SOLUTIONS TO HW #3

$\gamma_5^* = \gamma_5$

PROBLEM 1 $\quad \bar{u}\,(1-\gamma_5)\,v\; v^*\,(1-\gamma_5)$

$$M_{fi} = \frac{G_F}{\sqrt{2}}\left[\bar{u}_2\,(1-i\gamma_5)\,u_\mu\right]\left[\bar{u}_e\,(1-i\gamma_5)\,v_1\right]$$

$$|M_{fi}|^2 = \frac{G_F^2}{2}\,\mathrm{Tr}\left[u_2\bar{u}_2(1-i\gamma_5)\,u_\mu\bar{u}_\mu(1-i\gamma_5)\right]\times$$

$$\times\,\mathrm{Tr}\left[u_e\bar{u}_e\,(1-i\gamma_5)\;v_1\bar{v}_1\,(1-i\gamma_5)\right] =$$

$(1+i\gamma_5)(1-i\gamma_5) = 1+\gamma_5^2 = 2$

$$= \frac{G_F^2}{2}\,\mathrm{Tr}\left[\not{p}_2\,(\cancel{1}-i\gamma_5)(\not{p}_\mu+m_\mu)\left(\frac{1+\gamma_5\not{s}_\mu}{\cancel{2}}\right)(\cancel{1}-i\gamma_5)\right]\times$$

$$\times\,\mathrm{Tr}\left[\not{p}_e\left(\frac{1+\gamma_5\not{s}_e}{\cancel{2}}\right)(\cancel{1}-i\gamma_5)\,\not{p}_1\,(\cancel{1}-i\gamma_5)\right] =$$

$$= \frac{G_F^2}{2}\,\mathrm{Tr}\left[\not{p}_2(\not{p}_\mu+m_\mu)(1+\gamma_5\not{s}_\mu)\right]\times\mathrm{Tr}\left[\not{p}_e(1+\gamma_5\not{s}_e)\not{p}_1\right] =$$

$$= \frac{G_F^2}{2}\,\mathrm{Tr}\left[\not{p}_2\not{p}_\mu\right]\mathrm{Tr}\left[\not{p}_e\not{p}_1\right] = \frac{G_F^2}{2}\cdot 16\cdot(p_2\cdot p_\mu)(p_e\cdot p_1) =$$

2

$$\cancel{(m_\mu, \vec{0}) = (E_e}$$

$$\rightarrow d\Gamma = \frac{\overset{4}{\cancel{8}}\, G_F^2}{\cancel{2}\, m_\mu\, \cancel{2}^{8}\, \pi^5} (P_e)_\alpha (P_\mu)_\beta\, I_{\alpha\beta}\, \frac{d\vec{P}_e}{E_e}$$

$$8$$

$$I_{\alpha\beta} = \frac{\pi}{6}\left(k^2 g_{\alpha\beta} + 2 k_\alpha k_\beta\right)$$

$$d\Gamma = \frac{G_F^2}{8\pi^4 m_\mu} \frac{\pi}{6E_e}\Big[\underbrace{k^2}_{m_\mu^2 - 2 m_\mu E_e} \underbrace{(P_e \cdot P_\mu)}_{m_\mu E_e} + 2 \underbrace{(P_e \cdot k)}_{m_\mu E_e} \underbrace{(P_\mu \cdot k)}_{m_\mu^2 - m_\mu E_e}\Big] =$$

