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An Introduction to Particle Dark Matter

Lecture 2

Key ideas from last lecture

✓ Dark matter is a **key ingredient** to form the **non-linear** structures we observe – not enough time with matter only!

✓ Features of a good **DM candidate**:

(i) **Mass**: >**90** orders of magnitude for **bosons**, **70** for **fermions**
(ii) **Interactions**: ~**dark**, **self-interacting** at most ~ strong int.
(iii) **Abundance**

✓ **Thermal decoupling** very successful paradigm (CMB, BBN)

✓ **Hot** relics: $\Omega_\nu \sim m_\nu$

✓ **Cold** relics: $\Omega \sim 1/\sigma$

trick: **freeze-out** condition gives

$$\Gamma = n \cdot \sigma \cdot v$$
$$H \simeq T^2/M_P$$

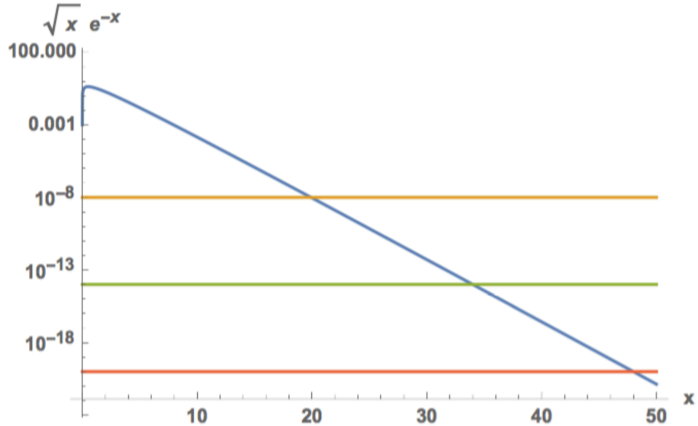
$$n_{\rm f.o.} \sim \frac{T_{\rm f.o.}^2}{M_P \cdot \sigma}$$

$$m_\chi/T \equiv x$$ (cold relic: **x>>1**)

Freeze-out condition (x) now reads

$$\frac{m_\chi^3}{x^{3/2}} e^{-x} = \frac{m_\chi^2}{x^2 \cdot M_P \cdot \sigma}$$

...so we gotta **solve** $$\sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma}$$



$$\sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma}$$

$$\sigma \sim G_F^2 m_\chi^2$$

Take e.g. a "**weakly interacting massive particle**"

$$m_\chi \sim 10^2 \text{ GeV.}$$

$$\sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma} \sim \frac{1}{10^2 \cdot 10^{18} \cdot 10^{-6}} \sim 10^{-14}.$$

thus ***$x = m_\chi / T \sim 35$***

Off to calculating the **thermal relic density**

$$\Omega_\chi = \frac{m_\chi \cdot n_\chi(T = T_0)}{\rho_c} = \frac{m_\chi\, T_0^3}{\rho_c} \frac{n_0}{T_0^3}$$

iso-entropic universe aT ~const $\qquad \frac{n_0}{T_0^3} \simeq \frac{n_{\rm f.o.}}{T_{\rm f.o.}^3}$

$$\Omega_\chi = \frac{m_\chi\, T_0^3}{\rho_c} \frac{n_{\rm f.o.}}{T_{\rm f.o.}^3} = \frac{T_0^3}{\rho_c} x_{\rm f.o.} \left(\frac{n_{\rm f.o.}}{T_{\rm f.o.}^2}\right) = \left(\frac{T_0^3}{\rho_c\, M_P}\right) \frac{x_{\rm f.o.}}{\sigma}$$

$$\left(\frac{\Omega_\chi}{0.2}\right) \simeq \frac{x_{\rm f.o.}}{20} \left(\frac{10^{-8}\ {\rm GeV}^{-2}}{\sigma}\right)$$

Notice we neglected relative **velocity**...
What is the velocity of a cold relic at freeze-out?

$$\frac{3}{2}T = \frac{1}{2}mv^2$$

...just use **equipartition** theorem... $v=(3/x)^{1/2}$ ~ 0.3

Now, back to **relic density**:

$$\left(\frac{\Omega_\chi}{0.2}\right) \simeq \frac{x_{\rm f.o.}}{20}\left(\frac{10^{-8}\ {\rm GeV}^{-2}}{\sigma}\right)$$

$$\sigma_{\rm EW} \sim G_F^2 T_{\rm f.o.}^2 \sim G_F^2\left(\frac{E_{\rm EW}}{20}\right)^2 \sim 10^{-8}\ {\rm GeV}^{-2},$$

$$\left(\frac{\Omega_\chi}{0.2}\right) \simeq \frac{x_{\rm f.o.}}{20}\left(\frac{10^{-8}\ {\rm GeV}^{-2}}{\sigma}\right)$$

Is this **unique** to **WIMPs**? **No**.

$$\sigma \sim \frac{g^4}{m_\chi^2}$$

"**WIMPless**" miracle... what did we use?

$$m_\chi \cdot \sigma \cdot M_P \gg 1$$

$$\sigma \sim 10^{-8}\ {\rm GeV}^{-2}$$

Substitute and find that ***m_χ >> 0.1 eV*** !

In practice various **constraints** on light thermal relics from structure formation, relativistic degrees of freedom at BBN, CMB... ***m_χ > MeV***

What is the **range** of **masses** expected for cold relics?

Cross section cannot be arbitrarily large: **unitarity** limit

$$\sigma \lesssim \frac{4\pi}{m_\chi^2}$$

$$\frac{\Omega_\chi}{0.2} \gtrsim 10^{-8}\ \mathrm{GeV}^{-2} \cdot \frac{m_\chi^2}{4\pi}$$

$$\left(\frac{m_\chi}{120\ \mathrm{TeV}}\right)^2 \lesssim 1$$

What is the **range** of **masses** expected for cold relics?

If you have a **WIMP**, defined by a cross section $\sigma \sim G_F^2 \, m_\chi^2$

$$\Omega_\chi h^2 \sim 0.1 \frac{10^{-8}\ \mathrm{GeV}^{-2}}{G_F^2 \, m_\chi^2} \sim 0.1 \left(\frac{10\ \mathrm{GeV}}{m_\chi} \right)^2$$

"**Lee-Weinberg**" limit

Discussion so far OK for a **qualitative** assessment of **relic density**

State of the art much more sophisticated: Solve **Boltzmann equation**

$$\hat{L}[f] = \hat{C}[f]$$

$$\hat{L}_{\rm NR} = \frac{\rm d}{{\rm d}t} + \frac{{\rm d}\vec{x}}{{\rm d}t}\vec{\nabla}_x + \frac{{\rm d}\vec{v}}{{\rm d}t}\vec{\nabla}_v$$

$$\hat{L}_{\rm cov} = p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma}\, p^\beta\, p^\gamma \frac{\partial}{\partial p^\alpha}$$

Looks ugly, but for the **FRW** metric **phase-space** density simplifies…

$$f(\vec{x},\vec{p},t) \rightarrow f(|\vec{p}|,t) \qquad\qquad f(E,t)$$

$$\hat{L}[f] = E\frac{\partial f}{\partial t} - \frac{\dot{a}}{a}|\vec{p}|^2\,\frac{\partial f}{\partial E}$$

Now, what we are interested in are **number** densities, which in terms of **phase-space** densities are simply…

$$n(t) = \sum_{\text{spin}} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} f(E,t)$$

…**integrate** the Liouville operator over **momentum space** and get

$$\int L[f] \cdot g \frac{\mathrm{d}^3 p}{(2\pi)^3} = \frac{\mathrm{d}n}{\mathrm{d}t} + 3H \cdot n,$$

Back to **Boltzmann** equation, suppose a **2-to-2** reaction, with 3, 4 **in eq.**

$$1 + 2 \leftrightarrow 3 + 4$$

Consider the **collision** factor, and again integrate over **momenta**...

$$g_1 \int \hat{C}[f_1] \frac{\mathrm{d}^3 p}{(2\pi)^3} = -\langle \sigma \cdot v_{\mathrm{M\o l}} \rangle \left(n_1 n_2 - n_1^{\mathrm{eq}} n_2^{\mathrm{eq}} \right)$$

...where the **cross section**

$$\sigma = \sum_f \sigma_{12 \to f}$$

$$g_1 \int \hat{C}[f_1] \frac{d^3p}{(2\pi)^3} = -\langle \sigma \cdot v_{\text{Møl}} \rangle \left(n_1 n_2 - n_1^{\text{eq}} n_2^{\text{eq}} \right)$$

let's understand the rest of the equation:

$$v_{\text{Møl}} \equiv \frac{\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}{E_1 \, E_2}$$

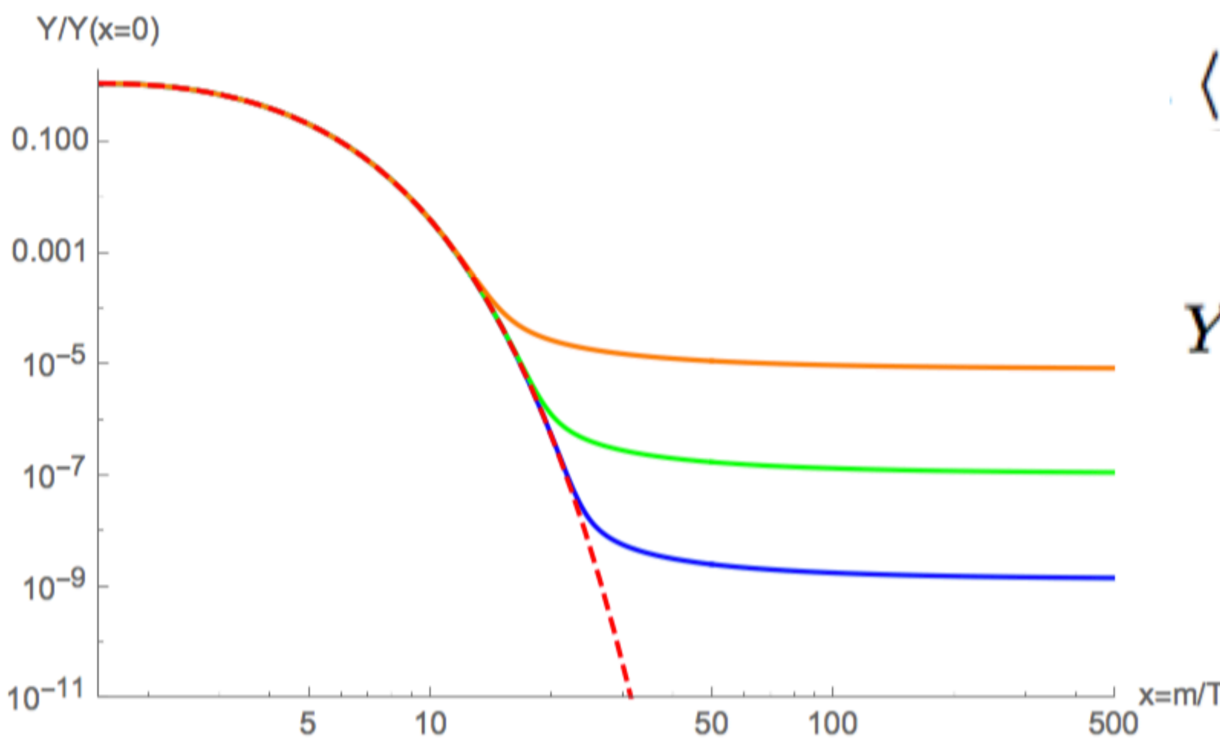
$$\langle \sigma \cdot v_{\text{Møl}} \rangle = \frac{\int \sigma \cdot v_{\text{Møl}} \, e^{-E_1/T} e^{-E_2/T} \, d^3p_1 \, d^3p_2}{\int e^{-E_1/T} e^{-E_2/T} \, d^3p_1 \, d^3p_2}$$

