SOLUTIONS TO HW #3

$\gamma_5^* = \gamma_5$

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PROBLEM 1

$\bar{u}\,(1-\gamma_5)\,v\,v^*\,(1-\gamma_5)$

$$M_{fi} = \frac{G_F}{\sqrt{2}}\left[\bar{u}_2\,(1-i\gamma_5)\,u_\mu\right]\left[\bar{u}_e\,(1-i\gamma_5)\,v_1\right]$$

$$|M_{fi}|^2 = \frac{G_F^2}{2}\,\mathrm{Tr}\left[u_2\bar{u}_2(1-i\gamma_5)\,u_\mu\bar{u}_\mu(1-i\gamma_5)\right] \times$$

$$\times\,\mathrm{Tr}\left[u_e\bar{u}_e\,(1-i\gamma_5)\,v_1\bar{v}_1\,(1-i\gamma_5)\right] =$$

$(1+i\gamma_5)(1-i\gamma_5) = 1+\gamma_5^2 = 2$

$$= \frac{G_F^2}{2}\,\mathrm{Tr}\left[\not{p}_2\,\cancel{(1-i\gamma_5)}(\not{p}_\mu+m_\mu)\left(\frac{1+\gamma_5\not{s}_\mu}{\cancel{2}}\right)\cancel{(1-i\gamma_5)}\right] \times$$

$$\times\,\mathrm{Tr}\left[\not{p}_e\left(\frac{1+\gamma_5\not{s}_e}{\cancel{2}}\right)\cancel{(1-i\gamma_5)}\,\not{p}_1\,\cancel{(1-i\gamma_5)}\right] =$$

$$= \frac{G_F^2}{2}\,\mathrm{Tr}\left[\not{p}_2(\not{p}_\mu+m_\mu)(1+\gamma_5\not{s}_\mu)\right] \times \mathrm{Tr}\left[\not{p}_e(1+\gamma_5\not{s}_e)\not{p}_1\right] =$$

$$= \frac{G_F^2}{2}\,\mathrm{Tr}\left[\not{p}_2\not{p}_\mu\right]\mathrm{Tr}\left[\not{p}_e\not{p}_1\right] = \frac{G_F^2}{2}\cdot 16\cdot(p_2\cdot p_\mu)(p_e\cdot p_1) =$$

$$d\Gamma = \frac{8G_F^2 (P_2 \cdot P_\mu)(P_e \cdot P_1)}{2m_\mu (2\pi)^5} \frac{d^3P_e}{2E_e} \frac{d^3P_1}{2E_1} \frac{d^3P_2}{2E_2} \delta^4(P_f - P_i) =$$

$$= \frac{G_F^2 (P_\mu)_\alpha (P_e)_\beta}{2(2\pi)^5 m_\mu} \frac{E_e^2 \, dE_e \, d\Omega_e}{E_e} I_{\alpha\beta}$$

WHERE $I_{\alpha\beta} = \frac{\pi}{6}\left(k^2 g_{\alpha\beta} + 2k_\alpha k_\beta\right)$

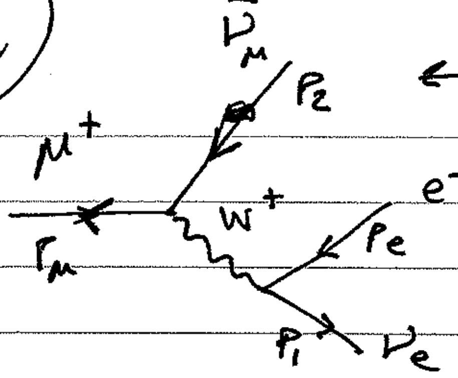
$$d\Gamma = \frac{G_F^2}{64\pi^5 m_\mu} E_e \frac{\pi}{6}\Big[\underbrace{k^2}_{m_\mu^2 - 2m_\mu E_e} \underbrace{(P_e \cdot P_\mu)}_{m_\mu E_e} + 2\underbrace{(P_e \cdot k)}_{m_\mu E_e}\underbrace{(P_\mu \cdot k)}_{m_\mu^2 - m_\mu E_e}\Big] dE_e \, d\Omega_e =$$

