ACCELERATOR PHYSICS

Brief history w/ some sketchy design features

(A) Electrostatic acceleration

Van de Graaf - high voltage generated by insulating belt charging conductor

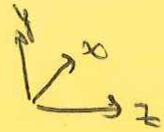
Tandem " " " - H^- accelerated to high voltage, stripped, protons accelerated back to ground (x2)

Cockcroft-Walton - high voltage obtained w/ diode rectifiers plus storage capacitors

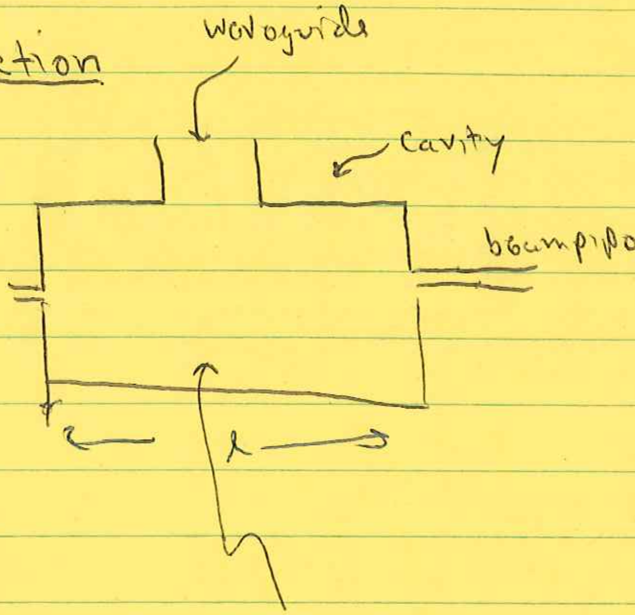
Limit (ultimately) ~ 10 MeV.

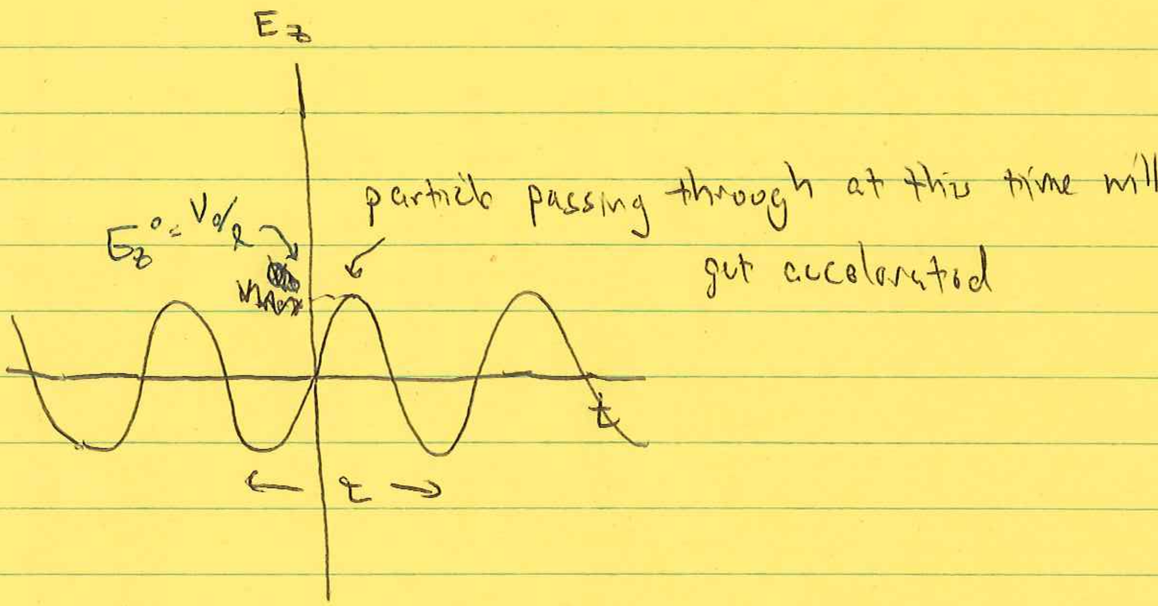
But first to split nucleus ⇒ 1932 (same year Chadwick discovered neutron, although he used an α source).