Final version of **Boltzmann Eq.**

$$\dot{n} + 3Hn = \langle \sigma v \rangle \left(n_{\text{eq}}^2 - n^2 \right)$$

$$\dot{n} + 3Hn = \langle\sigma v\rangle \left(n_{\rm eq}^2 - n^2\right)$$

$$\frac{dY(x)}{dx} = -\frac{xs\langle\sigma v\rangle}{H(m)} \left(Y(x)^2 - Y_{\rm eq}^2(x)\right)$$



$$\langle\sigma v\rangle = \langle\sigma v\rangle_0 x^{-n}$$

$$Y_{\rm today} \simeq \frac{n+1}{\lambda} x_{\rm f.o.}^{n+1}$$

$$\lambda = \frac{\langle\sigma v\rangle_0 s_0}{H(m)}$$

There exist important "**exceptions**" to this standard story:

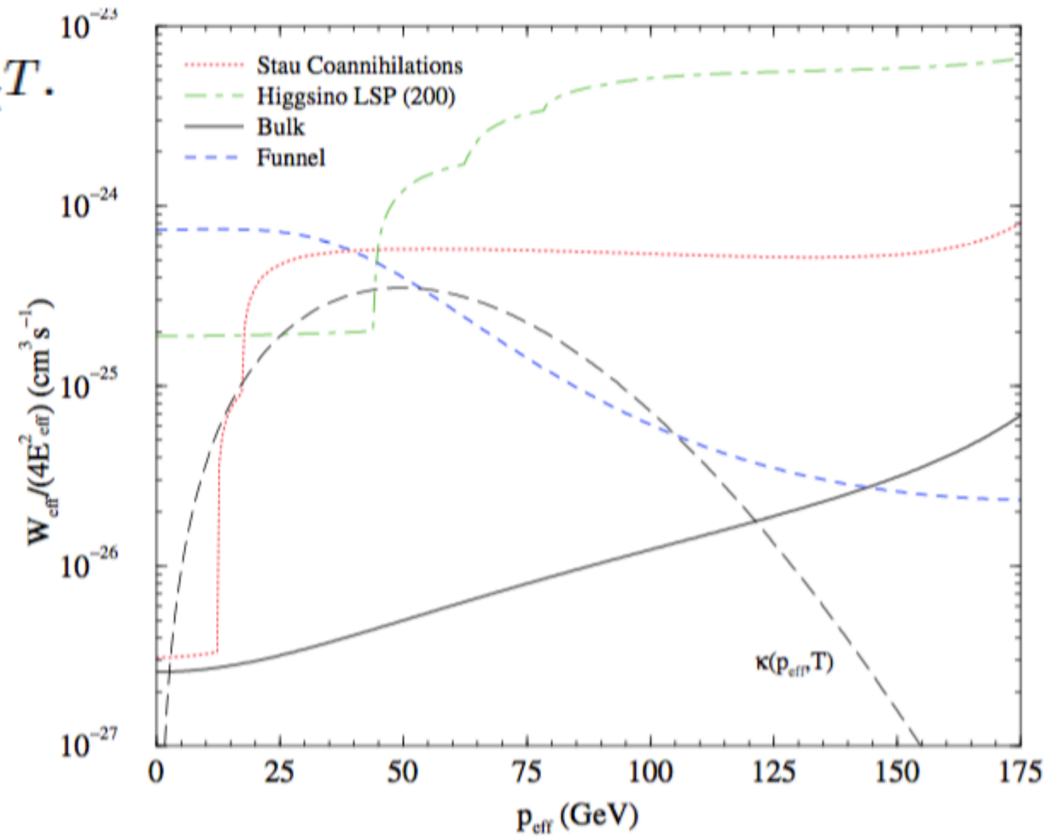
1. **Resonances**

$$\langle s \rangle \simeq 4m_\chi^2 + 6m_\chi T.$$

2. **Thresholds**

3. **Co-annihilation**

$$\langle \sigma v \rangle \; \to \; \langle \sigma_{\rm eff} v \rangle = \frac{\sum_{i<j=1}^{N} \sigma_{ij} \exp\left(-\frac{\Delta m_i + \Delta m_j}{T}\right)}{\sum_{i=1}^{N} g_i \exp\left(-\frac{\Delta m_i}{T}\right)}.$$



Affects what the **pair-annihilation** rate **today** is compared to what it was at **freeze-out**!

$$\langle \sigma_{\rm eff} v \rangle = \int_0^\infty dp_{\rm eff} \frac{W_{\rm eff}(p_{\rm eff})}{4E_{\rm eff}^2} \kappa(p_{\rm eff}, T) \qquad E_{\rm eff}^2 = \sqrt{p_{\rm eff}^2 + m^2}.$$

So far we looked into what happens if we fiddle with the left hand side of

$$\Gamma = n \cdot \sigma \sim H,$$

Consider a "**Quintessence**" dark energy model – homogeneous real scalar field

$$\rho_\phi = \frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}t}\right)^2 + V(\phi)$$

$$P_\phi = \frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}t}\right)^2 - V(\phi)$$

$$w = P_\phi/\rho_\phi \qquad \rho_\phi \sim a^{-3(1+w)} \qquad \rho \sim a^{-6}$$

$$H \sim \frac{T^2}{M_P} \frac{T}{T_{\rm KRE}} \qquad (T \gtrsim T_{\rm KRE})$$

$$n_{\rm f.o.} \langle \sigma \; v \rangle \sim \frac{T^2}{M_P} \frac{T}{T_{\rm KRE}}.$$

$$\frac{n_{\rm f.o.}}{T^2_{\rm f.o.}} \sim \frac{1}{M_P \; \langle \sigma \; v \rangle} \frac{T_{\rm f.o.}}{T_{\rm KRE}}. \qquad \Omega_\chi^{\rm quint} = \frac{T_0^3}{M_P \cdot \rho_c} x_{\rm f.o.} \left(\frac{n_{\rm f.o.}}{T^2_{\rm f.o.}} \right)$$

$$\frac{\Omega_\chi^{\rm quint}}{\Omega_\chi^{\rm standard}} \sim \frac{T_{\rm f.o.}}{T_{\rm KRE}} \lesssim \frac{m_\chi}{20} \frac{1}{T_{\rm BBN}} \sim 10^4 \frac{m_\chi}{100 \text{ GeV}}.$$

After **chemical** decoupling (number density freezes out),

DM can still be in **kinetic** equilibrium

(i.e. its **velocity** distribution is in equilibrium)

generically, this is the case, since for **cold** relics

$$\chi\chi \leftrightarrow ff \qquad \rightarrow \quad \Gamma = n_{\mathrm{non-rel}} \cdot \sigma$$
$$\chi f \leftrightarrow \chi f \qquad \rightarrow \quad \Gamma = n_{\mathrm{rel}} \cdot \sigma$$

Think of a **prototypical WIMP**:

$$\sigma_{\chi f \leftrightarrow \chi f} \sim G_F^2 T^2$$

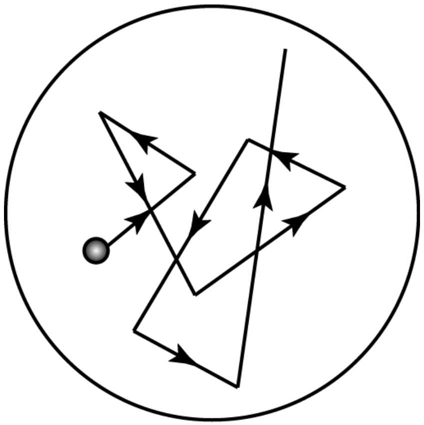
Problem: every collision has a **momentum transfer** $\delta p \sim T$

...but we need to keep the (cold) DM momentum in equilibrium, i.e.

$$\frac{p^2}{2m_\chi} \sim T \qquad p \sim \sqrt{m_\chi T}$$

so ***δp << p***, we need a bunch of kicks!