$$= \frac{G_F^2 m_\mu^4}{384\pi^4 m_\mu} E_e \left[(1-x)\frac{x}{2} + 2\frac{x}{2}\left(1-\frac{x}{2}\right)\right] dE_e \, d\Omega_e =$$

$$= \frac{G_F^2 m_\mu^5}{1536\pi^4} x\left[\frac{x}{2} - \frac{x^2}{2} + x - \frac{x^2}{2}\right] dx \underbrace{d\Omega_e}_{4\pi} =$$

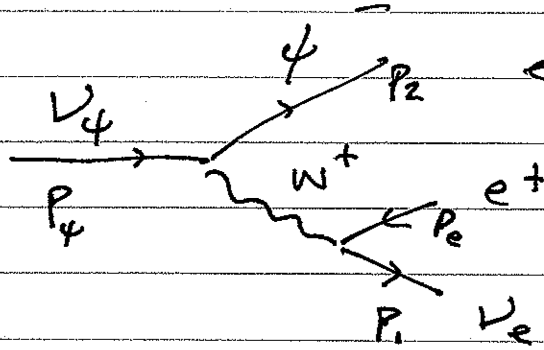
$$\Rightarrow \frac{d\Gamma}{dx} = \frac{G_F^2 m_\mu^5}{384\pi^3}\left[\frac{3}{2}x^2 - x^3\right]$$

PROBLEM #2



$\bar{v}_{\mu}\gamma(1-\gamma_5)\, v_2$

$u_1\gamma_\lambda(1-\gamma_5)\bar{v}_e$



$\bar{u}_2\gamma_\lambda(1-\gamma_5)u_\mu$

$u_1\gamma_\lambda(1-\gamma_5)\bar{v}_e$

$$|M_{fi}|^2_1 = \frac{G_F^2}{2}\, Tr\left[\cancel{v_2\gamma_\lambda(1-\gamma_5)\bar{v}_\mu\gamma_0 v_\mu(1+\gamma_5)\gamma_\sigma \bar{v}_2}\right]$$

$$v_\mu^+\gamma_0 \to \gamma_0$$

$$|M_{fi}|^2_1 = \frac{G_F^2}{2}\, Tr\left[\underbrace{v_\mu\bar{v}_\mu}_{\frac{1-\gamma_5 \not{s}_\mu}{2}}\gamma_\lambda(1-\gamma_5)\underbrace{v_2\bar{v}_2}_{\not{p}_2}\gamma_\sigma(1-\gamma_5)\right]\times\ldots$$

$$|M_{fi}|^2_2 = \frac{G_F^2}{2}\, Tr\left[\underbrace{u_2\bar{u}_2}_{\not{p}_2}\gamma_\lambda(1-\gamma_5)\underbrace{u_\mu\bar{u}_\mu}_{\frac{1+\gamma_5\not{s}_\mu}{2}}\gamma_\sigma(1-\gamma_5)\right]$$

$$\boxed{s_\mu \to -s_\mu} \to d\Gamma = \frac{G_F^2 m_\nu^5}{192\pi^3}\left[2x^2(3-2x)\right]\left[1-\left(\frac{1-2x}{3-2x}\right)\cos\theta_e\right]\frac{dx\, d\Omega}{4\pi}$$

PROBLEM #3

$$\frac{\Gamma_{D_s^- \to \tau^- \nu_\tau}}{\Gamma_{D_s^- \to \mu^- \nu_\mu}} = \left(\frac{m_\tau}{m_\mu}\right)^2 \left(\frac{m_{D_s^-}^2 - m_\tau^2}{m_{D_s^-}^2 - m_\mu^2}\right)^2 \simeq 9.7$$

