Santa Cruz, February 10, 2009

Midterm Exam

Solve the following problems (each problem is worth the same number of points)

1. Test the following infinite series for convergence:

$$\sum_{n=1}^{\infty}(-1)^{n-1}\; n^{-1/n}$$

2. Show that the following infinite series is convergent:

$$\sum_{n=1}^{\infty}\frac{2+(-1)^n}{n^2}$$

3. Find the interval of convergence of the following power series:

$$\sum_{n=0}^{\infty}\frac{n}{n+1}\left(\frac{x}{3}\right)^n, \qquad x\in\mathbb{R}$$

4. Determine the disk of convergence of the following complex series:

$$\sum_{n=0}^{\infty}(1+ni)(z+i)^n, \qquad x\in\mathbb{C}$$

5. Find the real and the imaginary part of

$$i\; e^{i|1+2i|}$$

6. Determine the angles in the triangle formed by the three vertexes $P_1=(2,2,2)$, $P_2=(3,1,1)$, and $P_3=(3,3,3)$.

7. A car drives in a horizontal circular track of radius R (to its center of mass). Find the speed at which the car will overturn, if h is the height of its center of mass and d is the distance between its left and right wheels.

8. Test the set of linear homogeneous equations

$$x + 3y + 3z = 0, \quad x - y + z = 0, \quad 2x + y + 3z = 0$$

to see if it possesses a nontrivial solution and find one.

9. Given the pair of equations

$$x + 2y = 3, \qquad 2x + 4y = 6$$

(a) show that the determinant of the coefficients vanishes;

(b) show that the numerator determinants also vanish;

(c) find at least two solutions

10. The matrix equation $A^2 = 0$ does not imply $A = 0$. Show that the most general 2×2 matrix whose square is zero may be written as

$$\begin{pmatrix} ab & b^2 \\ -a^2 & -ab \end{pmatrix},$$

where a and b are real or complex numbers.