Homework Set #1.

Due Date - Oral Presentation: Wednesday October 7, 2015
Due Date - Written Solutions: Wednesday October 14, 2015

1. **Relativistic Kinematics - a warm-up**

   (a) A particle of mass m_1 decays into two particles of mass m_2 and m_3. Calculate the energy of the two final-state particles in the center of mass frame.

   (b) A positron of energy E pair-annihilates with a stationary electron producing two gamma rays. The mass of the positron is the same as the mass of the electron m, while photons are massless. Calculate the energy of the photons in the center of mass frame, as a function of the impinging positron energy E in the laboratory frame.

   (c) Suppose one of the two photons is detected (in the laboratory frame) in the opposite direction to the incident positron: calculate the photon energy as a function of E and its limit for $E/mc^2 \gg 1$.

   (d) Suppose one of the two photons is detected in the orthogonal direction to the original positron direction: calculate the energy of this photon.

2. **Lorentz Invariance**

   (a) Show that
   $$\int_{-\infty}^{\infty} \mathrm{d}k^0 \delta(k^2 - m^2)\theta(k^0) = \frac{1}{2\omega_k},$$
   where $\theta(x)$ is the unit step function and $\omega_k \equiv \sqrt{\vec{k}^2 + m^2}$.

   (b) Show that the integration measure $\mathrm{d}^4 k$ is Lorentz invariant.

   (c) Finally, show that
   $$\int \frac{\mathrm{d}^3 k}{2\omega_k}$$
   is Lorentz invariant.