WITH $m_{D_s^-} = 1968$ MeV

$m_\mu = 105.66$ MeV

$m_\tau = 1777.0$ MeV

MEASURED : 10.65

GIVEN $\tau_{D_s} \simeq 500 \times 10^{-15}$ s $\longrightarrow$ $\Gamma_{TOT} \simeq 1.3 \times 10^{-9}$ MeV

$$\Gamma_{D_s \to \tau \nu_\tau} = \frac{|V_{cs}|^2 G_F^2 f_{D_s}^2 m_\tau^2 m_{D_s}}{8\pi} \left(1 - \frac{m_\tau^2}{m_{D_s}^2}\right)^2 =$$

$$= 8.2 \times 10^{-11} \text{ MeV}$$

$|V_{cs}| = 0.957$

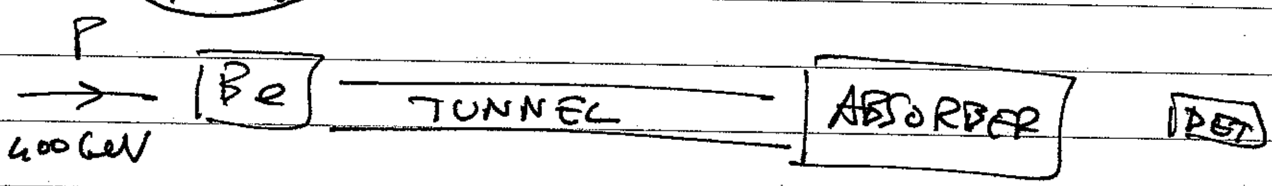
Hence, the predicted BR is : $\frac{\Gamma_{D_s \to \tau\nu_\tau}}{\Gamma_{TOT}} \sim 6.2\%$

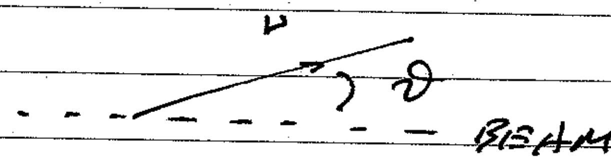
(MEASURED: $(6.6 \pm 0.6)\%$)

PROBLEM # 4

Tedious, direct substitution :>

PROBLEM # 5





LET E_ν = LAB ENERGY

$\bar{E}$: ν ENERGY IN THE MESON REST FRAME

A RELATIVISTIC TRANSFORMATION GIVES

$$(1) \qquad \bar{E} = \gamma E_\nu (1 - \beta \cos\vartheta) \quad ; \quad \bar{E}_{K,\pi} = \frac{M_{K,\pi}^2 - m_\mu^2}{2 M_{K,\pi}}$$

WHERE $\gamma_K \simeq \frac{200\text{ GeV}}{m_K} = 405$ $\qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$

$\gamma_\pi \simeq 1439$

FROM (1) WE GET $E_\nu = \frac{\bar{E}}{\gamma(1-\beta\cos\vartheta)}$

WHICH TELLS US THAT $E_\nu^{MAX} = \frac{\bar{E}}{\gamma(1-\beta)}$ (FOR $\cos\vartheta = 1$)

$E_\nu^{MIN} = \frac{\bar{E}}{\gamma(1+\beta)}$ (FOR $\cos\vartheta = -1$)

$$E_\nu^{MAX} = \frac{\bar{E}}{\gamma\left(1-\sqrt{1-\frac{1}{\gamma^2}}\right)} \simeq \frac{\bar{E}}{\gamma\left(1-1+\frac{1}{2\gamma^2}\right)} \simeq 2\gamma \bar{E}$$

