PHYSICS 221A SOLUTIONS TO HW#1

PROBLEM 1

(i) ALLOWED

(ii) NOT ALLOWED: CANNOT CONSERVE ENERGY & MOMENTUM

(iii) NOT ALLOWED: VIOLATES LEPTON NUMBER CONSERV.

(iv) NOT ALLOWED: $m(n) < m(p) + m(\pi^-)$ SO IT VIOLATES ENERGY CONSERVATION

(v) NOT ALLOWED: VIOLATES CHARGE CONSERVATION

PROBLEM 2

$K^-: I = 1/2,\ I_z = -1/2\ ;\quad p: I = 1/2,\ I_z = +1/2$

$|K^- p\rangle = \sqrt{\frac{1}{2}}\,|1,0\rangle - \sqrt{\frac{1}{2}}\,|0,0\rangle$

$\pi^{\pm,0}$ AND $\Sigma^{\pm,0}$: $I = 1,\ I_z = \pm 1, 0$

$|\pi^+\Sigma^-\rangle = \sqrt{\frac{1}{6}}\,|2,0\rangle + \sqrt{\frac{1}{2}}\,|1,0\rangle + \sqrt{\frac{1}{3}}\,|0,0\rangle$

$|\pi^0\Sigma^0\rangle = \sqrt{\frac{2}{3}}\,|2,0\rangle - \sqrt{\frac{1}{3}}\,|0,0\rangle$

$|\pi^-\Sigma^+\rangle = \sqrt{\frac{1}{6}}\,|2,0\rangle - \sqrt{\frac{1}{2}}\,|1,0\rangle + \sqrt{\frac{1}{3}}\,|0,0\rangle$

(1): $\frac{1}{2}A_1 - \sqrt{\frac{1}{6}}\,A_0$

$(2): \sqrt{\frac{1}{6}}\, A_0$

$(3): \ -\frac{1}{2} A_1 - \sqrt{\frac{1}{6}}\, A_0$

$$\sigma_1 : \sigma_2 : \sigma_3 = \left|\frac{1}{2}A_1 - \frac{1}{\sqrt{6}}A_0\right|^2 : \frac{1}{6}|A_0|^2 : \left|\frac{1}{2}A_1 + \frac{1}{\sqrt{6}}A_0\right|^2$$

PROBLEM 3

$\pi^+\pi^-: \ \sqrt{\frac{1}{6}}\,|2,0\rangle + \sqrt{\frac{1}{2}}\,|1,0\rangle + \sqrt{\frac{1}{3}}\,|0,0\rangle$

FROM C-G TABLES:

$$\pi^+\pi^-\pi^0 \quad \sqrt{\frac{1}{6}}\left(\sqrt{\frac{3}{5}}\,|3,0\rangle - \sqrt{\frac{2}{5}}\,|1,0\rangle\right) +$$

$$+ \sqrt{\frac{1}{2}}\left(\sqrt{\frac{2}{3}}\,|2,0\rangle - \sqrt{\frac{1}{3}}\,|0,0\rangle\right) +$$

$$+ \sqrt{\frac{1}{3}}\,|1,0\rangle$$

THEREFORE $I = 0, 1, 2, 3$

$\pi^0\pi^0: \ \sqrt{\frac{2}{3}}\,|2,0\rangle - \sqrt{\frac{1}{3}}\,|0,0\rangle$

$\pi^0\pi^0\pi^0: \ \sqrt{\frac{2}{3}}\left(\sqrt{\frac{3}{5}}\,|3,0\rangle - \sqrt{\frac{2}{5}}\,|1,0\rangle\right) - \sqrt{\frac{1}{3}}\,|1,0\rangle$

THEREFORE $I = 1, 3$

PROBLEM 4

$$\left([T^b, T^c]\right)_{ae} = \left(T^b T^c - T^c T^b\right)_{ae} =$$

$$= T^b_{ad} T^c_{de} - T^c_{ad} T^b_{de} =$$

$$= ii f^{abd} f^{dde} - ii f^{acd} f^{dbe} = ii f^{bcd} f^{ade}$$

CONSTRUCTION

HENCE: $f^{bcd} f^{ade} + f^{acd} f^{dbe} + f^{abd} f^{cde} = 0$

NOW, CONSIDERING JACOBI'S IDENTITY:

$$\underbrace{[X_a, [X_b, X_c]]}_{ii f^{bcd} f^{ade} X^e} + [X_c, [X_a, X_b]] + [X_b, [X_c, X_a]] = 0$$

$$ii f^{bcd} f^{ade} X^e + ii f^{abd} f^{cde} X^e + ii f^{cad} f^{bde} X^e = 0$$

TO HOLD $\forall X^e$, IT MUST BE THAT

$$f^{bcd} f^{ade} + \underbrace{f^{acd} f^{dbe}}_{\text{USING ANTISYM. OF STRUCTURE CONST}} + f^{abd} f^{cde} = 0$$

Q.E.D.

PROBLEM 5

SINCE $G(M_\pi) = (-1)^{M_\pi}$, $G(\rho) = +$

THE POSSIBLE ISOSPIN VALUES ARE 0, 1, 2

SINCE $|\pi^0\pi^0\rangle = \sqrt{\frac{2}{3}}|2,0\rangle - \sqrt{\frac{1}{3}}|0,0\rangle$

THE STRONG DECAY $\rho^0 \to \pi^0\pi^0$ IS PROHIBITED ONLY IF $I = 1$

SINCE PIONS ARE BOSONS, ANGULAR MOMENTUM CONSERVATION IMPLIES THAT ρ IS A BOSON

ITS TOTAL WAVEFUNCTION MUST THUS BE SYMMETRIC

THE $I = 1$ ISOSPIN WAVEFUNCTION IS ANTISYMMETRIC THEREFORE, THE SPIN WAVEFUNCTION NEEDS TO BE ALSO ANTISYMMETRIC. SINCE THE SPIN OF ρ IS EQUAL TO THE SPATIAL WAVE FUNCTION IN THE FINAL STATE, AND THIS NEEDS TO BE ANTISYMMETRIC, L = ODD, AND THE SPIN OF ρ IS INTEGER AND ODD.

THE PARITY OF THE FINAL STATE IS $P_f = (-1)^L (P(\pi))^2 =$
$= (-1)^L = -1$

HENCE, SINCE PARITY IS CONSERVED IN STRONG INTERACTIONS, $P(\rho) = -1$.

PROBLEM 6

(i) $S=-3$ IMPLIES sss, Q# OK; Ω^-

(ii) $C=1$ IMPLIES c; $Q=2 \longrightarrow uuc$ Σ_c^{++}

(iii) $C=1$, $S=-1$ IMPLIES cs; $Q=1$ IMPLIES ucs Ξ_c^+

(iv) $C=1$, $S=-2$ IMPLIES css; $Q=0$ OK Ω_c^0

(v) $B=-1$ AND $Q=0$ IMPLY udb Λ_b^0