Homework Set #2.

Due Date: Tuesday November 4, 2008

Solve the following 6 exercises:

1. Consider the decay of an initial particle of mass $M_{\rm in}$ into two final state particles of mass m_1 and m_2. Show that in the center of mass frame the final particles are produced with energies

$$E_1 = \frac{M_{\rm in}^2 + m_1^2 - m_2^2}{2M_{\rm in}}, \qquad E_2 = \frac{M_{\rm in}^2 + m_2^2 - m_1^2}{2M_{\rm in}}$$

and with momentum

$$P = |\vec{p}_1| = |\vec{p}_2| = \frac{1}{2M_{\rm in}}\sqrt{M_{\rm in}^4 + \left(m_1^2 - m_2^2\right)^2 - 2M_{\rm in}^2\left(m_1^2 + m_2^2\right)}.$$

2. Prove that in the case of non-zero neutrino mass m_ν the electron spectrum in a Kurie plot reads

$$N(p)dp \propto p^2 (E_0 - E)^2 \sqrt{1 - \left(\frac{m_\nu}{E_0 - E}\right)^2}\, dp.$$

3. The neutron has a lifetime of ~ 930 s and the muon of $\sim 2.2 \times 10^{-6}$ s. Using Sargent's rule, show that the couplings G_n and G_μ involved in the two cases are of the same order of magnitude when account is taken of phase space factors ($m_n \simeq 939.6$ MeV, $m_p \simeq 938.3$ MeV, $m_e \simeq 0.51$ MeV, $m_\mu \simeq 106$ MeV).

4. The Majorana representation of gamma matrices is given by the choices:

$$\gamma_0 = \begin{pmatrix} 0 & i\sigma_1 \\ -i\sigma_1 & 0 \end{pmatrix}, \qquad \gamma_1 = \begin{pmatrix} iI & 0 \\ 0 & -iI \end{pmatrix},$$

$$\gamma_2 = \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix}, \qquad \gamma_3 = \begin{pmatrix} 0 & iI \\ iI & 0 \end{pmatrix}$$

(a) Compute $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$.
(b) Show that this is a valid representation, namely that:

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu} I, \qquad \gamma_\mu^\dagger = \gamma_0 \gamma_\mu \gamma_0, \qquad \{\gamma_5, \gamma_\mu\} = 0, \qquad \gamma_5^2 = I.$$

(c) Show that $\gamma_\mu^* = -\gamma_\mu$.
(d) Show that if ψ is a solution to the Dirac equation, then in this representation ψ^* is also a solution.

5. Using the expression for the differential muon decay width $\mathrm{d}\Gamma$, compute the angular distribution of electrons as a function of $\cos\theta_e$, the angle between the muon spin and the electron momentum.

6. The GALLEX experiment at the Gran Sasso laboratory measured the ν_e flux from the Sun by counting the electrons produced in the reaction

$$\nu_e + {}^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge} + e^- .$$

   The energy threshold for the reaction is $E_{\mathrm{th}} \simeq 233$ keV. From the solar luminosity one expects a neutrino flux $\phi \simeq 6 \times 10^{14}$ m^{-2} s^{-1}. For a rough estimate, assume the whole flux to be above threshold and a constant cross section $\sigma \simeq 10^{-48}$ m^2. Assuming a detection efficiency $\epsilon \simeq 40\%$, how many ^{71}Ga nuclei are necessary to have one neutrino interaction per day? What is the corresponding ^{71}Ga mass? What is the natural gallium mass if the abundance of the ^{71}Ga isotope is $a \simeq 40\%$?

   (*The measured flux turned out to be about one-half of the expected value. This was a fundamental observation in the process of discovering neutrino oscillations.*)