1951 Nobel prize (Cockroft + Walton)



RF Acceleration





W/ ~~greater than~~ many such cavities, you achieve

$$E_{particle} >> q_{particle} V_0$$

and V_0 is no longer the limit.

(time varying fields $\Rightarrow$ non-conservative potential!)

Linear Accelerators

Put a bunch of RF cavities in a line.

1924 ⇒ Proposed by Ising (Sweden)

1928 ⇒ Widröe, two gaps, proof of principle

1931 ⇒ E.O. Lawrence 1.2 MeV Hg ions ⇒ first physics

But, energy densities at time (before WWII and development of radar) in RF too small to give high energies (recall electrostatic ⇒ many MeV)

How to get around this?

Circular Accelerators

Can use same RF gap many times

E.O. Lawrence 1930 - proposed CYCLOTRON.

Recall frequency of circular motion in uniform transverse magnetic field B

$$\omega_c = \frac{eB}{\gamma m_0} \equiv \text{cyclotron frequency}$$

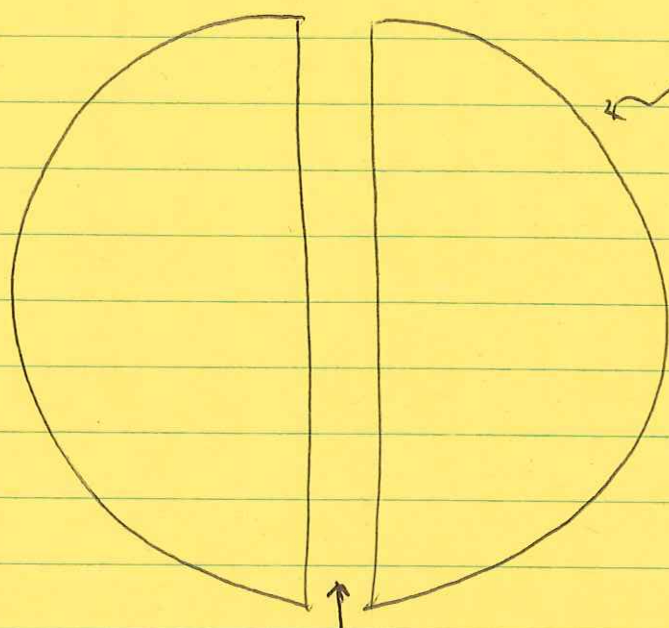
momentum-indep for $\gamma \Rightarrow 1$!

with a radius

$$\rho = \frac{pc}{eB}$$

which expands as p increases (for fixed B - more on this later).

Take a big, round permanent magnet & split it in two



RF frequency exactly ω_c ⇒ continuous acceleration, as long has

a) magnet big enough (magnetic field must decrease slightly w/ radius)

b) particles don't wander off orbit (subtle game of introducing gradients in magnetic fields ⇒ "weak focussing" ⇒ beginning of true science of accel. physics!!)

c) particles do not become relativistic (even slightly). APY

SHOW LIVINGSTON PLOT!

(b) is really the heart of accelerator physics: ~~how~~ whatever the method of acceleration, how does one maintain, or even condense, the ~~volume~~ spatial (actually, phase-spatial) volume occupied by the beam??

1932 ⇒ 1.2 MeV protons

1939 ⇒ 30 MeV protons, and Nobel Prize

and then

McMillan, Veksler (1945) ⇒ modulate RF to track relativistic effects, while maintaining orbital stability

1957 ⇒ 720 MeV SYNCHROCYCLOTRON (Berkeley)

Problem: Spatial control of beam (stability) as it moves out through magnet apertures, particularly transverse components

How to break 1 GeV barrier??

uniform field does not contain phase space!

±6×10^{-5}

±5cm

40μ

20μ

5×10^{11}

400

.01

10^{33}cm^{-2}sec^{-1}

20GJ

200MW

3GW?

