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**University of California, Santa Cruz**


# An Introduction to Particle Dark Matter

## Lecture 1

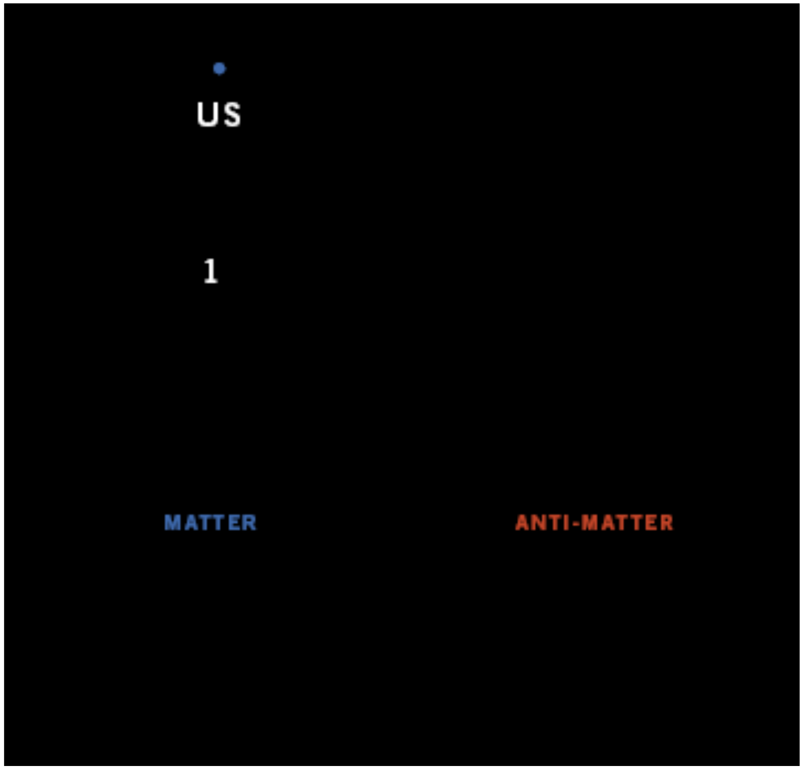
- ✓ PhD **Theoretical Particle Physics** (2004)
  *International School for Advanced Studies (SISSA-ISAS), Trieste, Italy*

- ✓ Postdoc, FSU and California Institute of Technology (2005-2007)
  *Theoretical Astrophysics and Particle Physics*

- ✓ Joined **UCSC Physics** Faculty (Assistant Professor, 2007-2011,
  Associate Professor, July 2011-2015
  Full Professor, July 2015-)

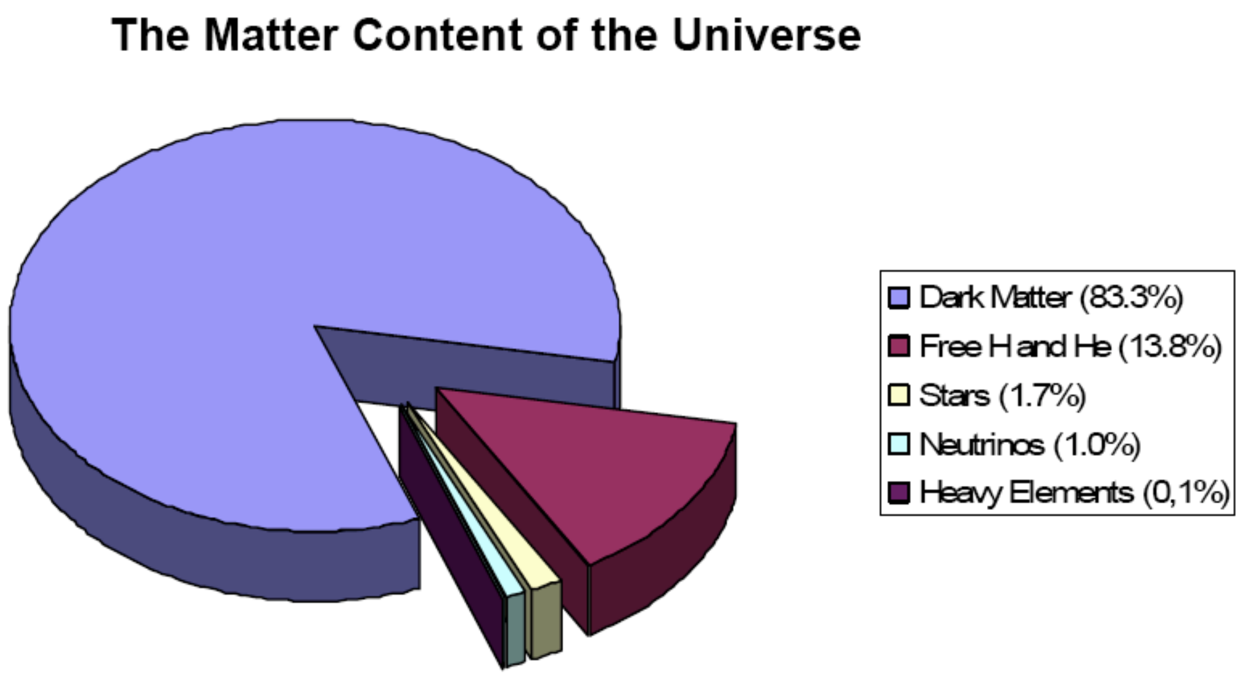
- ✓ Director of UCSC Physics **Graduate Studies** (2012-)

- ✓ **SCIPP Deputy Director** for **Theory** (July 2011-)


1. What is the origin of the tiny excess of matter over anti-matter?

# 2. What is the fundamental particle physics nature of Dark Matter?



Please come **introduce yourselves**!

