

Homework Set #1.

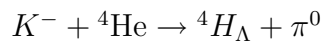
Due Date: Monday October 15, 2007

Solve at least 6 of the following 9 proposed exercises:

1. State which of the following reactions are allowed by conservation laws and which are forbidden, and motivate your statement in either case:

$$\begin{aligned}
 \text{(i)} \quad & \pi^0 \rightarrow e^+ + e^- \\
 \text{(ii)} \quad & p \rightarrow n + e^+ + \nu_e \\
 \text{(iii)} \quad & \mu^+ \rightarrow e^+ + e^- + e^+ \\
 \text{(iv)} \quad & K^+ + n \rightarrow \Sigma^+ + \pi^0
 \end{aligned}$$

2. Capture of negative kaons in helium sometimes leads to the formation of a hypernucleus (a nucleus where a neutron is replaced by a Λ hyperon) according to the reaction



Study of the decay branching ratios of ${}^4H_\Lambda$ and the isotropy of decay products establishes that $J({}^4H_\Lambda) = 0$. Show that this implies negative parity for the K^- , independent of the orbital angular momentum of the state from which the K^- is captured.

3. What restrictions does the decay mode $K^0 \rightarrow \pi^0\pi^0$ place on the neutral kaon spin and parity?
4. Find the relation between the total cross-sections (at a given center of mass energy) for the following reactions:

$$\begin{aligned}
 \text{(1)} \quad & \pi^- + p \rightarrow K^0 + \Sigma^0 \\
 \text{(2)} \quad & \pi^- + p \rightarrow K^+ + \Sigma^- \\
 \text{(3)} \quad & \pi^+ + p \rightarrow K^+ + \Sigma^+
 \end{aligned}$$

5. In which isospin states can (1) $\pi^+\pi^-\pi^0$ and (2) $\pi^0\pi^0\pi^0$ exist?
6. Deduce through which isospin channels the following reactions may proceed:

$$\begin{aligned}(1) \quad K^- + p &\rightarrow \Sigma^0 + \pi^0 \\(2) \quad K^- + p &\rightarrow \Sigma^+ + \pi^-\end{aligned}$$

and find out the ratio of the two cross sections assuming that one or other channel dominates.

7. The representation matrices for the adjoint representation of a Lie group are given by the structure constants:

$$(t_G^b)_{ac} = if^{abc}.$$

Show that the statement that the t_G^a satisfy the Lie algebra

$$\left([t_G^b, t_G^c]\right)_{ae} = if^{bcd}(t_G^d)_{ae}$$

is equivalent to the Jacobi identity.

8. Compute the magnetic moments of the following baryons in terms of the magnetic moments of quarks: p , n , Λ , Σ^+ , Σ^0 , Σ^- , Ξ^0 , Ξ^- .
9. Discuss the possible decay modes of the Ω^- hyperon allowed by conservation laws, and show that weak decay is the only possibility.