However, **subtlety**: kicks are in **random directions**!



$$N = \left(\frac{p}{\delta p}\right)^2 \sim \frac{m_\chi T}{T^2} = \frac{m_\chi}{T} \gg x_{\rm f.o.} \gtrsim 20$$

Let's estimate a typical WIMP **kinetic decoupling temperature**

$$n_{\rm rel} \cdot \sigma_{\chi f \leftrightarrow \chi f} \left(\frac{\delta p}{p}\right)^2 \sim T^3 \cdot G_F^2 T^2 \cdot \frac{T}{m_\chi} \sim H \sim \frac{T^2}{M_P}.$$

$$T_{\rm kd} \sim \left(\frac{m_\chi}{M_P \cdot G_F^2}\right)^{1/4} \sim 30 \text{ MeV } \left(\frac{m_\chi}{100 \text{ GeV}}\right)^{1/4}$$

What does this implies for **structure formation**?

$$M_{\rm ao} \sim \frac{4\pi}{3}\left(\frac{1}{H(T_{\rm kd})}\right)^3 \rho_{\rm DM}(T_{\rm kd}) \sim 30\ M_\oplus \left(\frac{10\ {\rm MeV}}{T_{\rm kd}}\right)^3$$

$$M_\oplus \simeq 3 \times 10^{-6} M_\odot$$

First structures that collapse are these tiny **minihalos**

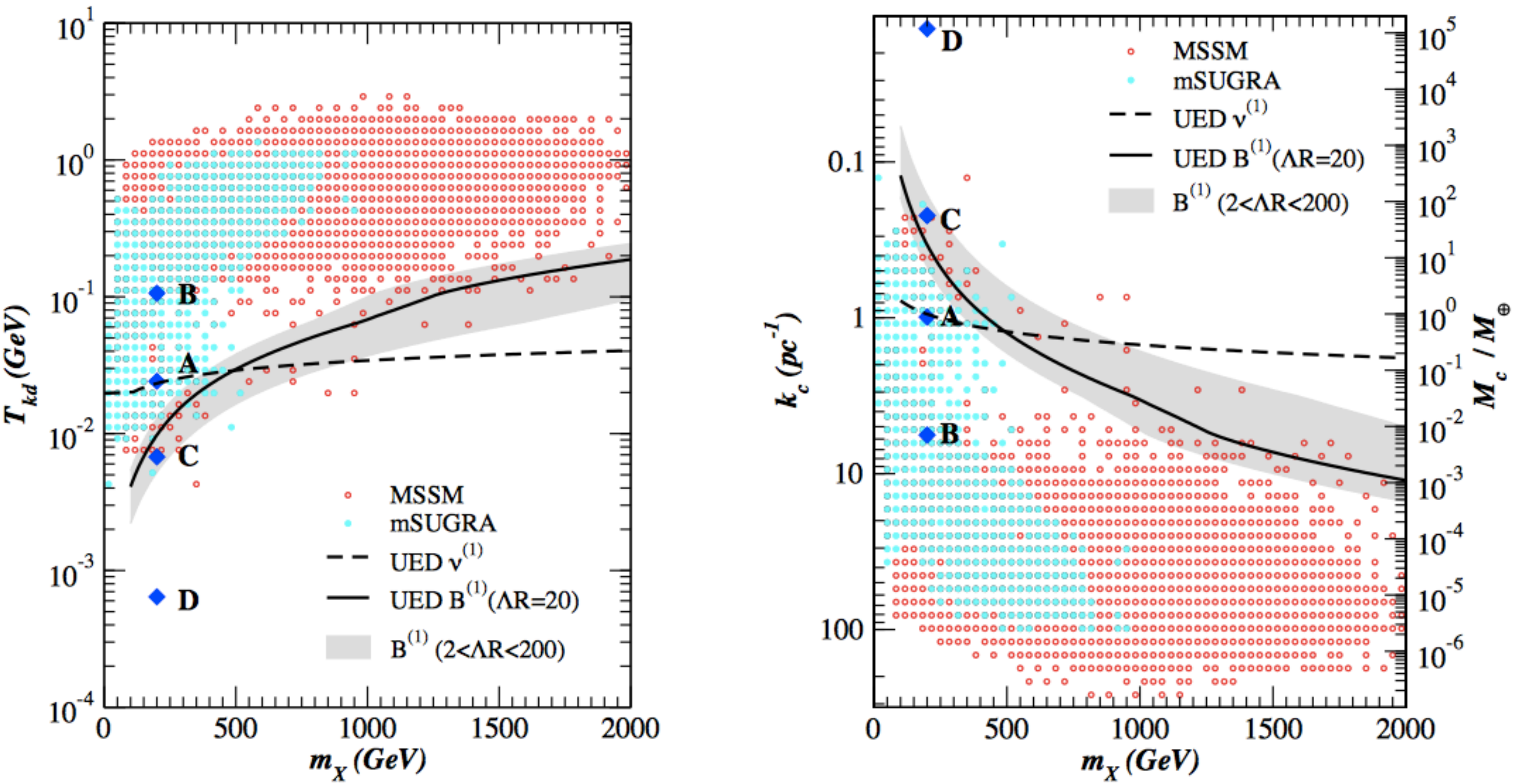
(maybe some survive today?)

Structures then **merge** into bigger and bigger halos

(**bottom-up** structure formation)

Notice that the kinetic decoupling/cutoff scale **varies** significantly even for a selected particle dark matter scenario!

e.g. for **SUSY, UED**



What happens instead for **hot relics**?

They decouple when **$T \gg m_\nu$**

Structures can only collapse when **$T \sim m_\nu$**
(i.e. when things slow down enough for gravitational collapse!)

Structures are cutoff to the **horizon size** at that temperature

$$d_\nu \sim H^{-1}(T \sim m_\nu) \qquad d_\nu \sim \frac{M_P}{m_\nu^2}$$

$$d_\nu \sim \frac{M_P}{m_\nu^2}$$

$$M_{\text{cutoff, hot}} \sim \left(\frac{1}{H(T=m_\nu)}\right)^3 \rho_\nu(T=m_\nu) \sim \left(\frac{M_P}{m_\nu^2}\right)^3 m_\nu \cdot m_\nu^3 = \frac{M_P^3}{m_\nu^2}$$

$$\frac{M_P^3}{m_\nu^2} \sim 10^{15}\ M_\odot \left(\frac{m_\nu}{30\ \text{eV}}\right)^{-2} \sim 10^{12}\ M_\odot \left(\frac{m_\nu}{1\ \text{keV}}\right)^{-2}$$

How does this compare with **observations**?

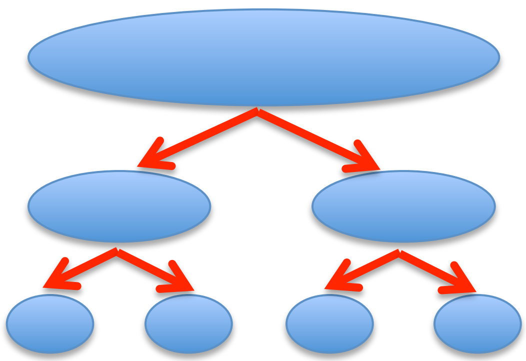
$$\frac{M_P^3}{m_\nu^2} \sim 10^{15}\, M_\odot \left(\frac{m_\nu}{30\ \mathrm{eV}}\right)^{-2} \sim 10^{12}\, M_\odot \left(\frac{m_\nu}{1\ \mathrm{keV}}\right)^{-2}$$

Observational **constraints** give

$$M_{\rm cutoff} \ll M_{\rm Ly-\alpha} \simeq 10^{10}\ M_\odot$$

So at best dark matter can be **keV** scale, if produced thermally

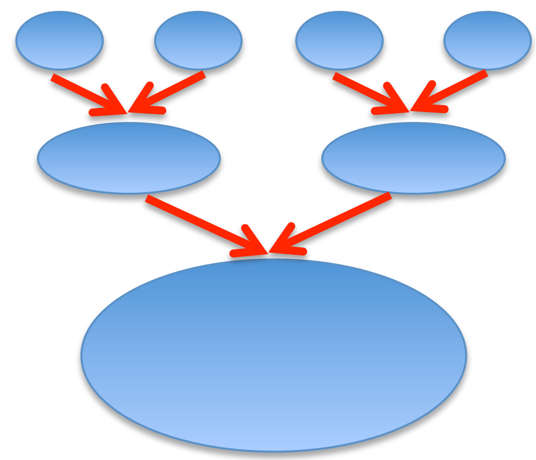
Structure formation looks strikingly different for hot and cold dark matter



Hot Dark Matter

Top-Down

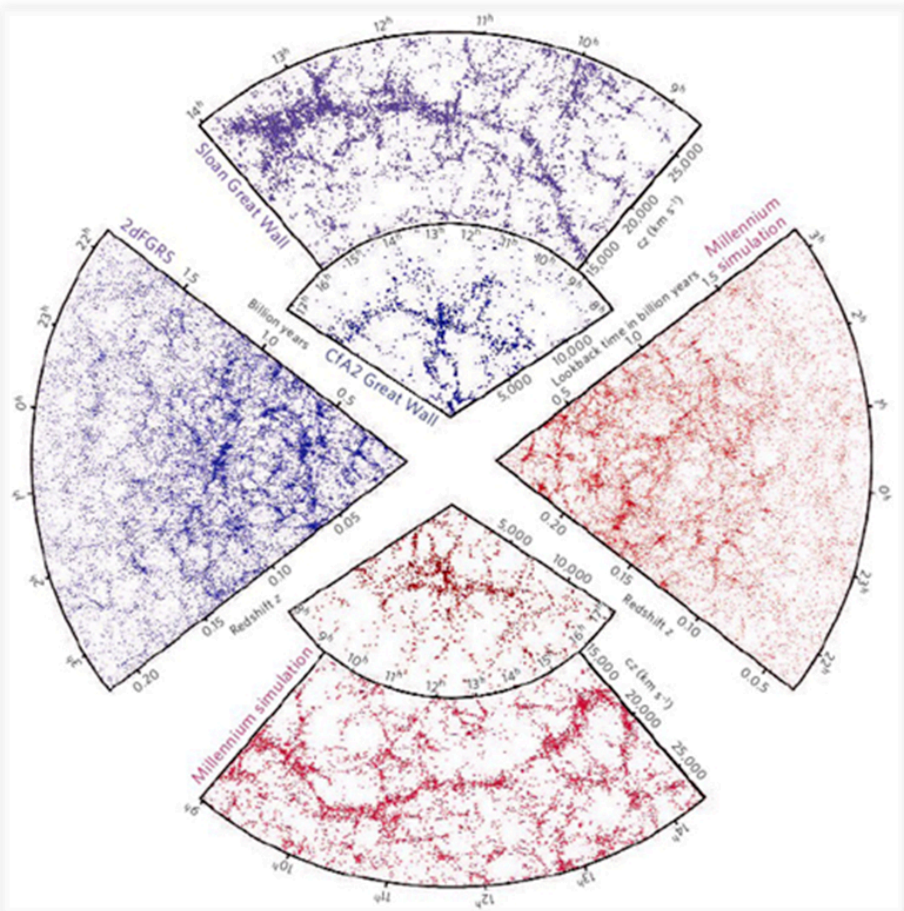
[doesn’t work!]