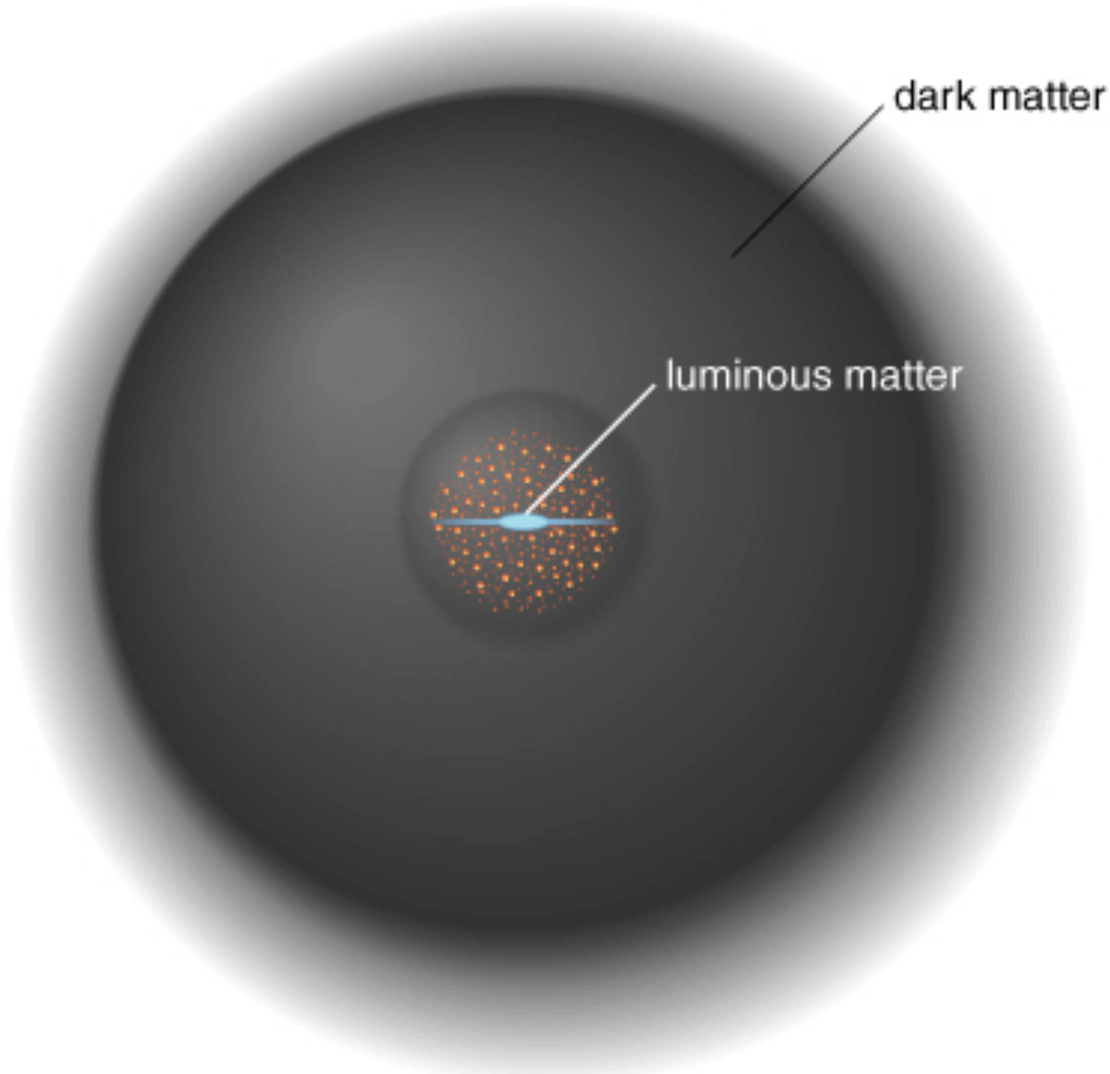
[to myself, other Instructors, to each other...]

If you are ever on the **US West Coast** please let me know!

Never **underestimate** the importance of **networking** in science!

Feel free to **interrupt** me during the lectures!

*(Rare occurrence, but I do occasionally make mistakes…)*


dark matter
luminous matter

**4/5**

a **new**
elementary particle

...as such it is of interest
to **particle physicists**!

## What this mini-lecture series **is not**:

✧ Review of "**evidences**" for dark matter

✧ Review of "**models**" for dark matter

✧ Review of possible claimed "**signals**" from dark matter

*(actually, **two** exceptions…)*

## What this mini-lecture series **will be**

- ✓ Gross **features** of dark matter *as a* ***particle***
- ✓ **Paradigms** for dark matter in the **early universe**
- ✓ **Schematics** of dark matter **searches**
- ✓ Selected **lessons** from **old and new** particle dark matter **models**

One thing we do **know well** about dark matter

**Global amount** of dark matter in the **universe**

**Reason**: very good handles on **total energy density**, total **matter** density, total **baryonic** matter

**CMB** data indicate the **universe** is nearly **flat**

→ energy density is close to **critical**...

What is the **critical density**? (very good number to have in mind!)

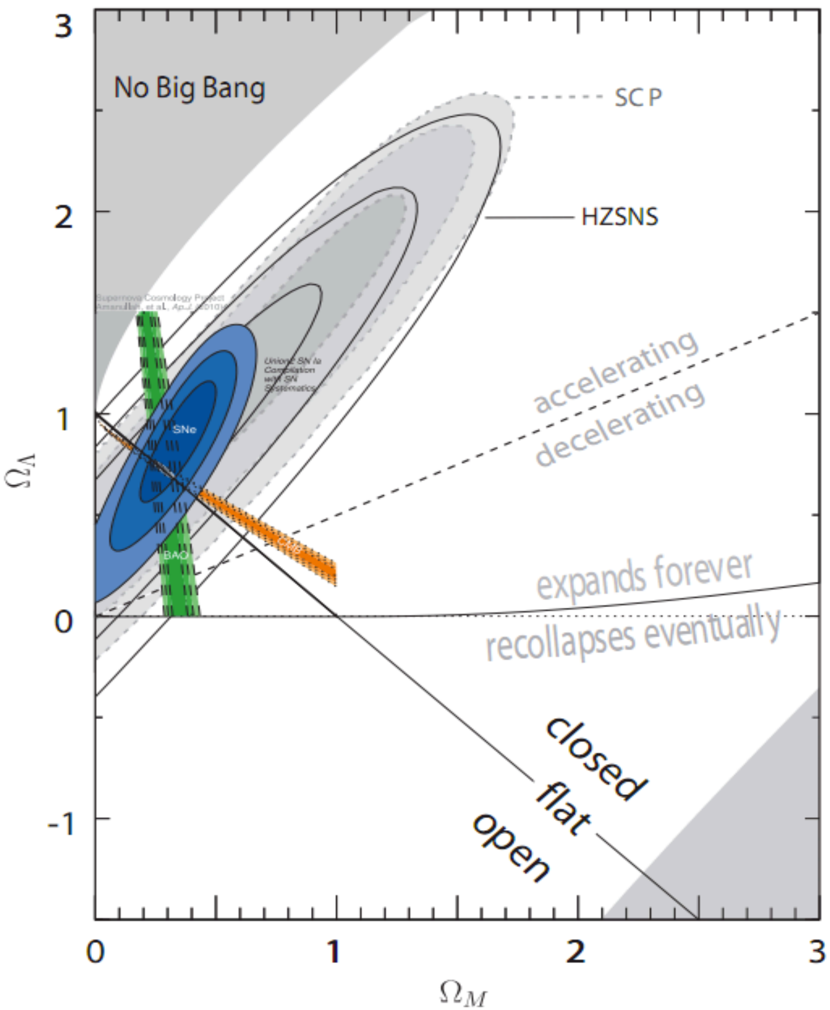
$$\rho_{\rm crit} \equiv \frac{3H_0^2}{8\pi G_N} \simeq 10^{-29}\ {\rm g/cm^3}$$

...since 1 GeV ~$10^{-24}$ g, **10 protons** per **cubic meter** (=tiny!)

Various ways to "**weigh**" **matter** versus dark energy (CMB+SN+BAO)

...and **ordinary** (baryonic) **matter** versus **non-baryonic** (BBN, CMB)

*(see Scott Dodelson's lectures)*



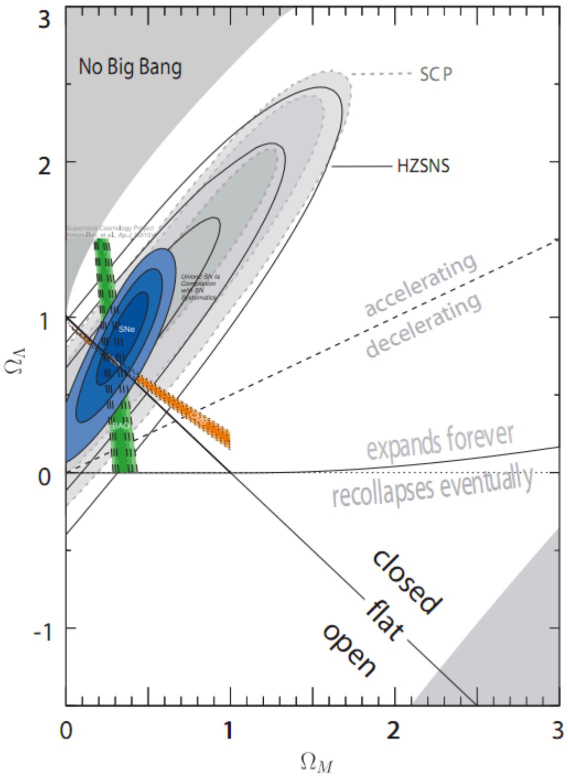

**Global amount** of **dark matter**
in the universe
from simple **subtractions**!

$$\bar{\rho}_{\rm DM} = \Omega_{\rm DM}\rho_{\rm crit} \simeq 0.3\rho_{\rm crit}.$$

DM average density in "**astro**" units...

$$\bar{\rho}_{\rm DM} \sim 10^{10}\ \frac{M_\odot}{\rm Mpc^3}$$

**clusters**... $10^5$ denser!




in "**particle physics**" units...

$$\bar{\rho}_{\rm DM} \sim 10^{-6}\ \frac{\rm GeV}{\rm cm^3}$$

**galaxies**... $10^6$ denser!

$$\delta\rho/\rho \gg 1$$

the Universe is highly **non-linear**!

