Final Exam

Due Date: Friday December 10, 2010, 5PM in the Instructor's mailbox

The K^0_{l3} decay

The decay process

$$K^0 \to \pi^- l^+ \nu_l$$

is well described by the Fermi interaction:

$$H = \frac{G}{\sqrt{2}} J^h_\mu \bar{\psi}_{\nu_l} \gamma^\mu (1 - \gamma^5) \psi_l$$

where the hadronic current J^h_μ has a matrix element

$$\langle \pi | J^h_\mu | K \rangle = f_1 P_\mu + f_2 q_\mu$$

with $f_{1,2}$ the form factors, which are functions of q^2, and $P_\mu = (p_K)_\mu + (p_\pi)_\mu$ and $q_\mu = (p_K)_\mu - (p_\pi)_\mu$.

(i) Calculate the matrix element squared for the process, summed over final state polarizations (take the neutrino as massless, but keep m_l)

(ii) Assuming the form factors are constant, and knowing that the 3-body phase space can be cast in this case as

$$\mathrm{d}\Phi^{(3)} = \frac{1}{32\pi^3} \mathrm{d}E_l \mathrm{d}E_\pi,$$

calculate the differential decay width

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_l \mathrm{d}E_\pi}.$$

[Bonus points: explicitly calculate $\mathrm{d}\Phi^{(3)}$]

(iii) Neglect terms proportional to the charged lepton mass and integrate in the E_l and E_π variables over the appropriate kinematic regions to calculate the K^0_{l3} decay width. [Suggestion: find the range for E_π, then the range for E_l as a function of E_π and $|\vec{p}_\pi|$]

(iv) Determine the numerical value of $|f_1|^2$ knowing that experimentally $\Gamma^{-1}_{K^0_{l3}} \simeq 1.3 \times 10^{-7}$ s.