L \cdot I \]

\[ \mathcal{E} = - \frac{d\Phi}{dt} = -L \frac{dI}{dt} \]

Inductor opposes the increase in current.

Inductor opposes the decrease in current, trying to keep the current going.
Light Bulb Demo Circuit

The two parallel paths see the same voltage $V$ when the switch is closed.

- The current $I_2$ flows immediately through the resistor $R$.  
- The current $I_1$ builds up slowly through the inductor $L$.

Opening the switch causes the current to decay slowly, flowing through $R$.  

$$I = I_1 + I_2$$
Inductance and Sparks

Suppose a constant current is flowing in the circuit below. What happens if you try to open the switch?

\[
\frac{dI}{dt} \rightarrow -\infty \quad \text{so} \quad \mathcal{E} \rightarrow \infty
\]

The EMF cannot, of course, really go to infinity.

Instead, it will go high enough (thousands of volts) to produce a spark.

The spark keeps the current flowing momentarily, so instead of being infinite, \(dI/dt\) is just very large.
Light Bulb Demo Circuit

- Disconnect the path with the resistor.

- Now, when we try to open the switch, the current in the inductor has nowhere to go.
  
  - When we try to force the current to change instantly to zero, we get a huge EMF and a spark.