...which is one of the **key** reasons why **modified gravity** as an alternative to dark matter does not work!

**CMB** sky is very **boring** – *T* fluctuations very **small**!

***T*** fluctuations proportional to (baryonic) **density** fluctuations,

$$\delta\rho/\rho \lesssim 10^{-4}$$

Matter **over-densities** in linear regime

grow **linearly** with scale factor

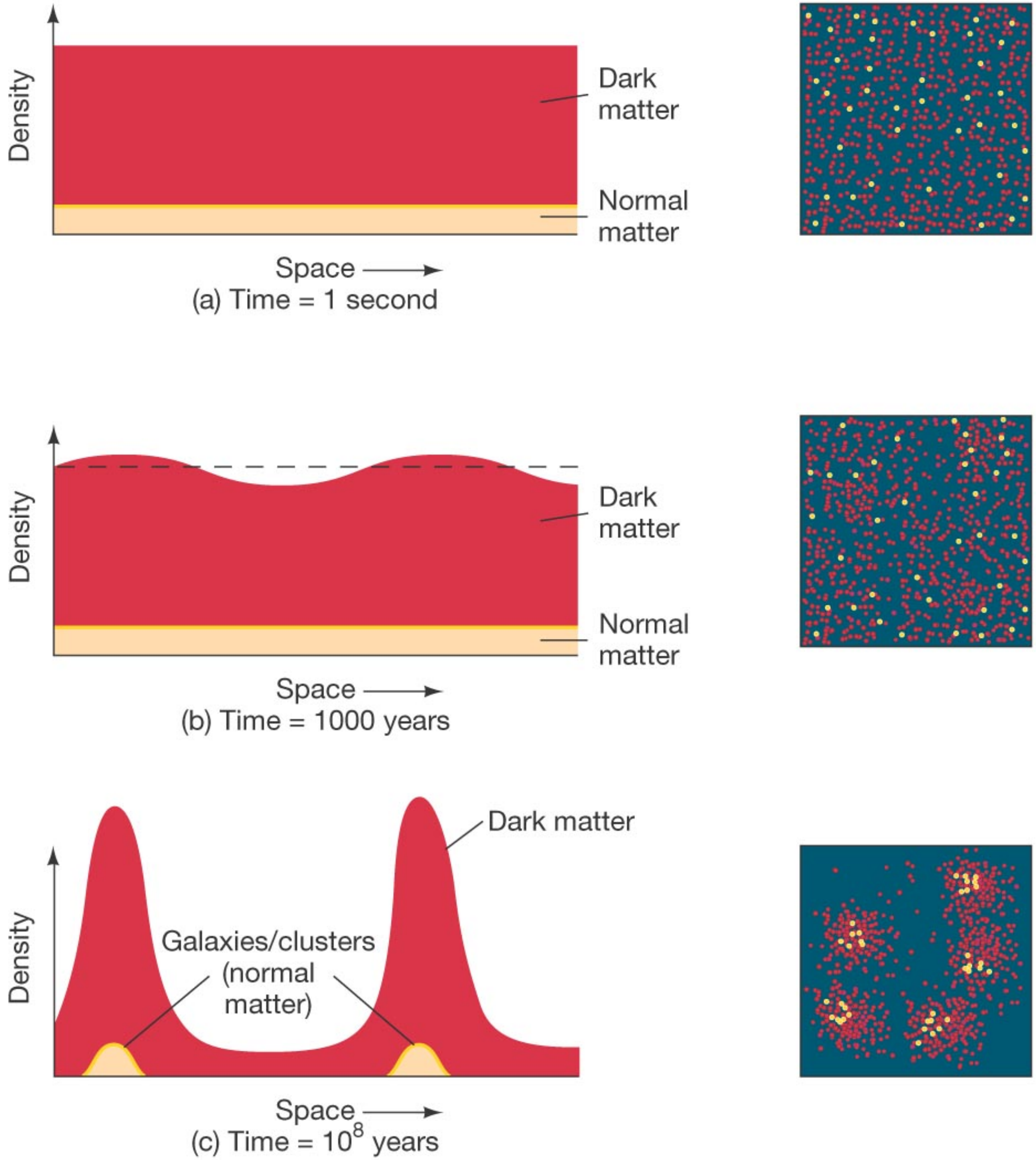
But the scale factor since CMB decoupling grew by ***$z_{rec}$~1,100***

Not enough time (since recombination(for structures to go **non-linear**!

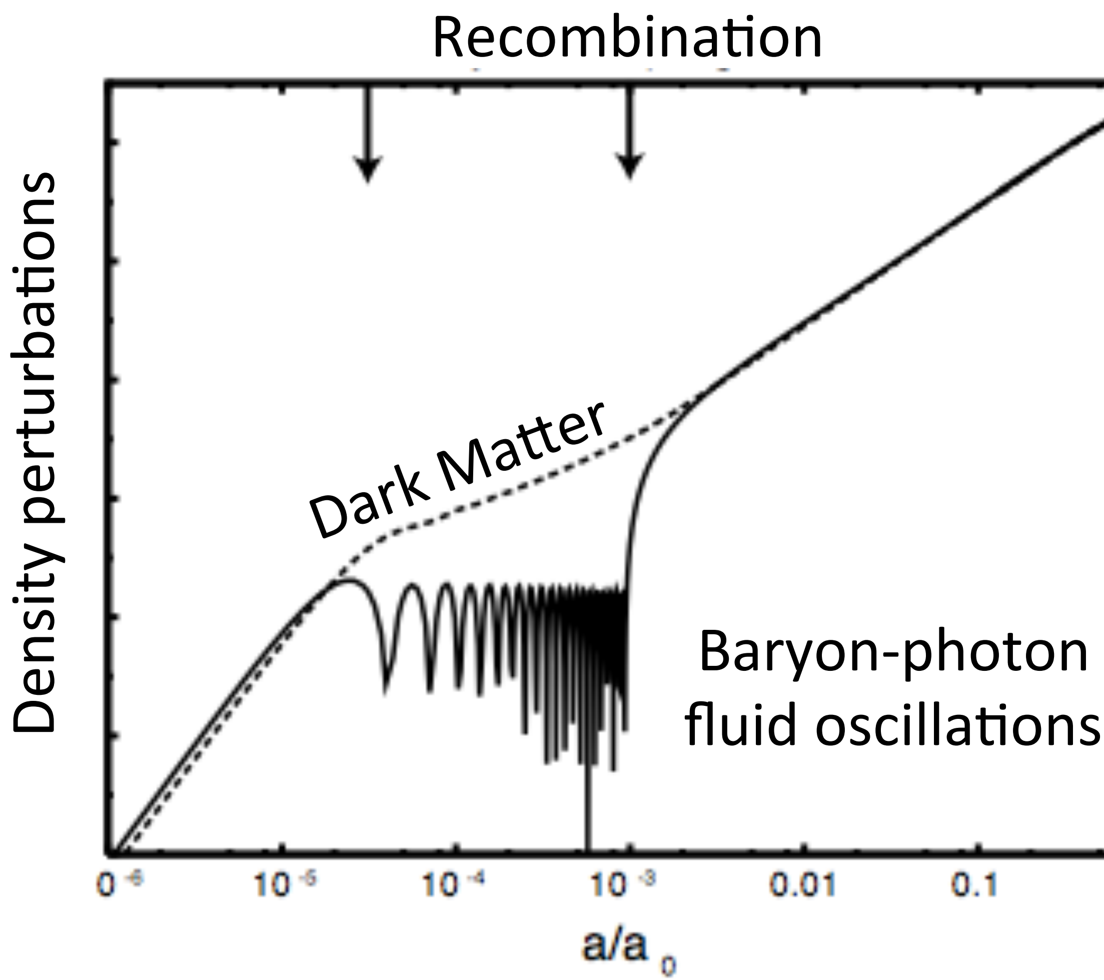
We need a **species** that has **decoupled** from photons much earlier (**Dark Matter**) so that its density **perturbations** are much **larger** at recombination!