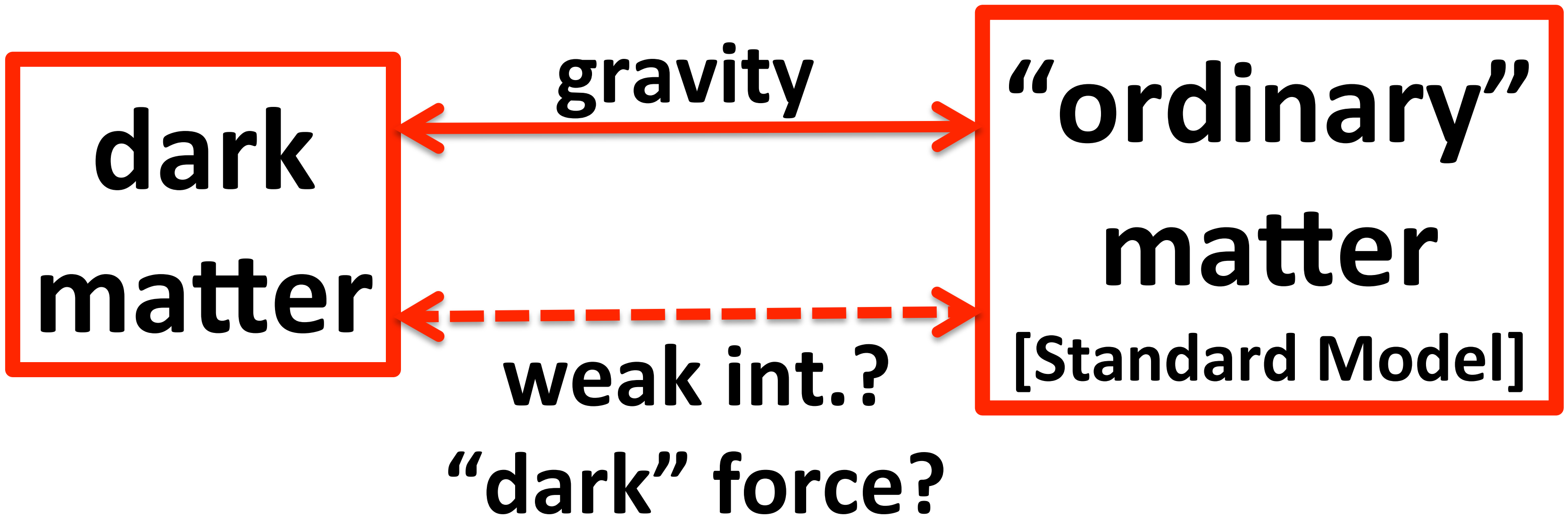
Cold Dark Matter

Bottom-Up

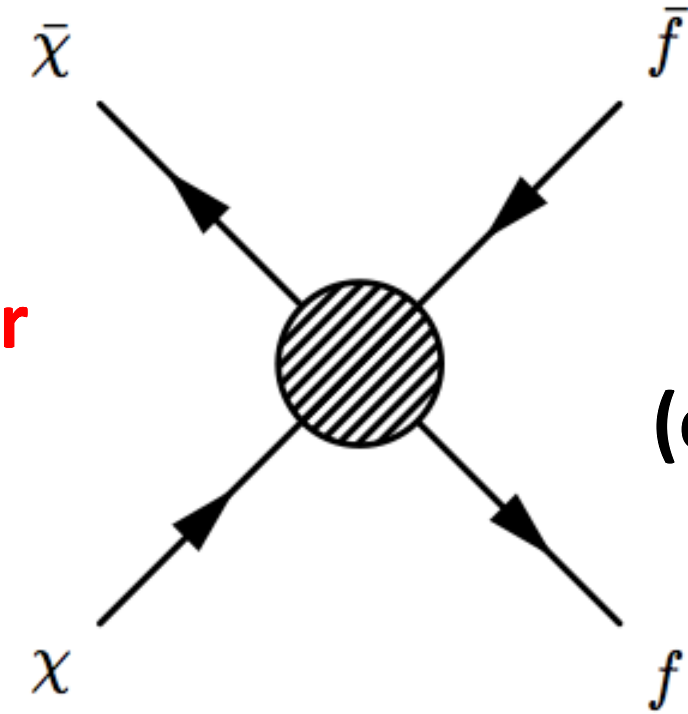
[Yeah!]



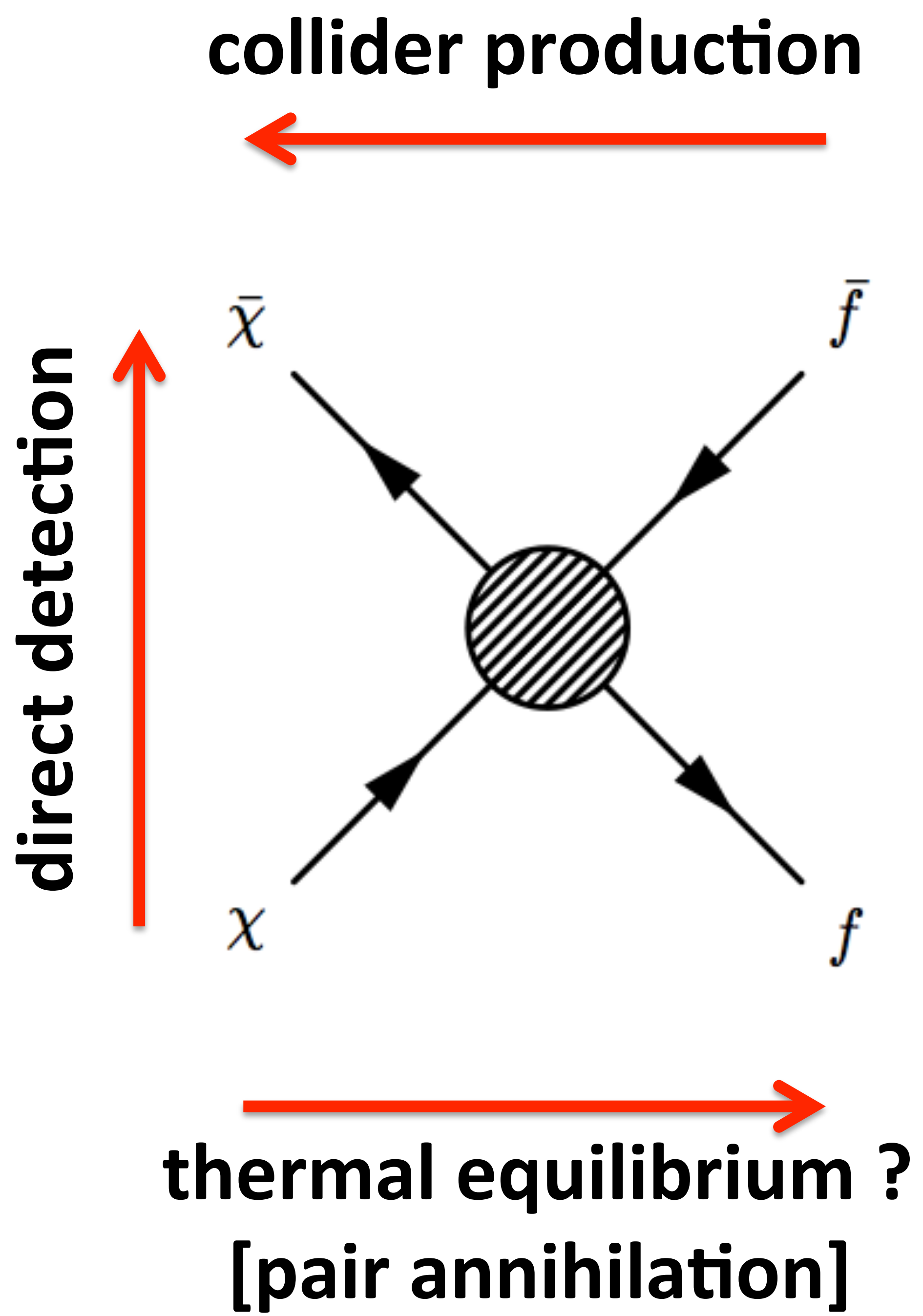
1980's: Davis, Efstathiou, Frenk and White show that simulations of structure formation in a universe with **cold dark matter** match observed structure incredibly well!!

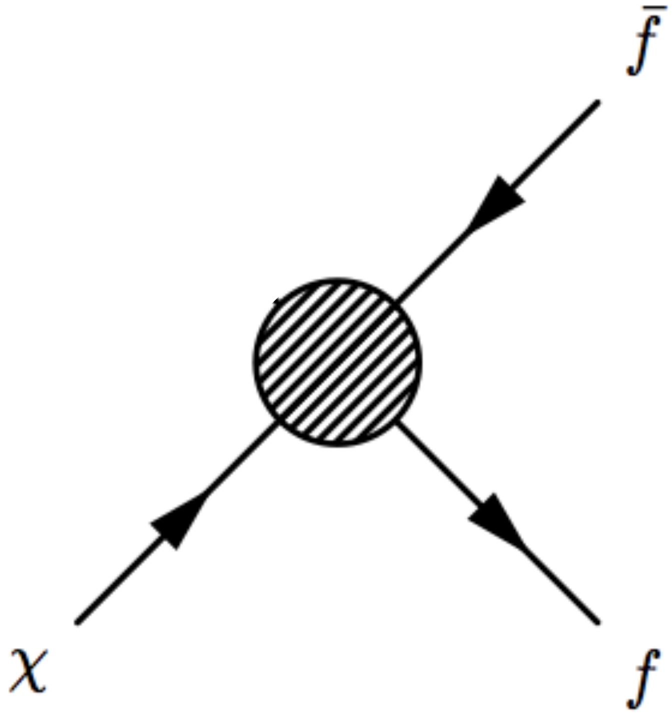

dark matter
gravity
"ordinary" matter
[Standard Model]
weak int.?
"dark" force?