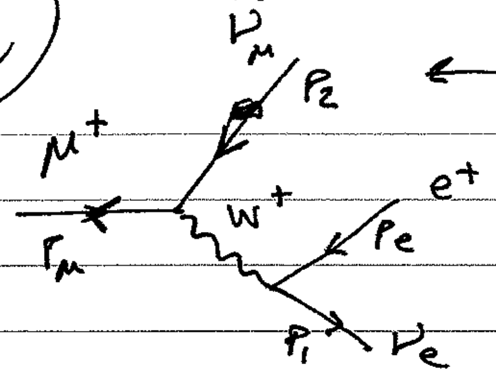
$$dx = \frac{2\, dE_e}{m_\mu}$$

$$= \frac{G_F^2}{48\pi^4 \cancel{m_\mu}} \cdot m_\mu^3 \left[(1 - \cancel{2}x)\frac{x}{2} + \cancel{2}\, \frac{x}{\cancel{2}}\left(1 - \frac{x}{2}\right)\right] \cdot dE_e\, d\Omega_e \cdot \frac{E_e^{\cancel{2}}}{\cancel{E_e}}$$

$$= \frac{G_F^2}{48\pi^4\, 3}\, m_\mu^5\, \frac{1}{4}\left[\frac{x}{2} - \frac{x^2}{2} + x - \frac{x^2}{2}\right] dx\, \underset{\downarrow\ \cancel{4\pi}}{d\Omega_e}$$

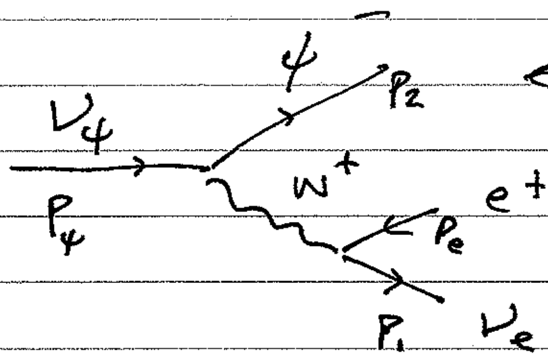
$$\Rightarrow \frac{d\Gamma}{dx} = \frac{G_F^2 m_\mu^5}{48\pi^3}\left(\frac{3}{2}x - x^2\right)$$

PROBLEM #2



$\bar{v}_{p_\mu}\gamma(1-\gamma_5)\,u_2$

$u_1\,\gamma_\lambda\,(1-\gamma_5)\,\bar{v}_e$



$\bar{u}_2\,\gamma_\lambda\,(1-\gamma_5)\,u_\mu$

$u_1\,\gamma_\lambda\,(1-\gamma_5)\,\bar{v}_e$

$$|M_{fi}|_1^2 = \frac{G_F^2}{2}\,Tr\left[\cancel{\bar{v}_2\gamma_\lambda(1-\gamma_5)\,\bar{v}_\mu\gamma_0 v_\mu(1+\gamma_5)\gamma_\sigma v_2}\right]$$

$$v_\mu^*\gamma_0 \to \gamma_0$$

$$|M_{fi}|_1^2 = \frac{G_F^2}{2}\,Tr\left[\underbrace{v_\mu\bar{v}_\mu}_{\frac{1-\gamma_5\not{s}_\mu}{2}}\gamma_\lambda(1-\gamma_5)\underbrace{v_2\bar{v}_2}_{\not{p}_2}\gamma_\sigma(1-\gamma_5)\right]\times\ldots$$

$$|M_{fi}|_2^2 = \frac{G_F^2}{2}\,Tr\left[\underbrace{u_2\bar{u}_2}_{\not{p}_2}\gamma_\lambda(1-\gamma_5)\underbrace{u_\mu\bar{u}_\mu}_{\frac{1+\gamma_5\not{s}_\mu}{2}}\gamma_\sigma(1-\gamma_5)\right]$$

$$\boxed{s_\mu \to -s_\mu} \to d\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}\left[2x^2(3-2x)\right]\left[1-\left(\frac{1-2x}{3-2x}\right)\cos\theta_e\right]\frac{dx\,d\Omega}{4\pi}$$

PROBLEM # 3

$$\frac{\Gamma_{D_s^- \to \tau^- \nu_\tau}}{\Gamma_{D_s^- \to \mu^- \nu_\mu}} = \left(\frac{m_\tau}{m_\mu}\right)^2 \left(\frac{m_{D_s^-}^2 - m_\tau^2}{m_{D_s^-}^2 - m_\mu^2}\right)^2 \simeq 9.7$$