$$(\delta\rho/\rho)_{\rm DM} \gg 10^{-4}$$

Dark matter **seeds** timely structure formation!

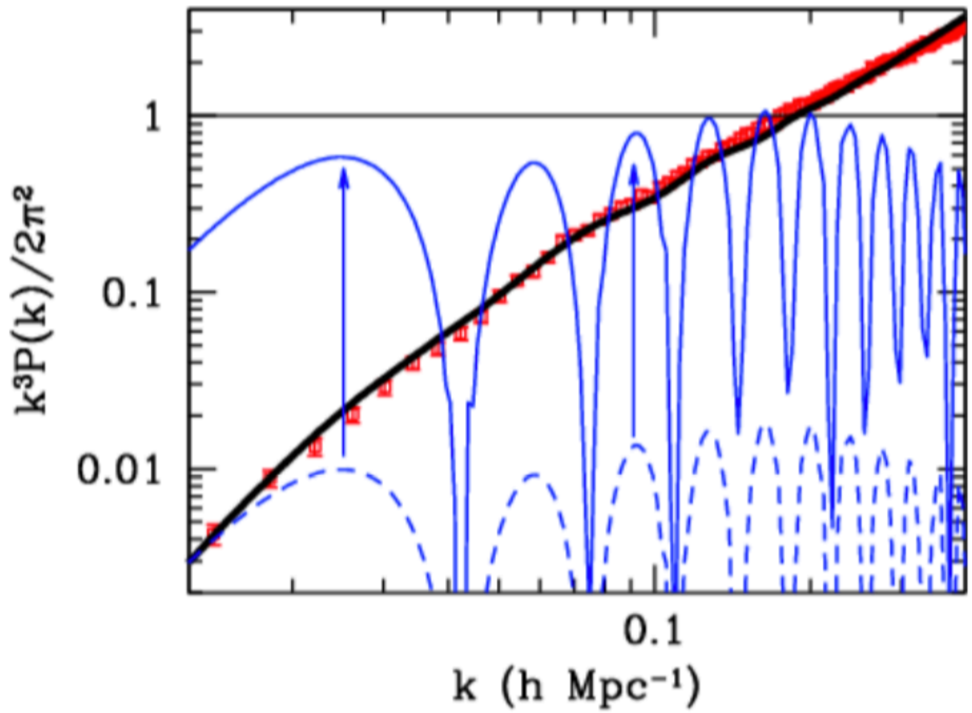


Density
Dark matter
Normal matter
Space
(a) Time = 1 second
Density
Dark matter
Normal matter
Space
(b) Time = 1000 years
Dark matter
Density
Galaxies/clusters (normal matter)
Space
(c) Time = 10^8 years


Recombination
Density perturbations
Dark Matter
Baryon-photon
fluid oscillations
0⁻⁶
10⁻⁵
10⁻⁴
10⁻³
0.01
0.1
1
a/a 0

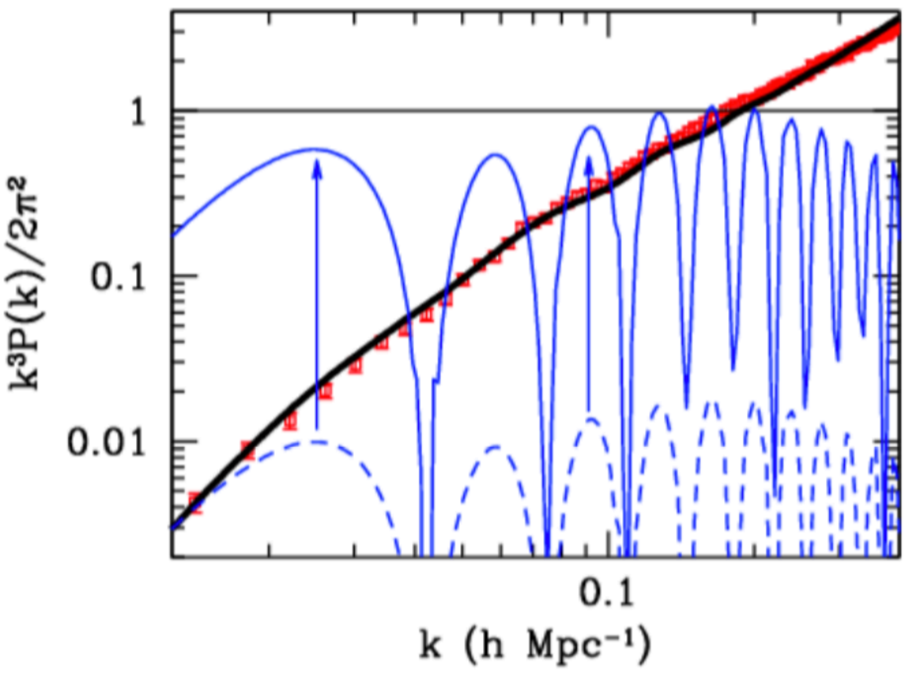
Things go **badly wrong without DM** for structure formation!

Power spectrum of density perturbations (credit: **Scott Dodelson**!)




$\delta\rho/\rho \sim 1$

Even with best (covariant) incarnation of modified gravity (TeVeS), structure goes non-linear, but the **power spectrum** of matter density fluctuation is **entirely wrong**...