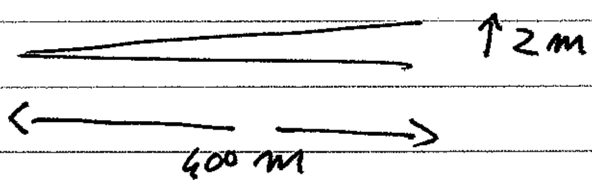
THUS: $E_\nu^{MAX}(k) = 2\gamma_k \bar{E}_k \simeq 192 \text{ GeV}$

$E_\nu^{MAX}(\pi) = 2\gamma_\pi \bar{E}_\pi \simeq 89 \text{ GeV}$

FOR $E_\nu^{MIN} = \dfrac{\bar{E}}{\gamma(1+\beta)} \simeq \dfrac{\bar{E}}{2\gamma_{\pi,k}} = \begin{cases} \pi: 0.01 \text{ GeV} \\ K: 0.29 \text{ GeV} \end{cases}$

(4.3)

REQUIRING THAT THE ν'S FROM k DECAY TRAVERSE THE DETECTOR AMOUNTS TO REQUIRING A CONDITION ON THE ANGLE ϑ:



$\bar{\vartheta} = \text{ArcTan}\left(\frac{2}{400}\right)$

$\bar{\vartheta} \simeq 0.286°$

HENCE ALL ν'S WITH ENERGIES $E_\nu > \dfrac{\bar{E}_k}{\gamma_k(1-\beta_k \cos\bar{\vartheta})}$

$E_\nu > 37.4 \text{ GeV}$

(4.4)

THE FRACTION OF PIONS DECAYING IN $L = 300\ m$

IS $$F_\pi = \frac{L}{\gamma_\pi c \tau_\pi} \simeq \frac{300\ m}{11,183\ m} \simeq 2.7\%$$

THE CONDITION ON ENERGY FOR ν FROM π DECAY TO TRAVERSE THE DETECTOR IS

$$E_\nu > \frac{\bar{E}_\pi}{\gamma_\pi (1 - \beta_\pi \cos\bar{\vartheta})} = 1.6\ GeV$$

THE NEUTRINO SPECTRUM IS ENERGY INDEPENDENT, AND COSTANT BETWEEN E_ν^{MIN} AND E_ν^{MAX}

TO SEE THIS, WRITE $$\frac{dN}{dE_\nu} = \underbrace{\left(\frac{dN}{d\cos\vartheta_0}\right)}_{ISOTROPIC}\left(\frac{d\cos\vartheta_0}{dE_\nu}\right) \Rightarrow$$

ϑ_0 : DECAY ANGLE IN THE MESON REST FRAME.

HENCE, THE FRACTION OF π ν'S CROSSING THE DETECTOR WILL BE:

$$Frac = F_\pi \cdot \frac{E_\nu^{MAX,\pi} - 1.6}{E_\nu^{MAX,\pi} - E_\nu^{MIN,\pi}} \simeq 2.6\%$$

(4.5)

THE MEAN ν ENERGY IS $\bar{E}_\nu \sim \frac{E_\nu^{MAX,\pi}}{2} \simeq 42.5$ GeV

WHICH YIELDS AN AVERAGE $\bar{\sigma} \simeq 2.5 \times 10^{-37}$ cm^2

THE INTERACTION MEAN FREE PATH IS $\lambda = \frac{1}{N_A \bar{\sigma}} \sim 6.5 \times 10^{+12} \frac{g}{cm^2}$

WITH $N_A = 6.022 \times 10^{23} \frac{1}{g}$

THE DEPTH OF THE NEUTRINO DETECTOR ALONG

AXIS IS $\delta = \frac{M}{\pi R^2} \simeq 796 \frac{g}{cm^2}$

HENCE THE NUMBER OF NEUTRINOS PER BURST THAT INTERACT IN THE DETECTOR IS

$$N_{INT} \simeq N_\nu (\delta / \lambda) = \underbrace{10^{10} \cdot F_{rel} \cdot \left(\frac{796}{6.5 \times 10^{12}} \right)}_{0.026} \simeq 0.03$$