WITH $m_{D_s^-} = 1968$ MeV

$m_\mu = 105.66$ MeV

$m_\tau = 1777.0$ MeV

MEASURED : 10.65

~~MEASURED~~

GIVEN $\tau_{D_s} \simeq 500 \times 10^{-15}$ s $\longrightarrow \Gamma_{TOT} \simeq 1.3 \times 10^{-9}$ MeV

$$\Gamma_{D_s \to \tau \nu_\tau} = \frac{|V_{cs}|^2 G_F^2 f_{D_s}^2 m_\tau^2 m_{D_s}}{8\pi} \left(1 - \frac{m_\tau^2}{m_{D_s}^2}\right)^2 =$$

$= 8.2 \times 10^{-11}$ MeV

$|V_{cs}| = 0.957$

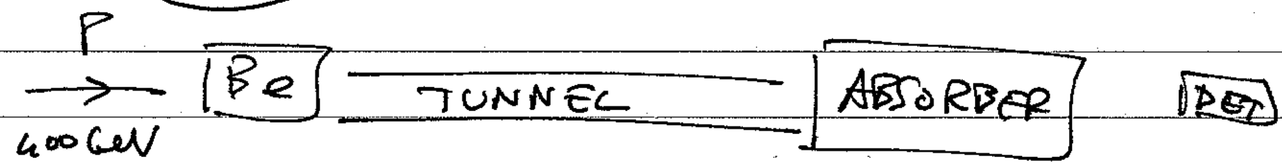
Hence, the predicted BR is : $\frac{\Gamma_{D_s \to \tau \nu_\tau}}{\Gamma_{TOT}} \sim 6.2\%$

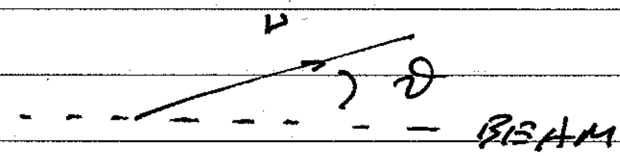
(MEASURED: $(6.6 \pm 0.6)\%$)

PROBLEM # 4

Tedious, direct substitution :>

PROBLEM # 5





LET E_ν = LAB ENERGY

$\bar{E}$: ν ENERGY IN THE MESON REST FRAME

A RELATIVISTIC TRANSFORMATION GIVES

$$(1) \quad \bar{E} = \gamma E_\nu (1 - \beta \cos\vartheta) \quad ; \quad \bar{E}_{K,\pi} = \frac{M_{K,\pi}^2 - m_\mu^2}{2 M_{K,\pi}}$$

WHERE $\gamma_K \simeq \frac{200\ \text{GeV}}{m_K} = 405$ $\qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$

$\gamma_\pi \simeq 1439$

FROM (1) WE GET $E_\nu = \dfrac{\bar{E}}{\gamma(1 - \beta\cos\vartheta)}$

WHICH TELLS US THAT $E_\nu^{MAX} = \dfrac{\bar{E}}{\gamma(1-\beta)}$ (FOR $\cos\vartheta = 1$)

$E_\nu^{MIN} = \dfrac{\bar{E}}{\gamma(1+\beta)}$ (FOR $\cos\vartheta = -1$)

$$E_\nu^{MAX} = \frac{\bar{E}}{\gamma\left(1-\sqrt{1-\frac{1}{\gamma^2}}\right)} \simeq \frac{\bar{E}}{\gamma\left(1-1+\frac{1}{2\gamma^2}\right)} \simeq 2\gamma\bar{E}$$