Don’t get fooled by the “**Vulcan**” versus “**Neptune**” analogy

*[Vulcan: No new planet between Mercury and the Sun, but GR*
*Neptune: New planet]*

**Modified Gravity** [MOND,TeVeS] actually **does not work** at all!!

Knowledge of the dark matter average **density**
is a powerful **model-building** tool

Models that **predict** the "right" **amount** of dark matter get kudos

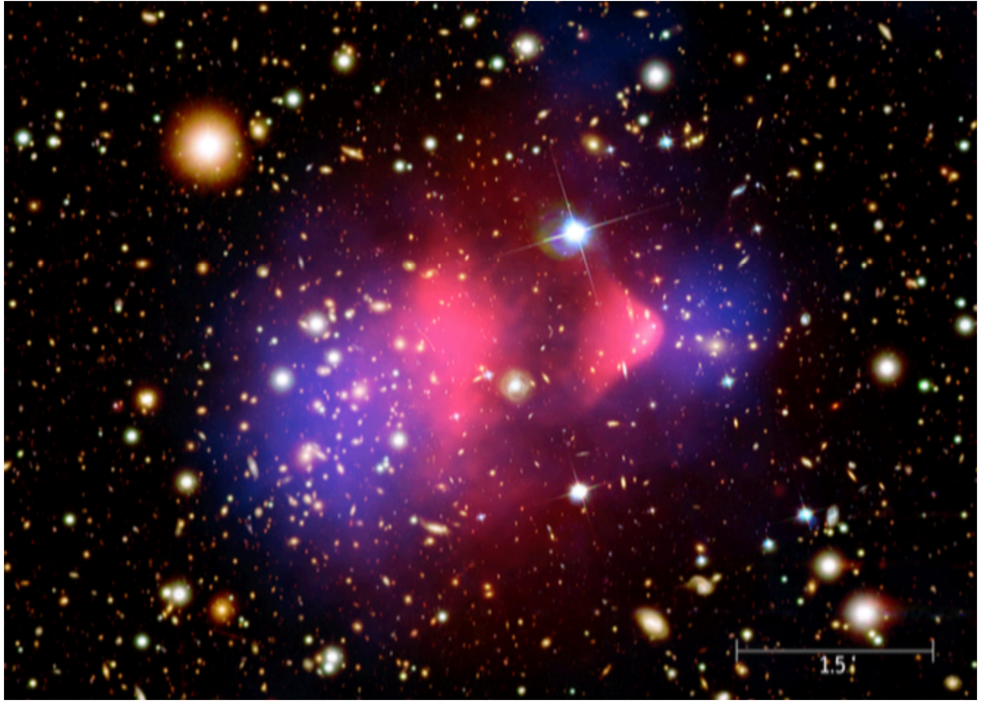
Dark Matter "**cosmogony**" well-motivated guideline to model building

prototypical example: dark matter
as a ***thermal relic***… more on this shortly

What else do we know about the **microscopic** nature of dark matter from its **macroscopic** features?

- "**Dark**": ...for the **reason above**! But detailed constraints on electric charge of dark matter are model-dependent... Milli-charge allowed... Phenomenologically: DM is nearly **dissipationless** (maybe not entirely though, see dark photons, dark disks...)

- **Collisionless**... really? Let's calculate the relevant constraints!