Dark Matter
Particles



Standard Model
(ordinary) Particles

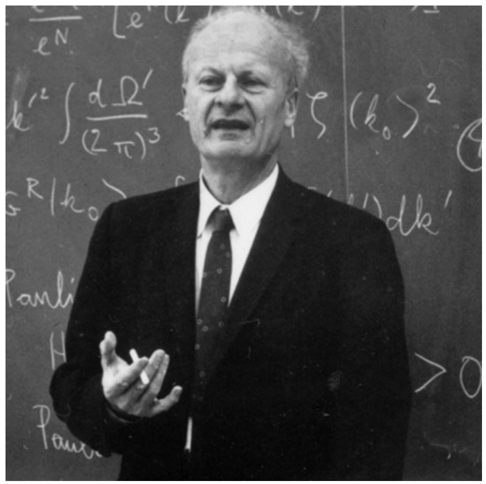

collider production
direct detection
$\bar{\chi}$
$\bar{f}$
χ
f
thermal equilibrium ?
[pair annihilation]



long-lived, but metastable

Consider **direct** detection

Detecting particles that interact **weakly** has always been known to be a **tough job**



H. Bethe

R. Peierls

After **estimating** in 1934 the **cross section** for $\bar{\nu}_e + p \rightarrow e^+ + n$

$$\sigma_{\bar{\nu}_e + p \rightarrow e^+ + n} \approx 10^{-43} \ (E_\nu/\mathrm{MeV})^2 \ \mathrm{cm}^2.$$

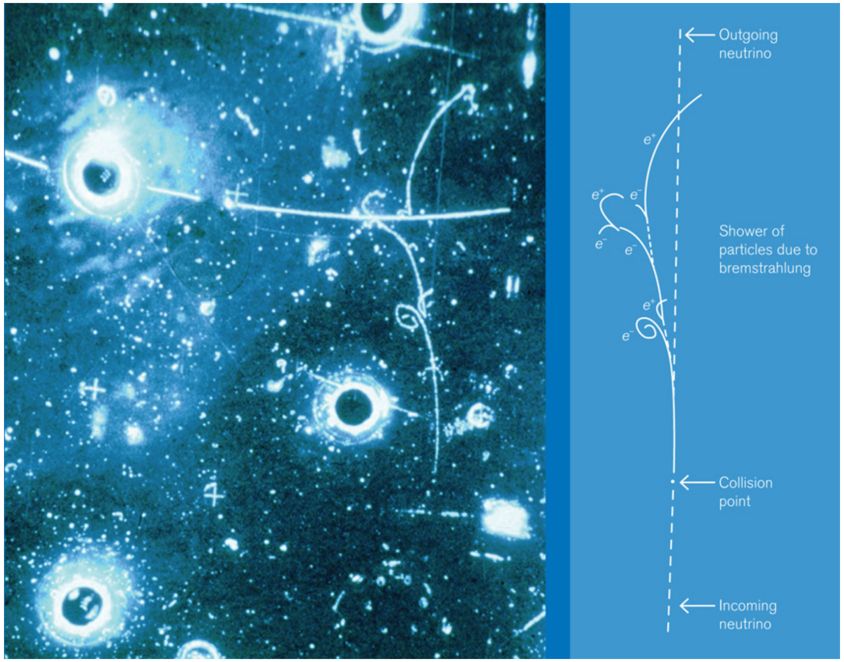
"It is therefore ***absolutely impossible*** to observe processes of this kind"

Bethe and Peierls were too **pessimistic**/**conservative**:
neutrinos were detected in 1953, abundantly in 1956

Inelastic process (maybe relevant for DM?)

$$\bar{\nu}_e + p \to e^+ + n \qquad \chi + X \to \chi' + Y,$$

Elastic neutrino scattering took **much longer** (Gargamelle 1973)



Let's use **WIMPs** again as **prototypical** DM particles

First, which **energies** and what **masses** are we talking about?

maximal recoil momentum for a DM particle

with velocity *v* is $\mathbf{2m_\chi v}$, so maximal energy

$$E_{\max} = (2m_\chi v)^2/(2m_N)$$

Now, the **maximal velocity** a DM particle can have in the Galaxy is the **escape velocity** v_{max} ~ *500-700 km/s*

→ ***E~ keV*** for GeV particles!

Plug in numbers for a detector with an energy threshold ~ keV...

minimal detectable **DM mass ~ GeV**

OK, now what about the **event rate**? $R = K\phi\sigma.$

$$K \simeq 6.0 \times 10^{26}/A \qquad \phi = v\rho_{\rm DM}/m_\chi$$

Plug in sensible **benchmark** values…

$$R = \frac{0.06\ \text{events}}{\text{kg day}}\left(\frac{100}{A}\right)\left(\frac{\sigma}{10^{-38}\ \text{cm}^2}\right)\left(\frac{\rho_{\rm DM}}{0.3\ \text{GeV/cm}^3}\right)\left(\frac{v}{200\ \text{km/s}}\right)$$

To have a detection need both enough **signal** events,

and enough **background** suppression

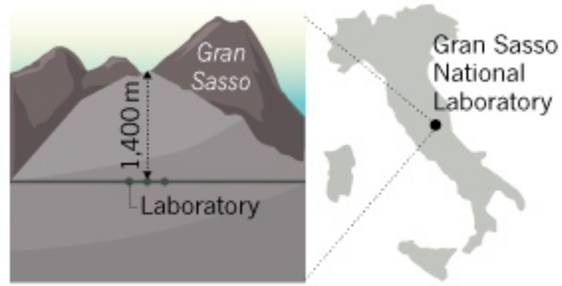
1. slowly decaying "primeval" nuclides (U, Th, ^{40}K),
   ab. 10^{-4}, half lives ~10^9 yr

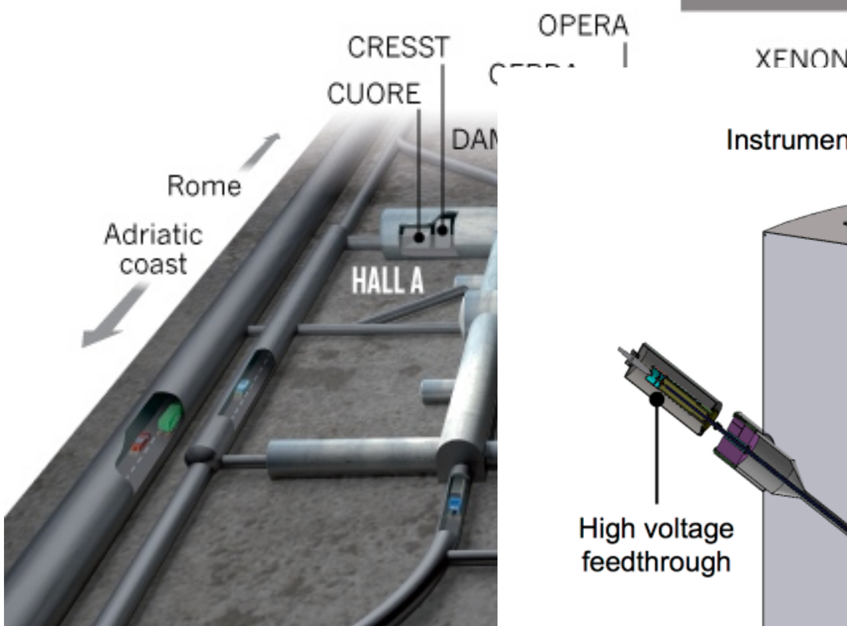
2. rare, fast decaying trace elements like tritium, ^{14}C:
   ab 10^{-18} , half lives 10 yr

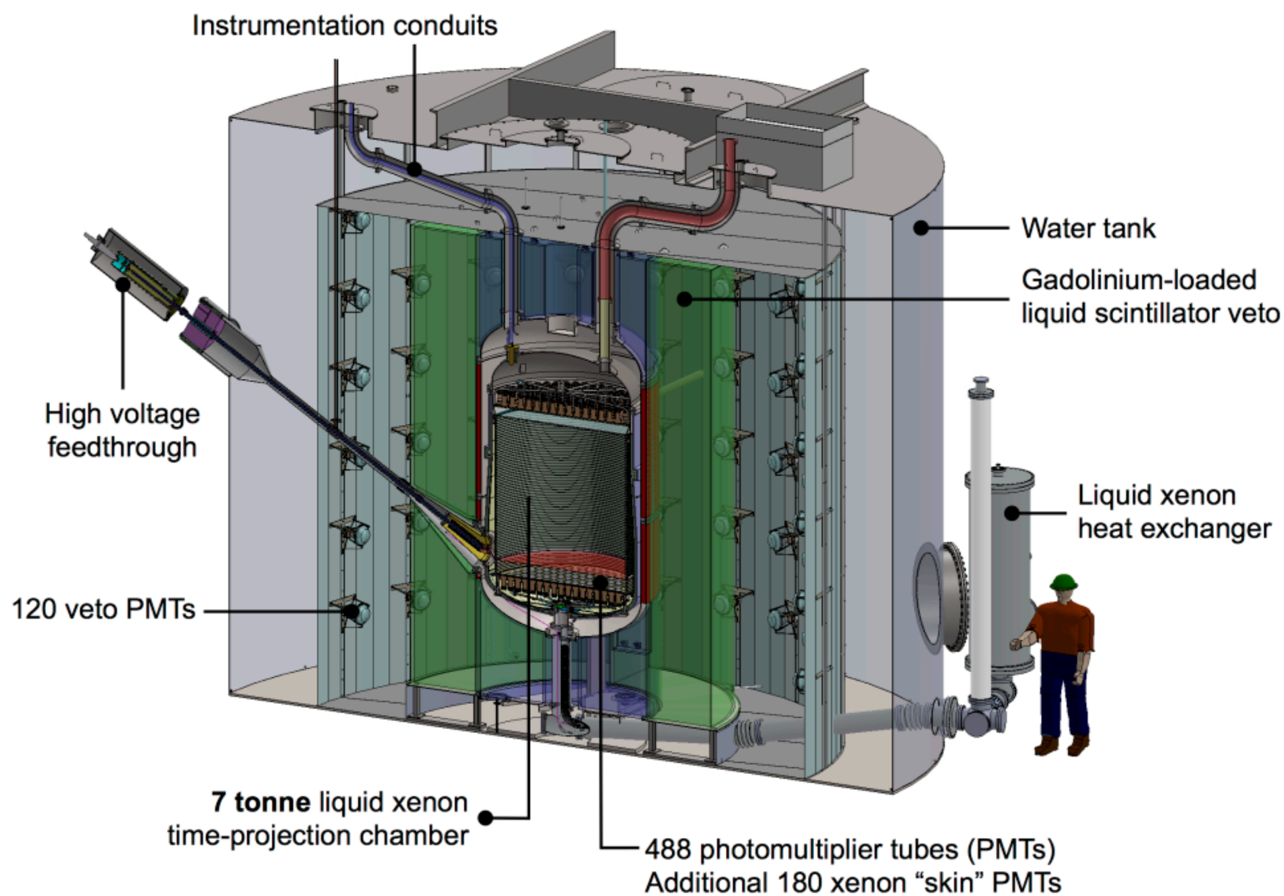
Big detectors, in **underground**, actively **shielded** environments...

