

Homework Set #3.

Due Date: Monday November 5, 2007

Solve the following 6 exercises:

1. Consider a matrix element that has $(1 - i\gamma_5)$ appearing instead of $\gamma_\mu(1 - \gamma_5)$ for $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$. Calculate the e^- spectrum in the μ^- rest frame for this hypothetical matrix element, neglecting m_e , and compare to the standard case.
2. Suppose that there exists a fourth generation lepton doublet made of a nearly massless charged lepton ψ^- and a heavy neutrino ν_ψ . The ν_ψ decay mode would then be

$$\nu_\psi \rightarrow \psi^- l^+ \nu_l.$$

- (2.1) Neglecting m_ψ and m_l compared to m_{ν_ψ} , find the l^+ spectrum in the ν_ψ rest frame.
- (2.2) How does the shape of the spectrum compare to the e^+ spectrum in μ^+ decay?
- (2.3) Calculate the ν_ψ decay width for the decay mode above, as a function of m_{ν_ψ} .
3. Consider the decays $D_s^- \rightarrow \tau^- \bar{\nu}_\tau$ and $D_s^- \rightarrow \mu^- \bar{\nu}_\mu$. These can be calculated in terms of one constant, f_{D_s} , analogous to the pion decay constant f_π . Calculate the ratio of the two decay amplitudes. Using the measured D_s lifetime calculate the expected branching ratio to $\tau^- \bar{\nu}_\tau$, assuming $f_{D_s} = 280$ MeV.
4. Show explicitly that the CKM matrix is unitary using the following parametrization (same as in Seiden's Eq. (8.17))

$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

where $c_{ij} \equiv \cos(\theta_{ij})$, $s_{ij} \equiv \sin(\theta_{ij})$ and θ_{ij} , $\delta \in \mathbf{R}$.

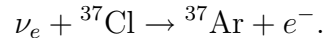
5. A “narrowband” neutrino beam is produced by bombarding a Be target with 400 GeV protons and forming a “pencil” secondary beam with a small spread in momentum centered at 200 GeV/c. This beam contains charged pions and kaons with one sign of charge and it traverses

an evacuated decay tunnel 300 m long, where a fraction of the mesons decay to muons and neutrinos. This is followed by an absorber consisting of 200 m of steel and 150 m of rock. A cylindrical detector of radius 2 m is placed 400 m beyond the end of the decay tunnel, and aligned with the beam axis.

- (4.1) Find a relation between the laboratory energy of a neutrino and its angle relative to the axis, for neutrinos from pion and from kaon decays.
- (4.2) What are the maximum and minimum energies of the neutrinos produced in the two cases?
- (4.3) Above what neutrino energy do all neutrinos from kaon decay traverse the detector?
- (4.4) If 10^{10} pions per burst enter the decay tunnel, how many neutrinos from pion decay traverse the detector?
- (4.5) If the detector has mass 10^5 kg, how many neutrinos from pion decay interact in it per burst, if the cross-section per nucleon at energy E is $\sigma_{\nu N} \approx 0.6(E/\text{GeV}) \times 10^{-38} \text{ cm}^2$?

$$[m_\pi \simeq 139 \text{ MeV}, m_K \simeq 494 \text{ MeV}, m_\mu \simeq 106 \text{ MeV}]$$

6. A way to detect solar neutrinos is to use the reaction



Suppose to have a detector containing approximately 4×10^5 liters of tetrachloroethylene (C_2Cl_4). Estimate how many atoms of ${}^{37}\text{Ar}$ per day would be produced, making the following assumptions:

- Solar constant = $2 \text{ cal cm}^{-2} \text{ min}^{-1}$
- 10% of thermonuclear energy of the Sun appears in neutrinos, of mean energy 1 MeV
- 1% of all neutrinos are energetic enough to induce the above reaction
- The cross section per ${}^{37}\text{Cl}$ nucleus is 10^{-45} cm^2
- The ${}^{37}\text{Cl}$ isotopic abundance is 25%
- The density of C_2Cl_4 is 1.5 g ml^{-1}

Do you expect any difference between the day and night time rates?

[Suggested reading: Davis et al., *Phys.Rev.Lett.* **20**, 1205 (1968)]