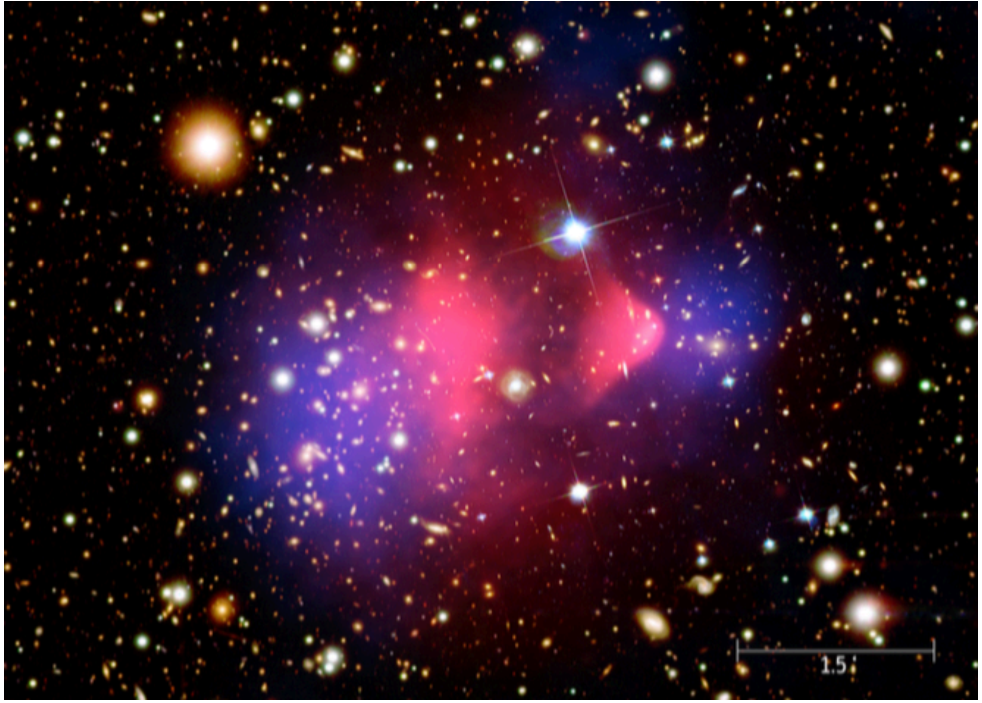



**mean free path $\lambda$** larger than cluster size, ~ 1 Mpc

cluster **density**: $\rho$ ~ 1 GeV/$cm^3$, thus…

$\lambda = 1/(\sigma\,(\rho/m)) > 1$ Mpc → $\sigma/m < 1$ Mpc / 1 GeV/$cm^3$

→ ***$\sigma/m < 1\ cm^2/g$***, or ***1 barn/GeV***



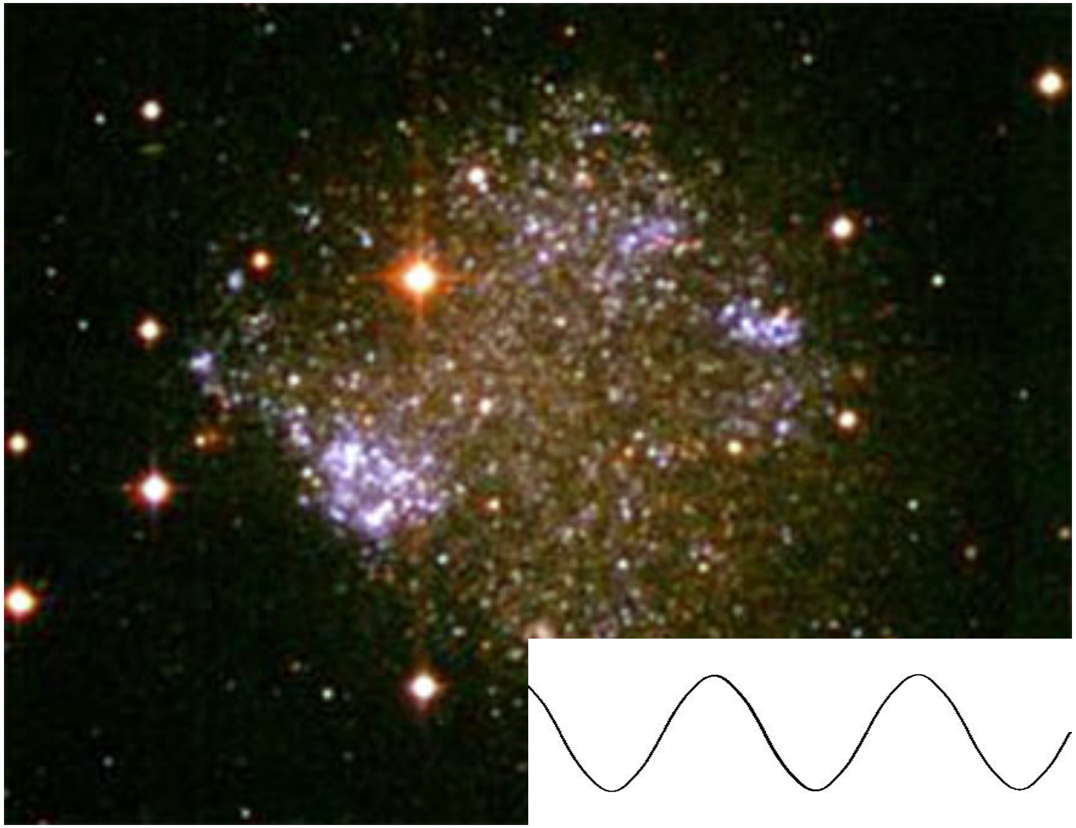

**1 barn/GeV**... which is **strong interaction**-size...

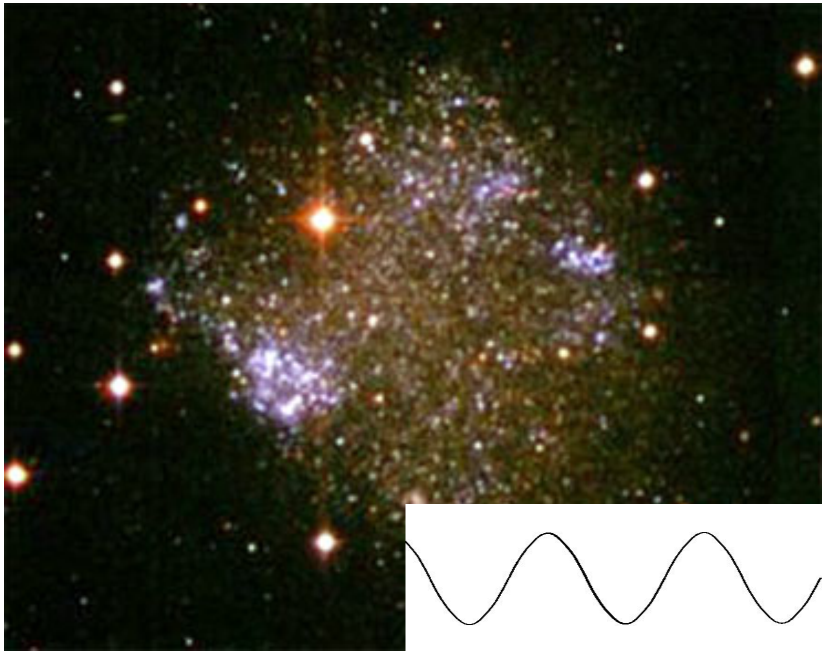
is this **small**?

Also, if cross section is **slightly smaller**, no **visible effect**...
if cross section **slightly larger**, **disaster**...

Begs the question: is "collisional" **self-interacting** dark matter a "**natural**" possibility??

- **Classical**: needs to be confined (gravitationally bound) on scales at least as large as dSph... if de Broglie wavelength is larger, disaster strikes!





little exercise: consider $v \sim 100\ km/s$, show that $\lambda = h/p$ is

$$\lambda \sim 3\ \mathrm{mm}\ \left(\frac{1\ \mathrm{eV}}{m}\right)$$

which means that to have $\lambda \ll$ kpc ~ $3 \times 10^{21}$ cm, **$m > 10^{-22}$ eV**

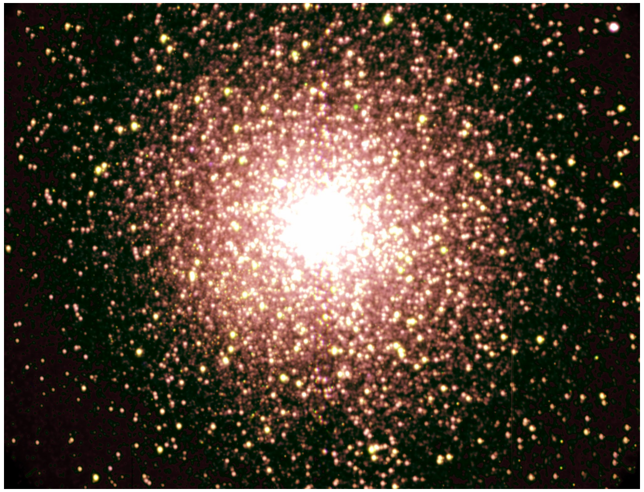
Much, much **better constraints** if the DM is a fermion – we know that the **phase space** density is bounded (Pauli blocking): ***$f = gh^{-3}$***

Using **observed** density and velocity dispersion of dSph, **Tremaine-Gunn** limit (1979): observed phase space density cannot exceed upper bound! (Liouville theorem) Exercise!

$\sigma \sim 150\ \mathrm{km/s}$ $\rho \gtrsim 1\ \mathrm{GeV/cm^3}$

$$m^4 > \frac{\rho h^3}{[g(2\pi\sigma^2)^{3/2}]} \sim (25\ \mathrm{eV})^4.$$

- **Fluid**: don't want to **disrupt** pretty (and old!) **clusters** of stars



Neat exercise to estimate the **energy exchanged** by encounters of GC and BH, in the impulse approximation, demand that that energy be smaller than binding energy, get maximal mass for BH

Also constraints on **disk stability** ("heating")

Bottom line: ***$m < 10^3$*** solar masses ***$\sim 10^{70}$ eV***

…here's the **name of the game**:

(i) **Mass**: >**90** orders of magnitude for **bosons**, **70** for **fermions**

(ii) **Interactions**: ~**dark**, **self-interacting** at most ~ strong interactions