THE A, B AND C OF GRAN SASSO

Experiments at the Gran Sasso National Laboratory are housed in and around three huge halls carved deep inside the mountain, where they are shielded from cosmic rays by 1,400 metres of rock.

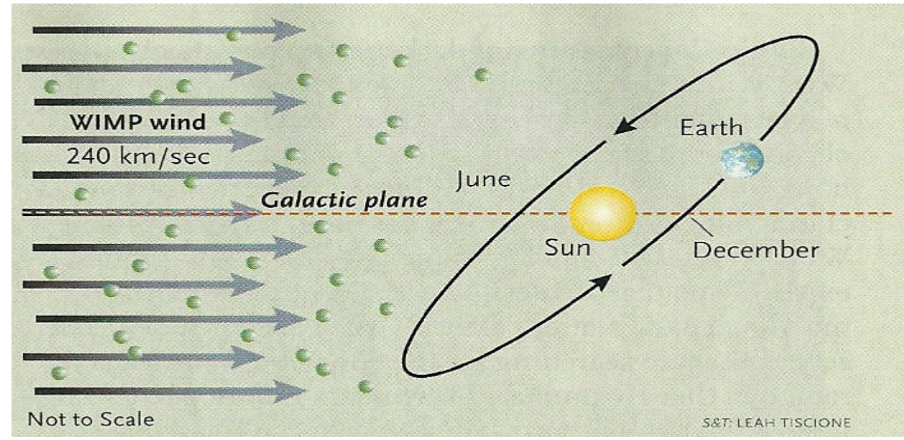




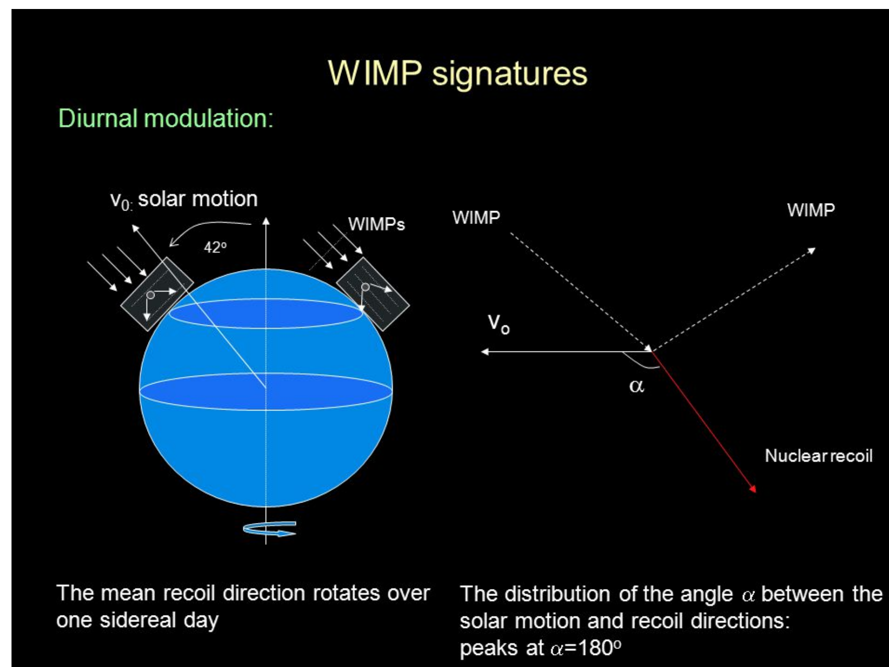


Other handles on a DM **signal** versus radioactive **background**:

1. **Seasonal** modulation



2. **Diurnal** modulation



3. **Directional** information

Now: direct detection **event rates**, for real!

$$\frac{\mathrm{d}R}{\mathrm{d}E_R} = N_T \, n_\chi \, \langle v_\chi \frac{\mathrm{d}\sigma}{\mathrm{d}E_R} \rangle$$

$$E_R = \frac{q^2}{2m_T} = \frac{\mu_T^2}{m_T} v_\chi^2 (1 - \cos\theta)$$

$$\mathrm{d}E_R = (\mathrm{d}\cos\theta)(\mu_T^2/m_T)v^2$$

$$\frac{\mathrm{d}R}{\mathrm{d}E_R} = N_T \frac{\rho_{\mathrm{DM}} m_T}{m_\chi \mu_T^2} \int_{v_{\mathrm{min}}}^{v_{\mathrm{esc}}} \mathrm{d}^3 v \frac{f(v)}{v} \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}$$

How do we calculate the scattering **cross section**?

Non-relativistic limit, the scattering **matrix element** is the Fourier transform of WIMP-nucleus potential

$$\mathcal{M}(q^2) \sim \int \langle f|V(\vec{r})|i\rangle e^{i\vec{q}\cdot\vec{r}} \mathrm{d}\vec{r},$$

to the lowest order in velocity, the potential is just a **contact interaction** of spin-independent and axial

$$V(\vec{r}) = \sum_{\text{nucleons } n} \left(G_s^n + G_a^n \vec{\sigma}_\chi \cdot \vec{\sigma}_n\right) \delta(\vec{r} - \vec{r}_n)$$

where the *G*'s are the effective DM-nucleon interactions for **scalar** and **axial** interactions

Coherence requires the nucleus size to be much smaller than the momentum transfer wavelength (*1/q*)

$$qR_{\text{nucleus}} \ll 1$$

Loss of coherence is phenomenologically accounted for by introducing **form factors** describing the nucleus response

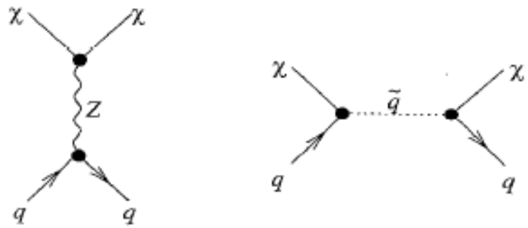
$$\mathcal{M}(q^2) = T(0)F(q^2)$$

Given a **microscopic** theory of dark matter,

how does one get to the **DM-nucleus cross section**?

An interesting **multi-layered** problem in **effective field theory**!

