First Midterm Exam

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Student ID: 
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Closed book; no notes. Final Numerical results should be given to 2 significant figures. Check that your exam includes all 6 pages and 13 problems or questions. Show all relevant work on this paper or, if necessary, attach extra sheets and indicate clearly where the work is to be found.

The following constants and equations may or may not be needed:

- Coulomb's law constant: \( K = \frac{1}{4\pi \varepsilon_0} = 9.0 \cdot 10^9 \text{ Nm}^2/\text{C}^2 \).
- Permittivity of free space: \( \varepsilon_0 = 8.9 \cdot 10^{-12} \text{ F/m} \).
- Electron mass: \( 9.1 \cdot 10^{-31} \text{ kg} \). Proton mass: \( 1.7 \cdot 10^{-27} \text{ kg} \).
- Electronic charge: \( e = 1.6 \cdot 10^{-19} \text{ C} \).

- \( V_b - V_a = -\int_E \cdot d\ell \) and \( E = -\nabla V = -\left( \frac{\partial V}{\partial x} i + \frac{\partial V}{\partial y} j + \frac{\partial V}{\partial z} k \right) \) \( \frac{\partial V}{\partial x} i + \frac{\partial V}{\partial y} j + \frac{\partial V}{\partial z} k \) assuming that \( \mathbf{E} \) is perpendicular to the plane.

- Planar conductor geometry: \( V(z) = -Ez \), \( \mathbf{E} = \frac{\sigma}{\varepsilon_0} \hat{z} \) assuming that \( \hat{z} \) is perpendicular to the plane.

- Electric dipoles: \( p = qd \), \( \mathbf{r} = \hat{p} \times \mathbf{E} \), \( U = -\hat{p} \cdot \mathbf{E} \)
- Work done by a conservative force: \( W_{a \rightarrow b} = U_a - U_b = -\Delta U \)
- Conservation of energy: \( \Delta K = -\Delta U \) where \( K = \frac{1}{2}mv^2 \)
1. (3 pts) Suppose that you need to determine whether a ball hanging from an insulating thread is negatively charged. Which of the following tests would give a definitive answer?
   
   a) See whether it is repelled by a negatively charged rubber rod.
   b) See whether it is attracted to a positively charged plastic rod.
   c) Both tests (a) and (b) would be equally effective in yielding the answer.

2. (3 pts) The electric field at an insulating surface is
   
   a) always zero.
   b) always perpendicular to the surface.
   c) always parallel to the surface.
   d) in various directions depending on how the charge is distributed on and around the surface.

3. (3 pts) Two charged objects have opposite charges with relative magnitudes in the ratio 4:1. Which of the following diagrams best represents the force vectors on the charges?

   ![Diagram of charged objects and force vectors]

   a)  
   b)  
   c)  
   d)  

4. (3 pts) Metallic conducting surfaces can be charged either negative or positive.
   
   a) True.
   b) False.

5. (3 pts) A conducting spherical shell of radius 2 m is at a potential of 100 V. What is the potential \( V \) at the center of the empty sphere?
   
   a) 0  
   b) 50 V  
   c) 100 V  
   d) 200 V

6. (5 pts) A static uniformly charge rod with length 2a and total charge \( Q \) lies on the y axis as shown below. Which of the following expressions will correctly give the potential at point \( P \), a distance \( x \) from the y axis along the perpendicular bisector of the rod? Show some work for possible partial credit. (Note: I worked this example in lecture.)

   ![Diagram of charge rod and potential calculating]

   ![Potential expressions]

   a)  
   b)  
   c)  
   d)  
   e)  
   f)  

   \[ V = \frac{Q/2a}{4\pi \varepsilon_0} \cdot \int \frac{dy}{\sqrt{x^2 + y^2}} \]
7. (5 pts) A thin plastic rod with uniform linear charge density $\lambda$ is bent into the quarter circle shown below. Which of the following integrals will properly give the $x$ component of the electric field at the origin? Show some work for possible partial credit.

a) $E_x = \frac{1}{4\pi \varepsilon_0} \frac{\lambda}{R} \int_0^{\pi/2} d\theta$

b) $E_x = \frac{1}{4\pi \varepsilon_0} \frac{\lambda}{R^2} \int_0^{\pi/2} \sin \theta \, d\theta$

c) $E_x = \frac{1}{4\pi \varepsilon_0} \frac{\lambda}{R} \int_0^{\pi/2} \sin \theta \, d\theta$

d) $E_x = \frac{1}{4\pi \varepsilon_0} \frac{\lambda}{R} \int_0^{\pi/2} \cos \theta \, d\theta$

e) $E_x = \frac{1}{4\pi \varepsilon_0} \frac{\lambda}{R^2} \int_0^{\pi/2} \int_0^r r \sin \theta \, r \, dr \, d\theta$

f) $E_x = \frac{1}{4\pi \varepsilon_0} \frac{\lambda}{R^2} \int_0^{\pi/2} d\ell$

g) $E_x = \frac{1}{4\pi \varepsilon_0} \frac{\lambda}{R^2} \int_0^{\pi/2} \frac{R \, dy}{\sqrt{R^2 + y^2}}$

8. (5 pts) In the left-hand figure below, at each of the two black points draw field vectors representing the electric field contributions from each of the 4 charges, all of which are equal in magnitude. Then, in the right-hand figure, draw the resultant field vector at each black point. The lengths of the vectors should roughly indicate the relative field strengths.