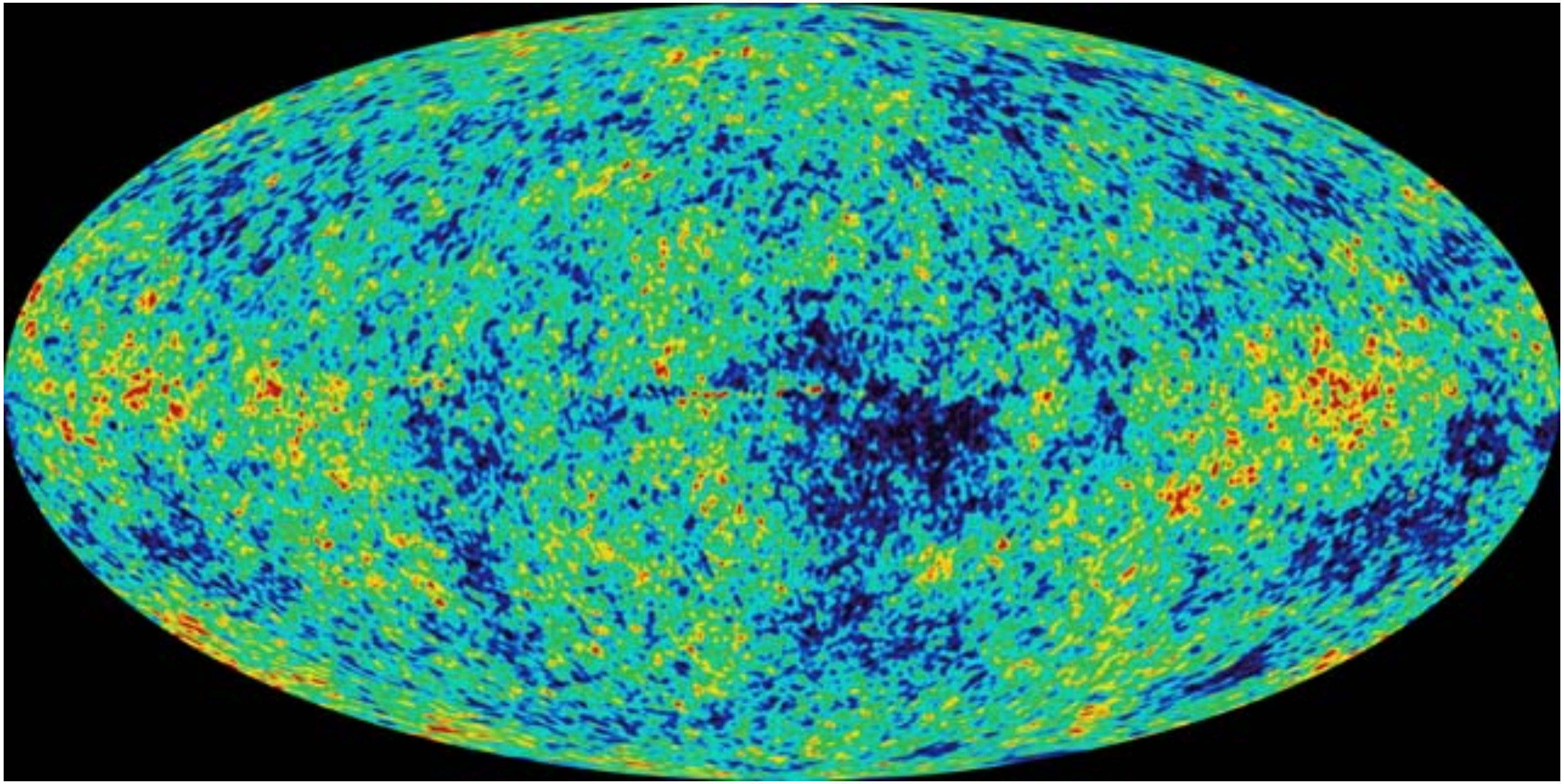
(iii) **Abundance**

Think left and think right and think low and think high.

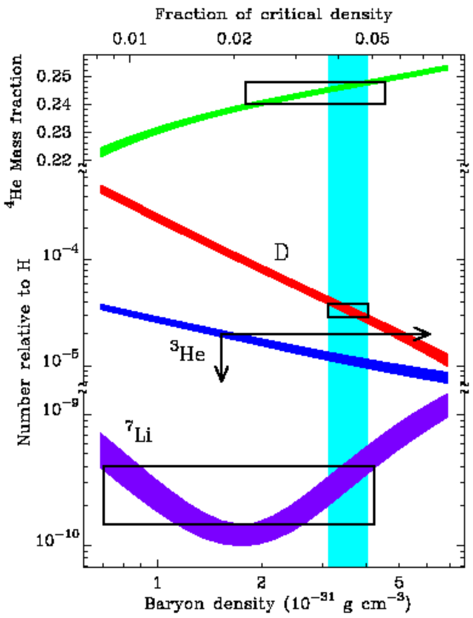
Oh the things you can think up, if only you try!

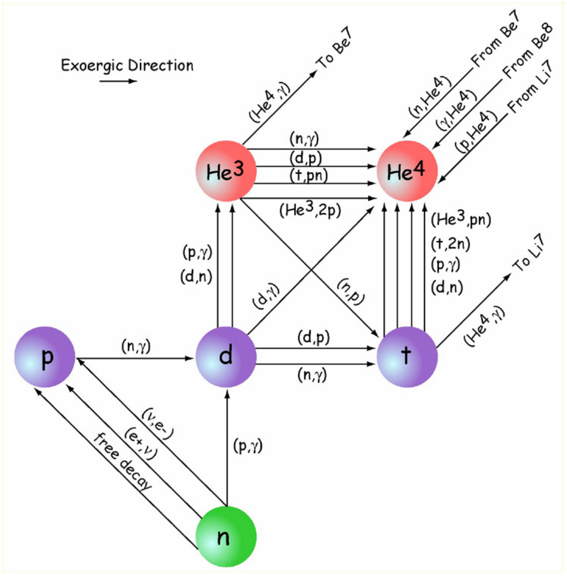
*Dr. Seuss*

A successful framework for the **origin of species** in the early universe: **thermal decoupling**



A successful framework for the **origin of species** in the early universe: **thermal decoupling**




A successful **synergy** of ***statistical mechanics***, ***general relativity***, and of ***nuclear and particle physics*** making **predictions** testable to exquisite accuracy with ***astronomical*** observations!

Key **idea** of thermal decoupling:

if the **reaction** keeping a species in equilibrium is **faster** than the **expansion rate** of the universe, the reaction is in **statistical equilibrium**; if it's **slower**, the species **decouples** ("freeze-out")

$$\Gamma \ll H(T) \qquad \qquad \Gamma(T_{\mathrm{t.o.}}) \sim H(T_{\mathrm{t.o.}})$$

the **reaction rate** (from definition of cross section!)

$$\Gamma = n \cdot \sigma \cdot v$$

(1) borrow **equilibrium number densities** from stat mech

$$n_{\rm rel} \sim T^3 \quad \text{for} \;\; m \ll T,$$
$$n_{\rm non-rel} \sim (mT)^{3/2} \exp\left(-\frac{m}{T}\right) \quad \text{for} \;\; m \gg T.$$

(2) borrow **Hubble rate** from general relativity
(FRW **solution** to Einstein's eq.)

$$H^2 = \frac{8\pi G_N}{3}\rho.$$

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GR+SM: **energy density** in radiation

$$\rho \simeq \rho_{\rm rad} = \frac{\pi^2}{30} \cdot g \cdot T^4 \quad \longrightarrow \quad H \simeq T^2/M_P$$

first application: **hot** thermal relic

language definition: **hot** = relativistic at $T_{f.o}$

**cold** = $v<c=1$. (actually not by much, typically!)

simple **application**: **relic** SM **neutrinos** (cosmo ν background)

$$\nu + \bar{\nu} \leftrightarrow f + \bar{f},$$

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$$n(T_\nu) \cdot \sigma(T_\nu) = H(T_\nu) \qquad \sigma \sim G_F^2 T_\nu^2$$

suppose this is a hot relic... ***$n \sim T_\nu^3$***

$$T_\nu^3 G_F^2 T_\nu^2 = T_\nu^2 / M_P$$

$$T_\nu = (G_F^2 M_P)^{-1/3} \simeq (10^{-10} \times 10^{18})^{-1/3}\ \mathrm{GeV} \sim 1\ \mathrm{MeV}$$

**happy** about **two** things in particular:

1. **hot** relic assumption works! $T_\nu \gg m_\nu$

2. **Fermi** effective theory OK! $T_\nu \ll m_W$

$$T_\nu = (G_F^2 M_P)^{-1/3} \simeq (10^{-10} \times 10^{18})^{-1/3}\ \mathrm{GeV} \sim 1\ \mathrm{MeV}$$

now, how do we calculate the **relic** thermal **abundance** of this prototypical hot relic?

Introduce ***Y=n/s*** (number and entropy **density**, $V=a^3$)

If universe is iso-entropic, ***$s \times a^3 = S$*** is conserved

***$Y \sim n\, a^3$*** is thus ~ **comoving number density**, and (without entropy injection)

$$Y_{\rm today} = Y_{\rm freeze-out} = Y(T_\nu)$$

$$Y_{\rm freeze-out} = \frac{n(T_\nu)}{s(T_\nu)} = \frac{\rho_\nu(T_\nu)}{m_\nu \cdot s(T_\nu)}$$

$$Y_{\text{today}} = Y_{\text{freeze-out}} = Y(T_\nu)$$

$$Y_{\text{freeze-out}} = \frac{n(T_\nu)}{s(T_\nu)} = \frac{\rho_\nu(T_\nu)}{m_\nu \cdot s(T_\nu)}$$

$$n_{\text{today}} = s_{\text{today}} \times Y_{\text{today}} = s_{\text{today}} \times Y_{\text{freeze-out}}$$

$$\rho_{\nu,\text{today}} = m_\nu \times Y_{\text{freeze-out}} \times s_{\text{today}}$$

$$\Omega_\nu h^2 = \frac{\rho_\nu}{\rho_{\text{crit}}} h^2 \simeq \frac{m_\nu}{91.5\text{ eV}}$$

**Cowsik-Mc-Clelland** limit

That was **fun**! Let's see if it works for something else...

