# Homework Set #5.

**Due Date - Oral Presentation**: Tuesday November 2, 2010
**Due Date - Written Solutions**: Tuesday November 9, 2010

**Bilinears** Consider bilinear products of a Dirac field $\psi(x)$ and its "conjugate" $\overline{\psi}(x)$. All possible combinations are given by:

$$S = \overline{\psi}\psi, \quad V^\mu = \overline{\psi}\gamma^\mu\psi, \quad T^{\mu\nu} = \overline{\psi}\gamma^{[\mu}\gamma^{\nu]}\psi, \quad , A^\mu = \overline{\psi}\gamma^5\gamma^\mu\psi \quad \text{and} \quad P = \overline{\psi}i\gamma^5\psi.$$

In the expressions above, $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$, and $\gamma^{[\mu}\gamma^{\nu]} \equiv \frac{1}{2}\left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\nu\right)$

1. Show that all bilinears are Hermitian (*Hint: first, show that* $(\overline{\psi}\Gamma\psi)^\dagger = \overline{\psi\Gamma}\psi$*, where* $\overline{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0$ *is the Dirac conjugate of* $\Gamma$).

2. Show that under continuous Lorentz symmetries, the $S$ and $P$ transform as scalars, the $V^\mu$ and the $A^\mu$ as vectors, and the $T^{\mu\nu}$ as an antisymmetric tensor. (*Hint: remember what you showed in HW#3...*)

3. Find the transformation rules of the bilinears under parity and show that while $S$ is a true scalar and $V$ is a true (i.e. polar) vecotr, $P$ is a pseudo-scalar and $A$ is an axial vector.