![Vectors from Individual Charges](image1)

![Resultant Vectors](image2)

9. (6 pts) An isolated parallel plate capacitor has charge $Q$ on one plate and $-Q$ on the other. The plates have size $L \times L$ and are a distance $s$ apart.

a) If the dimension $L$ of the plates is doubled, the potential difference $\Delta V$ between the plates will change by a factor of

(i) $1/4$  
(ii) $1/2$  
(iii) $1$  
(iv) $2$

b) If instead the charge $Q$ is doubled, the electric field between the plates will change by a factor of

(i) $1/2$  
(ii) $1$  
(iii) $2$  
(iv) $4$

c) If instead the distance $s$ between the plates is doubled, the potential difference $\Delta V$ between the plates will change by a factor of

(i) $1/4$  
(ii) $1/2$  
(iii) $1$  
(iv) $2$
10. (4 pnts) A wire carrying current consists of two segments that differ in diameter by a factor of 2 but are made from the same metal.

\[ \mathbf{J} = \frac{I}{A} \quad A = \pi r^2 \]

a) The current densities in the two segments are related as
   i) \( J_2 = \frac{1}{4} J_1 \)  \quad ii) \( J_2 = \frac{1}{2} J_1 \)  \quad iii) \( J_2 = J_1 \)  \quad iv) \( J_2 = 2J_1 \)

b) The electric field strengths in the two segments are related as
   i) \( E_2 = \frac{1}{4} E_1 \)  \quad ii) \( E_2 = E_1 \)  \quad iii) \( E_2 = 2E_1 \)  \quad iv) \( E_2 = 4E_1 \)

\[ E = \frac{\mathbf{J}}{\sigma} \]

11. (10 pnts) Consider the following parallel plate capacitor, in which an electron is launched with an initial velocity to the right, as indicated by the arrow.

a) Draw an electric field vector to represent the field between the plates.

b) Draw a vector representing the acceleration of the electron.

c) Sketch the electrons path until it hits one of the plates.

d) For the electron, how does the potential energy of point A compare with that of point B?
   i) \( U_A < U_B \)
   ii) \( U_A > U_B \)
   iii) \( U_A = U_B \)

e) How does the electric potential at point A compare with that at point B?
   i) \( V_A < V_B \)
   ii) \( V_A > V_B \)
   iii) \( V_A = V_B \)
12. In the figure below, point \( P \) is on the \( x \) axis, 1.0 cm from the origin. Two identical charges of \( q = 5.0 \text{ nC} \) lie on the \( y \) axis, 2 cm above and below the \( x \) axis.

\[
\begin{align*}
\text{a)} & \quad (5 \text{ pts}) \text{ On the figure above, sketch the electric field vector at point } P \text{ from the upper charge alone and another field vector from the lower charge alone. Label them } E_1 \text{ and } E_2 \text{ respectively. Then sketch the resultant field vector } \vec{E}_1 + \vec{E}_2 \text{ and label it } \vec{E}. \\

\text{b)} & \quad (15 \text{ pts}) \text{ What is the magnitude of the electric field at point } P? \\
E_1 &= E_2 = K \frac{q^2}{r^2} \\
E_x &= E_x^1 + E_x^2 = 2K \frac{q^2}{r^2} \cos \theta \\
E_x &= 2 \cdot 9 \cdot 10^9 \cdot \frac{5 \cdot 10^{-9}}{0.02^2 + 0.01^2} \cdot \frac{1}{\sqrt{5}} \\
E_x &= 8.05 \cdot 10^4 \text{ N/C} \\
E_y^1 &= -E_y^2 \Rightarrow E_y = 0 \\
E_y &= 0 \\
\Rightarrow |\vec{E}| &= E_x = 8.0 \cdot 10^5 \text{ N/C} \\

\text{c)} & \quad (5 \text{ pts}) \text{ What is the angle between the electric field vector and the positive } x \text{ axis?} \\
\theta_E &= 0
\end{align*}
\]
A particle with charge \( q_1 = 8 \ \text{nC} \) and mass \( m_1 = 1.0 \ \text{g} \) is shot directly at a second particle of charge \( q_2 = 200 \ \text{nC} \) that is immobile. If the particles are initially very far apart and the initial speed of the moving particle is \( v_0 = 2 \ \text{m/s} \), how close do the two particles get to each other? (Hint: think about energy.)

\[
\Delta U = -\Delta K \quad \text{Conservation of Energy}
\]

\[
U_f - U_i = -K_f + K_i
\]

\[
U_i = 0 \quad \text{at} \quad r = \infty
\]

\[
K_f = 0 \quad \text{at} \quad r_{\min}
\]

\[
\Rightarrow \quad U_f = K_i
\]

\[
K = \frac{q_1 q_2}{r_{\min}} = \frac{1}{2} m v_0^2
\]

\[
r_{\min} = 2 K \frac{q_1 q_2}{m v_0^2}
\]

\[
= 2 \cdot 9 \cdot 10^9 \cdot 8 \cdot 10^{-9} \cdot 200 \cdot 10^{-9} \quad \frac{(0.001)}{(2)^2}
\]

\[
r_{\min} = 7.2 \cdot 10^{-3} \ \text{m}
\]

\[
r_{\min} = 7.2 \ \text{mm}
\]