Mechanical Analogy

The “charge escalator” inside a battery continuously “lifts” electrons from the positive to the negative terminal. This renewal of charge sustains the electron current.

High PE

Negative terminal

Electron current

Low PE

Positive terminal

A battery is an example of Electro-Motive Force (EMF).

The battery supplies a definite potential energy to each charge carrier.

It does NOT provide a predetermined current (electrons per second).

The current depends on the overall circuit (it is zero if a wire is disconnected).
Electric Field in a Wire

A non-uniform surface charge helps shape the field in the wire, but for our purposes, all we need to know is that the net result when the circuit is complete is a field pointing parallel to the wire direction and parallel to the flow of current.
Current

Volume $V = A \cdot v \cdot \Delta t$ flows out of the pipe in time $\Delta t$.

$\Delta N_w = nV = nAv\Delta t$

$\frac{\Delta N_w}{\Delta t} = nAv$ molecules per second

$\Delta N_e = n_eV = n_eAv_d\Delta t$

$\frac{\Delta N_e}{\Delta t} = n_eAv_d$ electrons per second
Current & Current Density

Electric Current:  \[ I = \frac{\Delta Q}{\Delta t} = \frac{\Delta N_e \cdot e}{\Delta t} = n_e A v_d \cdot e \quad \text{ampere=C/s} \]

Current Density:  \[ J = \frac{I}{A} = n_e v_d e \quad \vec{J} = n_e e \vec{v}_d \quad \text{vector!} \]

Example: copper wire 0.5 mm in radius carries 1 ampere of current

\[ J = I/(\pi r^2) = 1/(3.14 \cdot 0.0005^2) = 1.3 \times 10^6 \text{ A/m}^2 \]

\[ v_d = J/(n_e e) = 1.3 \times 10^6/(8.5 \times 10^{28} \cdot 1.6 \times 10^{-19}) = 9.6 \times 10^{-5} \text{ m/s} \sim 0.1 \text{ mm/s} \]
Current I flows from wire A into wire B. The current in wire B is

- $a) \ I$
- $b) \ 2I$
- $c) \ \frac{1}{2} I$
- $d) \ \frac{1}{4} I$

The current densities are related as

- $a) \ J_B = J_A$
- $b) \ J_B = 2J_A$
- $c) \ J_B = \frac{1}{2} J_A$
- $d) \ J_B = \frac{1}{4} J_A$
Current Conservation

Kirchhoff’s junction law: the total current flowing out of a junction must equal the total current flowing into the junction.
• Note, in Chapter 28, don’t worry about the detailed mathematical model of conduction that is presented. I will not cover that.

• For example, eqn. 28.7:  
  \[ v_d = \frac{e\tau}{m} E \]

• This is a classical model, anyway, which is not quite correct. Quantum mechanics is really required to do it right.

• However, it is true that in a conductor  
  \[ v_d \propto E \]
  – (In vacuum \( v \propto t^2 \))
Conductivity

- Conductivity: $\sigma$
- Resistivity: $\rho$

\[
\vec{J} = \sigma \vec{E}
\]

<table>
<thead>
<tr>
<th>Material</th>
<th>Resistivity ($\Omega \text{ m}$)</th>
<th>Conductivity ($\Omega^{-1} \text{ m}^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>$2.8 \times 10^{-8}$</td>
<td>$3.5 \times 10^7$</td>
</tr>
<tr>
<td>Copper</td>
<td>$1.7 \times 10^{-8}$</td>
<td>$6.0 \times 10^7$</td>
</tr>
<tr>
<td>Gold</td>
<td>$2.4 \times 10^{-8}$</td>
<td>$4.1 \times 10^7$</td>
</tr>
<tr>
<td>Iron</td>
<td>$9.7 \times 10^{-8}$</td>
<td>$1.0 \times 10^7$</td>
</tr>
<tr>
<td>Silver</td>
<td>$1.6 \times 10^{-8}$</td>
<td>$6.2 \times 10^7$</td>
</tr>
<tr>
<td>Tungsten</td>
<td>$5.6 \times 10^{-8}$</td>
<td>$1.8 \times 10^7$</td>
</tr>
<tr>
<td>Nichrome*</td>
<td>$1.5 \times 10^{-6}$</td>
<td>$6.7 \times 10^5$</td>
</tr>
<tr>
<td>Carbon</td>
<td>$3.5 \times 10^{-5}$</td>
<td>$2.9 \times 10^4$</td>
</tr>
</tbody>
</table>

*Nickel-chromium alloy used for heating wires
The electric field strengths in the wires are related by

\[ a) \ E_B = E_A \]
\[ b) \ E_B = 2E_A \]
\[ c) \ E_B = \frac{1}{2} E_A \]
\[ d) \ E_B = \frac{1}{4} E_A \]
Electric Potential Energy

- The electrostatic force is a **conservative** force, just like gravity.
- As usual, we calculate work by integrating force times distance.

\[
W_{\vec{a} \rightarrow \vec{b}} = \int_{\vec{a}}^{\vec{b}} \vec{F} \cdot d\vec{\ell}
\]

- Consider the work done by the field on a positive test charge \(q\) when the charge moves from point \(a\) to point \(b\).
- The integral will be the same, no matter what path we chose.
- The integral around any closed path is zero.
Electric Potential Energy

- Because the force is conservative, we usually don’t have to do the nasty line integral.
- Instead, we define a potential energy $U$ at each point in space and just subtract to find the work.

$$W_{a \rightarrow b} = U(a) - U(b) = -(U(b) - U(a)) \equiv -\Delta U$$

$U(a)$ = the potential energy of our test charge $q$ at the point $a$.

$$\Delta U = -W_{a \rightarrow b} = -q \int_a^b \vec{E} \cdot d\vec{\ell}$$
**PE: Gravity vs. Electricity**

**Simplest examples: uniform field**

- **Gravity**
  - cannon ball
  - \( U = mgy \)

- **Electricity**
  - Parallel charged conducting plates
  - \( U = qEy \)

**The location where we call \( U = 0 \) is arbitrary!**