: synchrotron damping.
[, and PS were hard for me
iave been dropped for the

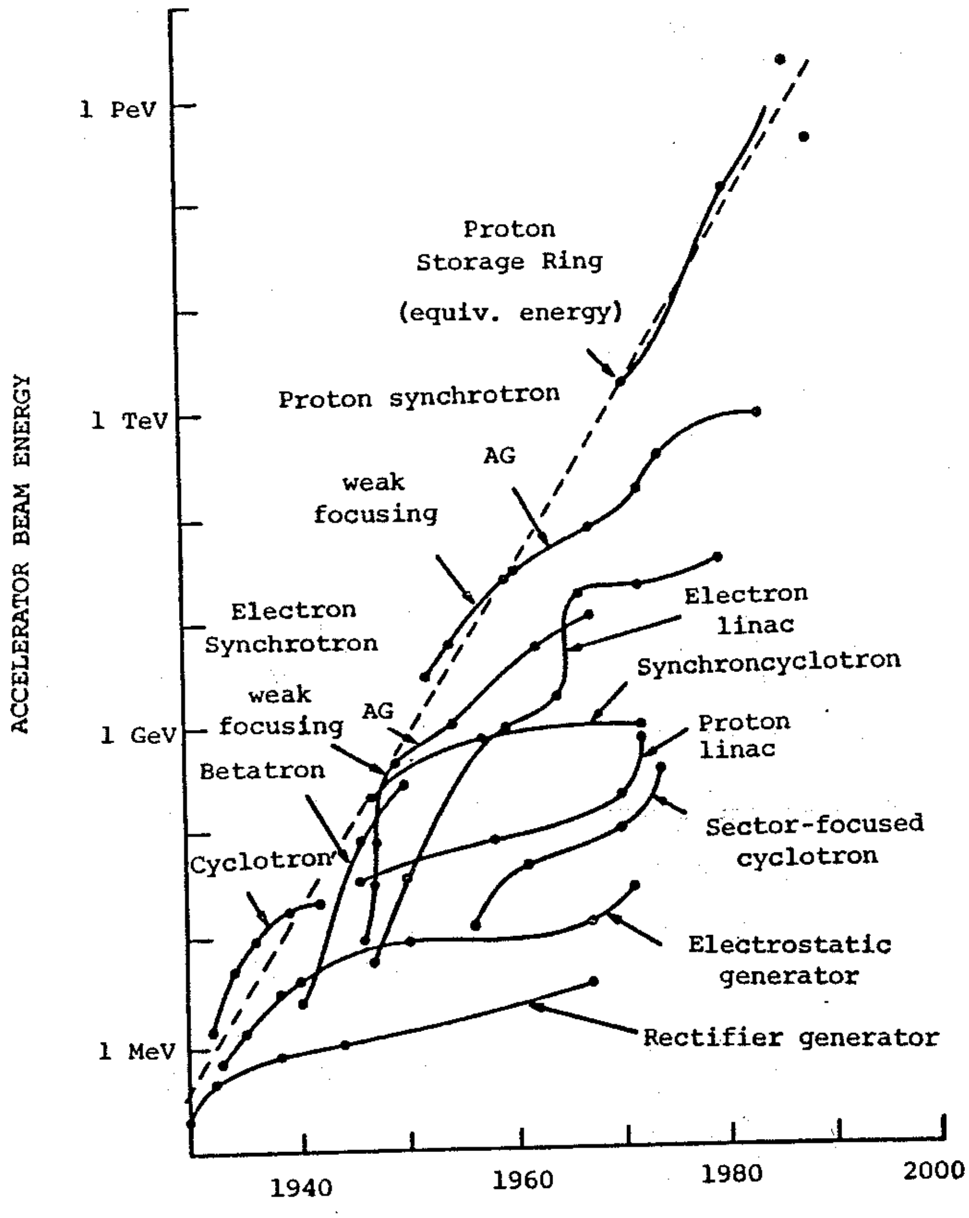


Figure 1

~~THE SYNCHROTON~~

~~(A) Weak focussing ~10 GeV~~ ~~$$$~~
~~(B) Strong focussing (alternating gradient) Many ~100 TeV (pipetron)~~

~~1~~ Basic idea: (also McMillan + Veksler, 1945)

Ramp field up so that particle orbit remains constant ; synchronize RF (in this case, ~~s~~ increase frequency of oscillation) to retain appropriate phase relation. Keeps beam on same, well controlled path. Allows focussing elements.