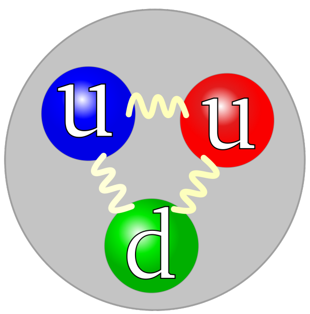
Dark Matter-quark



Low-energy EFT

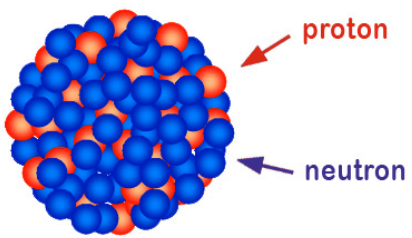
Nucleon matrix elements

Dark Matter-nucleon



Form factors

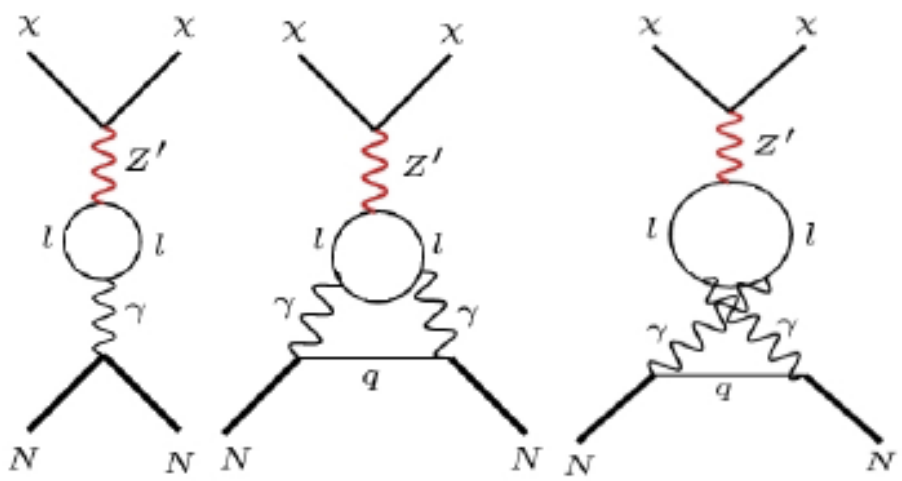
Dark Matter-nucleus

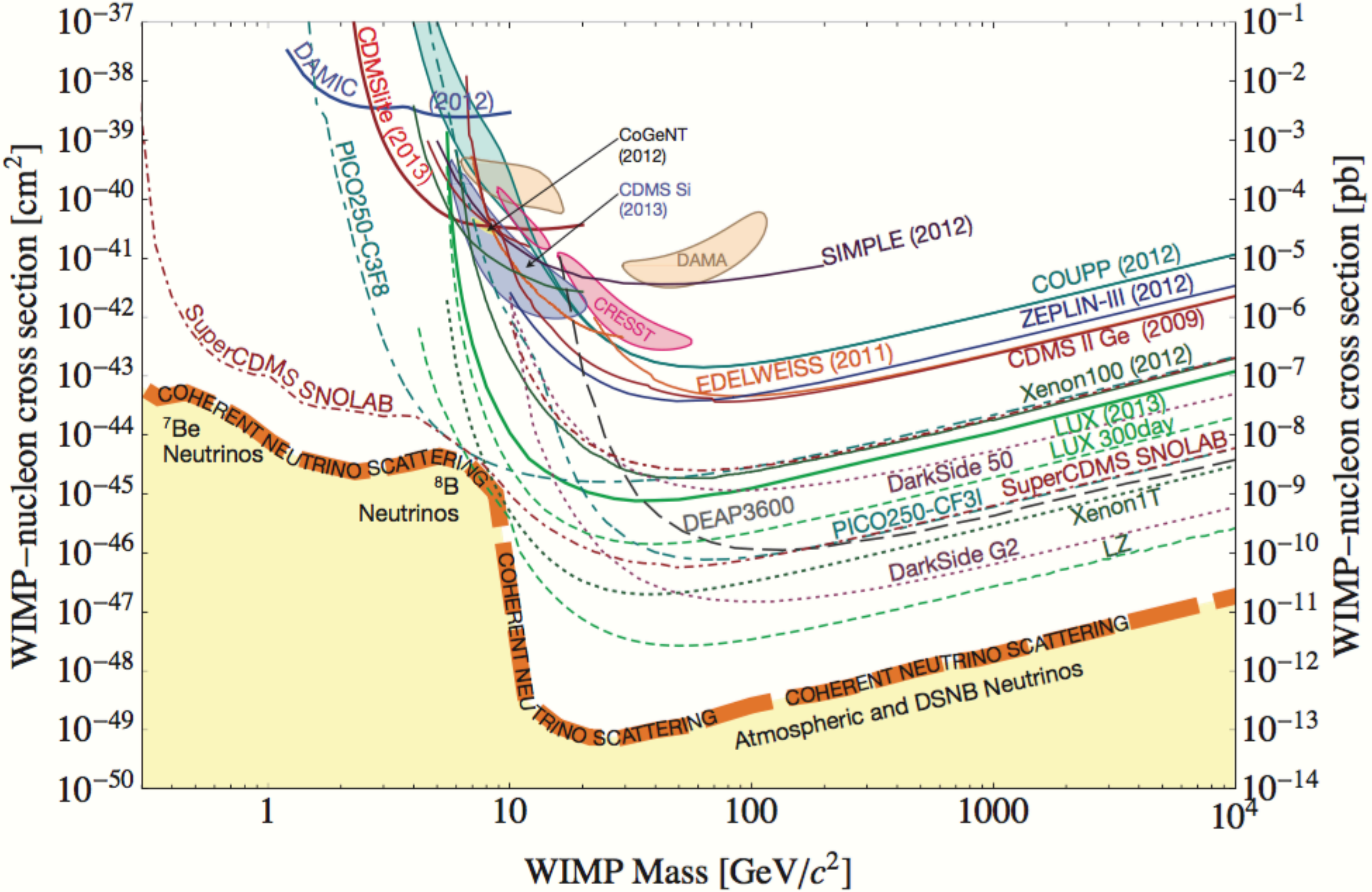


Sometimes life is simpler, e.g. if DM is (**milli**-electric-)**charged**

$$\sigma_N = \frac{16\pi\alpha^2\varepsilon^2 Z^2 \mu_N^2}{q^4}$$

Sometimes life is nastier, e.g. if DM is **lepto-philic**





WIMP–nucleon cross section [cm^2]
WIMP–nucleon cross section [pb]
WIMP Mass [GeV/c^2]
DAMIC
(2012)
CDMSlite (2013)
PICO250-C3F8
CoGeNT (2012)
CDMS Si (2013)
DAMA
SIMPLE (2012)
CRESST
COUPP (2012)
ZEPLIN-III (2012)
CDMS II Ge (2009)
EDELWEISS (2011)
Xenon100 (2012)
LUX (2013)
LUX 300day
SuperCDMS SNOLAB
DarkSide 50
DEAP3600
PICO250-CF3I
Xenon1T
DarkSide G2
LZ
SuperCDMS SNOLAB
COHERENT NEUTRINO SCATTERING
^{7}Be Neutrinos
^{8}B Neutrinos
Atmospheric and DSNB Neutrinos

Now off to **indirect** dark matter detection

Idea: use the **debris** of DM **pair-annihilation**
(likely large if thermal relic) or **decay**

$$\Gamma_{\text{SM, ann}} \sim \left(\int_V \frac{\rho_{\text{DM}}^2}{m_\chi^2} \text{d}V\right) \times (\sigma v) \times (N_{\text{SM, ann}}),$$

$$\Gamma_{\text{SM, dec}} \sim \left(\int_V \frac{\rho_{\text{DM}}}{m_\chi} \text{d}V\right) \times \left(\frac{1}{\tau_{\text{dec}}}\right) \times (N_{\text{SM, dec}})$$

What do we know about these **rates**?
σv from **thermal production** (with caveats!)

How about **decay rate**?

Suppose DM decay mediated by **high-scale** physics at scale ***M***

$$\Gamma_5 \sim \frac{1}{M^2} m_\chi^3$$

$$\tau_5 \sim 1 \text{ s} \left(\frac{1 \text{ TeV}}{m_\chi}\right)^3 \left(\frac{M}{10^{16} \text{ GeV}}\right)^2$$

Dimension-5 operator doesn't work – would be too **short lived**!

$$\Gamma_6 \sim \frac{1}{M^4} m_\chi^5,$$

Interesting, well motivated!

$$\tau_6 \sim 10^{27} \text{ s} \left(\frac{1 \text{ TeV}}{m_\chi}\right)^5 \left(\frac{M}{10^{16} \text{ GeV}}\right)^4$$

What about annihilation **final state**?

Very **model-dependent**

1. if DM belongs to an SU(2) **multiplet**, then well-defined combination of *ZZ*, *WW* final states...

2. In UED, DM is KK-1 mode of **hypercharge gauge boson**, thus

$|M|^2 \propto |Y_f|^4$ $Y_{u_L} = 4/3$ $Y_{e_R} = 2$

3. Special "**selection rule**", e.g. helicity suppression for Marjorana fermion (analogous to charged pion decay)

$$|M|^2 \propto m_f^2$$

Annihilation (or decay) of DM can be **detected** or **constrained** in a variety of ways

Here's one possible **classification**:

1. **Very Indirect**: effects induced by dark matter on **astrophysical objects** or on **cosmological observations**

2. **Pretty Indirect**: probes that don't "trace back" to the annihilation event, as their trajectories are bent as the particles propagate: **charged cosmic rays**

3. **Not-so-indirect**: **neutrinos** and **gamma rays**, with the great added advantage of traveling in straight lines

Very indirect probes include e.g.

- **Solar Physics** (dark matter can affect the Sun's core temperature, the sound speed inside the Sun,...)
- **Neutron Star Capture**, possibly leading to the formation of black holes (notably e.g. in the context of asymmetric dark matter)
- **Supernova** and **Star** cooling
- **Protostars** (e.g. WIMP-fueled population-III stars)
- **Planets warming**
- **Big Bang Nucleosynthesis**, on the **cosmic microwave background**, on **reionization**, on **structure formation**…

Pretty Indirect Probes: **charged cosmic rays**

Good idea is to use **rare** cosmic rays, such as **anti-matter**

antiprotons, **positrons** relatively abundant
(mostly from inelastic processes CR p on ISM p)

Interesting probe: **antideuterons** (or even **anti-^{3}He** !!)

$$\bar{D}: \quad p + p \rightarrow p + p + \bar{p} + p + \bar{n} + n$$

large energy **threshold** (~17 GeV), so typically large momentum, while from DM produced at very low momentum! Select **low-energy antideuterons**

positrons (and in part antiprotons) have attracted attention because of "**anomalies**" reported by PAMELA, AMS-02

general scheme for Galactic CR's: **diffusion** (leaky-box) models

$$\frac{\mathrm{d}n}{\mathrm{d}E} = \psi\left(\vec{x}, E, t\right)$$

$$\frac{\partial}{\partial t}\psi \;=\; D(E)\Delta\psi \;+\; \frac{\partial}{\partial E}\left(b(E)\;\psi\right) \;+\; Q\left(\vec{x}, E, t\right)$$

Things can be made arbitrarily more **complicated/sophisticated**:

- *Cosmic-ray convection*; recipe: add: $\frac{\partial}{\partial z}(v_c \cdot \psi)$;
- *Diffusive re-acceleration*; recipe: add: $\frac{\partial}{\partial p}\, p^2\, D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} \psi$;
- *Fragmentation and decays*; recipe: add: $-\frac{1}{\tau_{\mathrm{f,d}}}\psi$.

Boundary conditions:

$$R \sim \mathcal{O}(1) \times 10 \text{ kpc},$$
$$h \sim \mathcal{O}(1) \times 1 \text{ kpc}.$$

$$D(E) \sim D_0 \left(\frac{E}{E_0}\right)^{\delta}$$

Useful to **simplify** the diffusion equation assuming steady-state, using typical diffusion and energy loss **time-scales**, defined by

$$\tau_{\text{diff}} \sim \frac{R^2}{D_0} \cdot E^{-\delta}, \qquad \tau_{\text{loss}} \sim \frac{E}{b(E)}$$

Diff. Eq. then looks like $$0 = -\frac{\psi}{\tau_{\text{diff}}} - \frac{\psi}{\tau_{\text{loss}}} + Q$$

with **solution** $$\psi \sim Q \cdot \min[\tau_{\text{diff}}, \tau_{\text{loss}}]$$

If the source is cosmic rays accelerated via a **Fermi mechanism**,

$$Q \sim E^{-2} \longrightarrow \psi \sim E^{-2} \cdot E^{-\delta} \sim E^{-2.7}$$

…in agreement with **CR protons** (where en. losses are irrelevant)

For CR **electrons**, energy losses are efficient above a certain **energy**,

$$b_e(E) \simeq b^0_{\rm IC} \left(\frac{u_{\rm ph}}{1 \ {\rm eV/cm^3}} \right) \cdot E^2 + b^0_{\rm sync} \left(\frac{B}{1 \ \mu{\rm G}} \right)^2 \cdot E^2$$

$$b^0_{\rm IC} \simeq 0.76, \qquad b^0_{\rm sync} \simeq 0.025 \ \ 10^{-16} \ {\rm GeV/s}$$