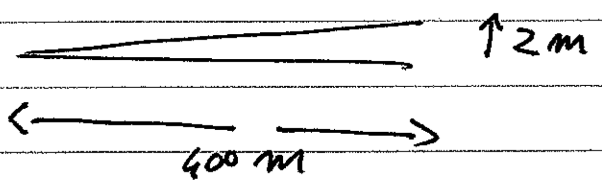
THUS: $\bar{E}_\nu^{MAX}(k) = 2\gamma_k \bar{E}_k \simeq 192 \text{ GeV}$

$$E_\nu^{MAX}(\pi) = 2\gamma_\pi \bar{E}_\pi \simeq 89 \text{ GeV}$$

FOR $\bar{E}_\nu^{MIN} = \frac{\bar{E}}{\gamma(1+\beta)} \simeq \frac{\bar{E}}{2\gamma_{\pi,k}} = \begin{cases} \pi: 0.01 \text{ GeV} \\ K: 0.29 \text{ GeV} \end{cases}$

(4.3)

REQUIRING THAT THE ν'S FROM k DECAY TRAVERSE THE DETECTOR AMOUNTS TO REQUIRING A CONDITION ON THE ANGLE ϑ:



$$\bar{\vartheta} = \text{ArcTan}\left(\frac{2}{400}\right)$$

$$\bar{\vartheta} \simeq 0.286°$$

HENCE ALL ν'S WITH ENERGIES $E_\nu > \frac{\bar{E}_k}{\gamma_k(1-\beta_k\cos\bar{\vartheta})}$

$E_\nu > 37.4 \text{ GeV}$

(4.4)

THE FRACTION OF PIONS DECAYING IN $L = 300\,m$

IS $$F_{\pi} = \frac{L}{\gamma_{\pi} c \tau_{\pi}} \simeq \frac{300\ m}{11{,}183\ m} \simeq 2.7\%$$

THE CONDITION ON ENERGY FOR ν FROM π DECAY TO TRAVERSE THE DETECTOR IS

$$E_{\nu} > \frac{\overline{E}_{\pi}}{\gamma_{\pi}(1-\beta_{\pi}\cos\overline{\vartheta})} = 1.6\ GeV$$

THE NEUTRINO SPECTRUM IS ENERGY INDEPENDENT, AND COSTANT BETWEEN E_{ν}^{MIN} AND E_{ν}^{MAX}

TO SEE THIS, WRITE $$\frac{dN}{dE_{\nu}} = \underbrace{\left(\frac{dN}{d\cos\vartheta_0}\right)}_{\text{ISOTROPIC}}\left(\frac{d\cos\vartheta_0}{dE_{\nu}}\right) = \text{const}$$

ϑ_0 : DECAY ANGLE IN THE MESON REST FRAME.

HENCE, THE FRACTION OF π ν'S CROSSING THE DETECTOR WILL BE:

$$\text{Frac} = F_{\pi} \cdot \frac{E_{\nu}^{MAX,\pi} - 1.6}{E_{\nu}^{MAX,\pi} - E_{\nu}^{MIN,\pi}} \simeq 2.6\%$$

(4.5)

THE MEAN ν ENERGY IS $\bar{E}_\nu \sim \frac{E_\nu^{MAX,\pi}}{2} \simeq 42.5$ GeV

WHICH YIELDS AN AVERAGE $\bar{\sigma} \simeq 2.5 \times 10^{-37}$ cm^2

THE INTERACTION MEAN FREE PATH IS $\lambda = \frac{1}{N_A \bar{\sigma}} \sim 6.5 \times 10^{+12} \frac{g}{cm^2}$

WITH $N_A = 6.022 \times 10^{23} \frac{1}{g}$

THE DEPTH OF THE NEUTRINO DETECTOR ALONG

AXIS IS $\delta = \frac{M}{\pi R^2} \simeq 796 \frac{g}{cm^2}$

HENCE THE NUMBER OF NEUTRINOS PER BURST THAT INTERACT IN THE DETECTOR IS

$$N_{INT} \simeq N_\nu (\delta / \lambda) = \underbrace{10^{10} \cdot F_{REL} \cdot \left(\frac{796}{6.5 \times 10^{12}} \right)}_{0.026} \simeq 0.03$$