Physics 221A  Solutions to HW #1

**Problem 1**

(i) **Allowed**

(ii) **Not allowed**: cannot conserve energy + momentum

(iii) **Not allowed**: violates lepton number conserv.

(iv) **Not allowed**: \( m(m) < m(p) + m(\nu^-) \) so it violates energy conservation

(v) **Not allowed**: violates charge conservation

**Problem 2**

\( K^- : I = \frac{1}{2}, I_2 = -\frac{1}{2} \); \( P : I = \frac{1}{2}, I_2 = +\frac{1}{2} \)

\( K^- p > - \sqrt{2} \left| 1,0 > - \sqrt{2} \left| 0,0 > \right. \)

\( \pi^{\pm, 0} \) and \( \Sigma^{\pm, 0} : I = 1, I_2 = \pm 1, 0 \)

\( |\pi^+ \Sigma^- > = \sqrt{\frac{1}{6}} |2,0 > + \sqrt{\frac{1}{2}} |1,0 > + \sqrt{\frac{1}{3}} |0,0 > \)

\( |\pi^0 \Sigma^0 > = \sqrt{\frac{1}{3}} |2,0 > + \sqrt{\frac{1}{2}} |0,0 > \)

\( |\pi^- \Sigma^+ > = \sqrt{\frac{1}{6}} |2,0 > - \sqrt{\frac{1}{2}} |1,0 > + \sqrt{\frac{1}{3}} |0,0 > \)

(1): \( \frac{1}{2} A_i - \sqrt{\frac{1}{6}} A_0 \)
(2): $\sqrt{\frac{1}{6}} A_0$

(3): $-\frac{1}{2} A_1 - \sqrt{\frac{5}{6}} A_0$

$$\Gamma_1: \Gamma_2: \Gamma_3 = \left| \frac{1}{2} A_1 - \frac{1}{\sqrt{6}} A_0 \right|^2 : \frac{1}{6} |H_0|^2 : \left| \frac{1}{2} A_1 + \frac{1}{\sqrt{6}} A_0 \right|^2$$

**Problem 3**

$$\pi^+\pi^- : \sqrt{\frac{1}{6}} |2,0> + \sqrt{\frac{1}{2}} |1,0> + \sqrt{\frac{1}{3}} |0,0>$$

From C-G Tables:

$$\pi^+\pi^- \rightarrow \pi^0 \pi^0$$

$$\sqrt{\frac{1}{6}} \left( \sqrt{\frac{3}{5}} |3,0> - \sqrt{\frac{2}{5}} |1,0> \right) +$$

$$+ \sqrt{\frac{1}{2}} \left( \sqrt{\frac{2}{3}} |2,0> - \sqrt{\frac{1}{3}} |0,0> \right) +$$

$$+ \sqrt{\frac{1}{3}} |1,0>$$

Therefore $I = 0, 1, 2, 3$

$$\pi^0 \pi^0 : \sqrt{\frac{2}{3}} |2,0> - \sqrt{\frac{1}{3}} |0,0>$$

$$\pi^0 \pi^0 \pi^0 : \sqrt{\frac{1}{3}} \left( \sqrt{\frac{2}{3}} |3,0> - \sqrt{\frac{1}{3}} |1,0> \right) - \sqrt{\frac{1}{3}} |1,0>$$

Therefore $I = 1, 3$
Problem 4

\[
(T^b, T^c)_{ae} = (T^b T^c - T^c T^b)_{ae} = T^b T^c_{de} - T^c T^b_{de} = ii f_\text{abd f dde} - ii f_\text{acd f dbe} = ii f_\text{bcd f ade} / \text{CONSTRUCTION}
\]

Hence: \( f_\text{bcd f ade} + f_\text{acd f dbe} + f_\text{abd f cde} = 0 \)

Now, considering Jacobi's identity:

\[
[X_a, [X_b, X_c]] + [X_c, [X_a, X_b]] + [X_b, [X_c, X_a]] = 0
\]

\[ ii f_\text{bcd f ade X e} + ii f_\text{abd f cde X e} + ii f_\text{caod f bde e} = 0 \]

To hold \( \forall X^e \), it must be that

\[ f_\text{bcd f ade} + f_\text{acd f dbe} + f_\text{abd f cde} = 0 \]

using antisym. of structure const.  Q.E.D.
Problem 5

Since \( G(M_\pi) = (-1)^{M_\pi} \), \( G(p) = + \)

The possible isospin values are 0, 1, 2

Since \( (\pi^0 \pi^0) = \sqrt{\frac{2}{3}} |2, 0\rangle - \sqrt{\frac{1}{3}} |0, 0\rangle \)

The strong decay \( p^0 \rightarrow \pi^0 \pi^0 \) is prohibited only if \( I = 1 \)

Since pions are bosons, angular momentum conservation implies that \( p \) is a boson

Its total wavefunction must thus be symmetric

The \( I = 1 \) isospin wavefunction is antisymmetric. Therefore, the spin wavefunction needs to be also antisymmetric. Since the spin of \( p \) is equal to the spatial wave function in the final state, and this needs to be antisymmetric, \( L = \text{odd} \), and the spin of \( p \) is integer and odd.

The parity of the final state is \( P_f = (-1)^L (P(p))^2 \)

\( = (-1)^{\text{odd}} = -1 \)

Hence, since parity is conserved in strong interactions, \( P(p) = -1 \).
Problem 6

(i) \( S = -3 \) IMPLIES 555, \( Q \neq 0 \# \) OK \( \Omega \)

(ii) \( C = 1 \) IMPLIES \( C \); \( Q = 2 \rightarrow \) MAKE \( \Sigma \)

(iii) \( C = 1 \)
\( S = -1 \)
IMPLIES \( CS \); \( Q = 1 \) IMPLIES \( MC \)

(iv) \( C = 1 \)
\( S = -2 \)
IMPLIES \( CSS \); \( Q = 0 \) OK \( \eta \)

(v) \( B = -1 \) and \( Q = 0 \) IMPLY \( \mu dB \) \( \lambda \)