Therefore (as observed) we expect a **broken power-law**

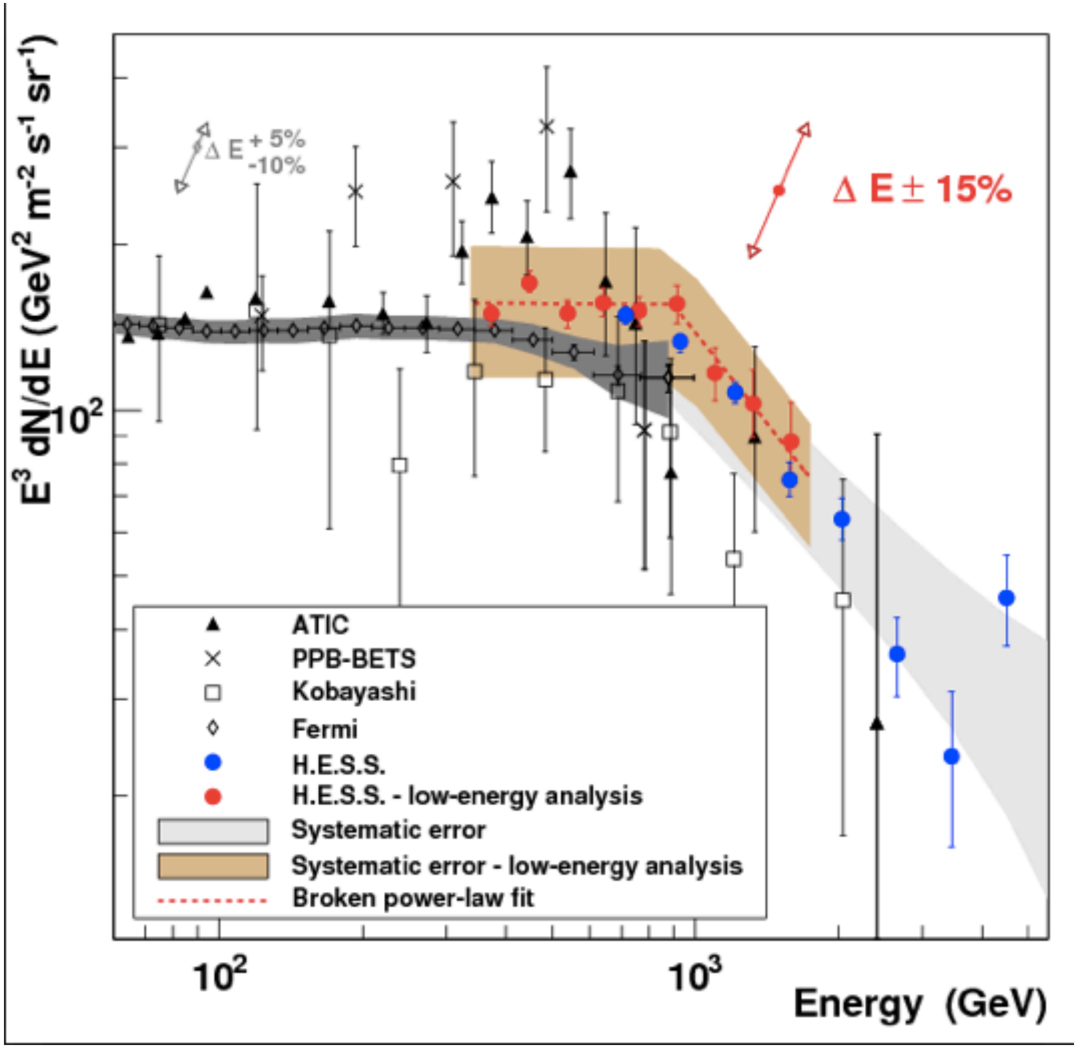
$$\psi_{\rm primary,\ low-energy} \sim Q \cdot \tau_{\rm diff} \sim E^{-2} \cdot E^{-\delta} \sim E^{-2.7}$$

$$\psi_{\rm primary,\ high-energy} \sim Q \cdot \tau_{\rm loss} \sim E^{-1} \cdot \frac{E}{E^2} \sim E^{-3}$$

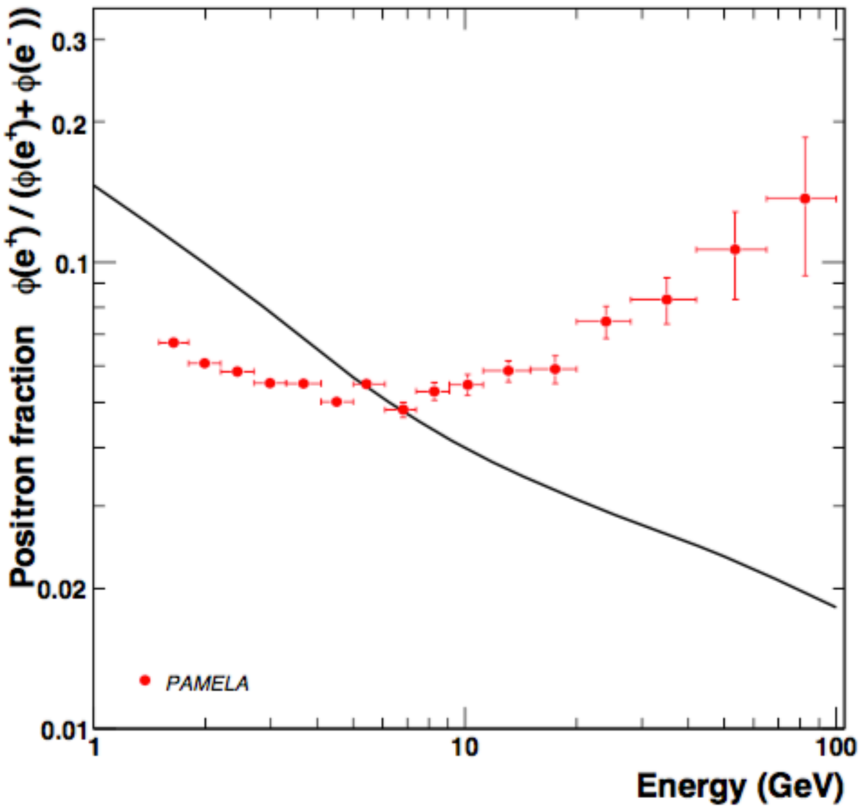
Also, **secondary-to-primary** ratios are generically

$$\frac{\psi_{e^+}}{\psi_{e^-}} \sim E^{-\delta}.$$

Electron spectrum looks pretty good



but the **secondary-to-primary ratio** prediction is at **odds** with observed rising positron fraction



Much **hype** about this possibly being from **DM** – but very **problematic**

- No excess **anitprotons** – must be "leptophilic" (possible but not generic)

- No observed **secondary radiation** from brems or IC

- Needed **pair-annihilation rate** very large for thermal production, leads to unseen gamma-ray or radio emission

$$\langle \sigma v \rangle \sim 10^{-24} \frac{\mathrm{cm}^3}{\mathrm{s}} \cdot \left(\frac{m_\chi}{100\ \mathrm{GeV}} \right)^{1.5}$$

Alternate explanation: nearby **point source** injecting a burst of **positrons** (a.k.a. Green's function, a.k.a. **PSR**)

$$\psi \propto Q \cdot \exp\left(-\left(\frac{r}{r_{\rm diff}}\right)^2\right)$$

Estimate **Age** and **Distance** of putative source

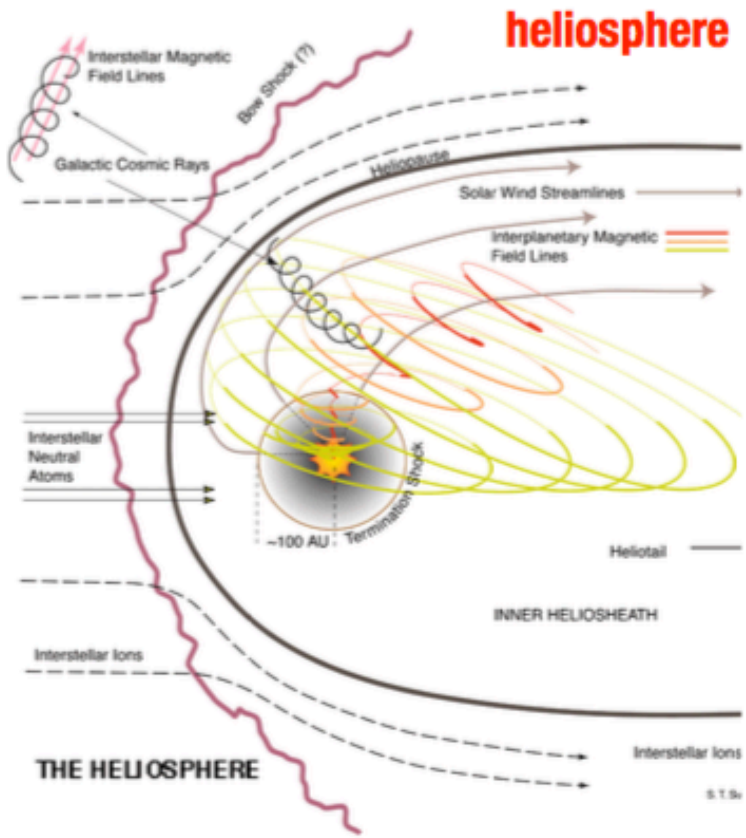
$$t_{\rm psr} \ll \tau_{\rm loss} = \frac{E}{b(E)}; \text{ for } E = 100 \text{ GeV}, \tau_{\rm loss} \sim \frac{100}{10^{-16} \cdot 100^2} \text{ s} \sim 10^{14} \text{ s} \sim 3 \text{ Myr}.$$

$$r_{\rm diff} \simeq \sqrt{D(E) \cdot t}.$$

$$\sqrt{D(E) \cdot t_{\rm psr}} \gg \text{ distance} \rightarrow \text{distance} \ll (3 \times 10^{28} \cdot 100^{0.7} \cdot 10^{14})^{1/2} \text{ cm} \sim 10^{22} \text{ cm} \sim 3 \text{ kpc}.$$

One possible way to **disentangle** PSR from DM: **anisotropy**

Complication: Larmor radius for **heliospheric** magnetic fields ***B~ nT***, is of the order of the **solar system size** (exercise)



Not-so-indirect DM detection: **neutrinos**!

Only **two** observed astrophysical sources of neutrinos!

Hard (but not impossible) to detect particles

flip side: neutrinos have very **long mean free paths** in matter!

idea: DM can be **captured** in celestial bodies, **accrete** in sizable densities, start pair-annihilating

if the process of capture and annihilation is in **equilibrium**, large **fluxes** of neutrino can escape