Try **proton-antiproton** freeze-out:
what's the **relic** matter **abundance** in a baryon-symmetric Universe?

$$\sigma \sim \Lambda_{\rm QCD}^{-2}$$

$$n\,\sigma = H \rightarrow T^3 \Lambda^{-2} = T^2/M_P \rightarrow T = \Lambda^2/M_P$$

doesn't quite work, we're way **outside**
the regime of validity for **hot relics**, since $T \lll\lll m_p$ ...

Need to work out the case of **cold relics**, which looks nastier by eye

$$n \sim (m_\chi T)^{3/2} \exp\left(-\frac{m_\chi}{T}\right)$$

Here's the trick: **freeze-out** condition gives

$$n_{\mathrm{f.o.}} \sim \frac{T^2_{\mathrm{f.o.}}}{M_P \cdot \sigma}$$

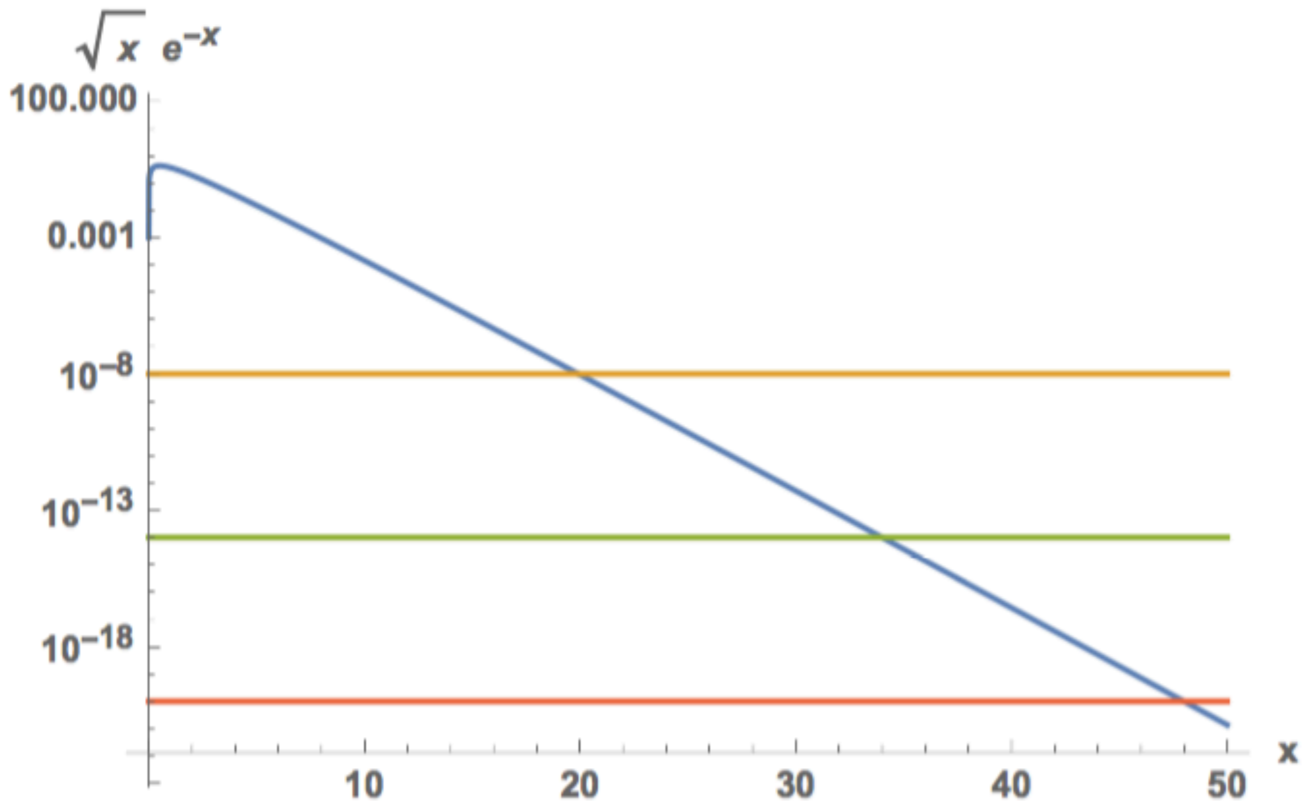
now define $m_\chi / T \equiv x$ (cold relic: **x>>1**)

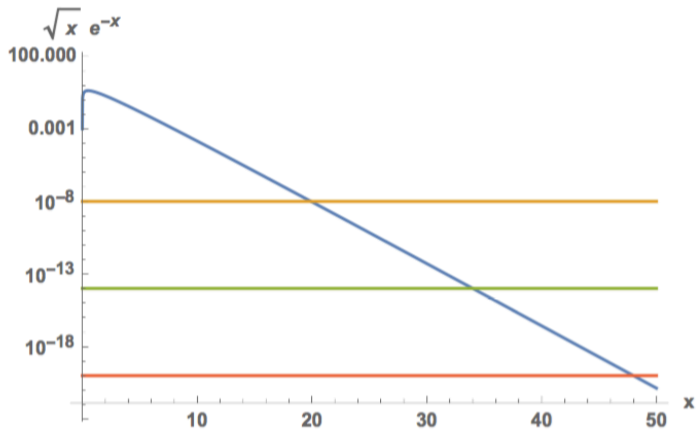
**Freeze-out** condition (x) now reads

$$\frac{m_\chi^3}{x^{3/2}} e^{-x} = \frac{m_\chi^2}{x^2 \cdot M_P \cdot \sigma}$$

…so we gotta **solve** $\sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma}$

$$\sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma}$$






$$\sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma}$$

$$\sigma \sim G_F^2 m_\chi^2$$

Take e.g. a "**weakly interacting massive particle**"

$$m_\chi \sim 10^2 \text{ GeV}.$$

$$\sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma} \sim \frac{1}{10^2 \cdot 10^{18} \cdot 10^{-6}} \sim 10^{-14}.$$

thus ***$x = m_\chi / T \sim 35$***

Off to calculating the **thermal relic density**

$$\Omega_\chi = \frac{m_\chi \cdot n_\chi(T = T_0)}{\rho_c} = \frac{m_\chi\, T_0^3}{\rho_c} \frac{n_0}{T_0^3}$$

iso-entropic universe $aT$ ~const

$$\frac{n_0}{T_0^3} \simeq \frac{n_{\rm f.o.}}{T_{\rm f.o.}^3}$$

$$\Omega_\chi = \frac{m_\chi\, T_0^3}{\rho_c} \frac{n_{\rm f.o.}}{T_{\rm f.o.}^3} = \frac{T_0^3}{\rho_c} x_{\rm f.o.} \left(\frac{n_{\rm f.o.}}{T_{\rm f.o.}^2}\right) = \left(\frac{T_0^3}{\rho_c\, M_P}\right) \frac{x_{\rm f.o.}}{\sigma}$$

$$\left(\frac{\Omega_\chi}{0.2}\right) \simeq \frac{x_{\rm f.o.}}{20} \left(\frac{10^{-8}\ {\rm GeV}^{-2}}{\sigma}\right)$$