Must contain:

(A) Longitudinal phase space

Phase stability principle

(B) Transverse phase space

1) Weak focussing → subtle shaping of dipole fields to contain phase space ~10 GeV

2) Strong focussing (alternating gradient) ~100 TeV (pipetron)

~~(Courant, Livingston, Snyder) 1952~~ Christofilos

#2 is the domain of a large part of modern accelerator physics.

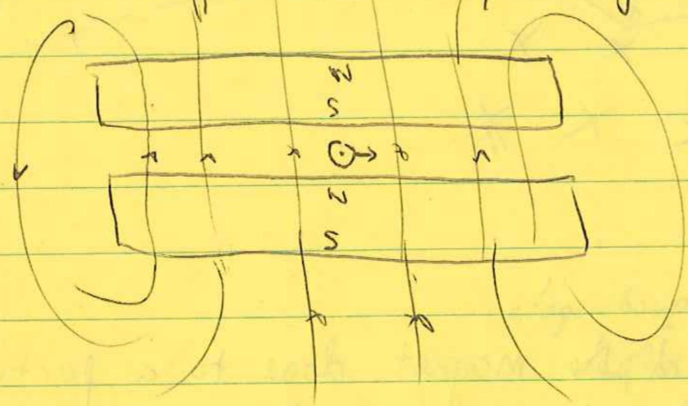
Synchotron keeps beam on ~~single~~ path @ fixed radius ⇒ development of focussing mag field gradients; AG

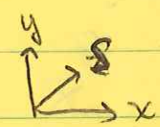
Storage rings
Linear Accl.
Livingston Plot

Let's get a bit ahead of ourselves for a moment and discuss (B).

DIPOLE FIELDS

Particles maintain circular orbit in uniform "guide field" provided by magnetic dipole





+ Particle into page experiences uniform force in +x direction which induces uniform circular motion.

$$B_y = B_y^0 \quad \text{(dipole)}$$

$\text{MeV}/c \rightarrow p = 300\, \rho B q$ ← electron charges

$p = \rho_{orbit} B q$

independent of (x, y, z). m T electron charge

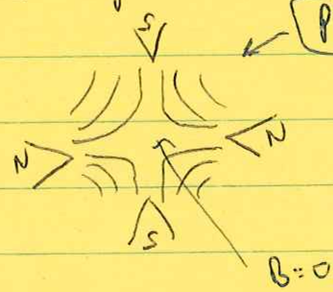
300 MeV = 1 T-m

e in units of electron charge

QUADRUPOLE FIELDS

Next lowest multipole introduces linear dependence to magnetic field, e.g.,

PURE QUAD



$$B_y = B_y^0 + G(x - x_0)$$

dipole term

quadrupole term,

APT

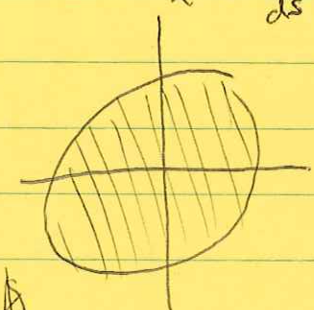
(*) Show this! (but perhaps trivial)? Perhaps: just how then calculate wavelength + frequency of oscillations given the gradient.

where x_0 is the x location of the nominal orbit.
(usually pick coord. system so that $x_0 = 0$!)

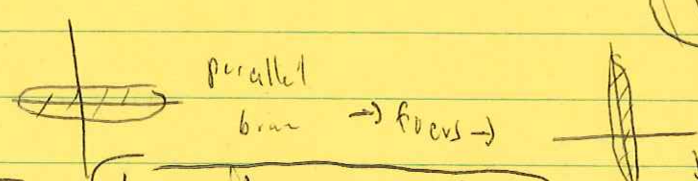
This provides a linear restoring force, leading to sinusoidal oscillations about the nominal orbit

$\Rightarrow$ transverse stability, i.e., particles off orbit either by displacement, or angle, will execute stable oscillations about the nominal orbit.

NOTE that the ensemble of particles, at any point in the synchrotron, occupies some volume in phase space. As you travel around the ring, this volume changes shape as $x' \rightleftarrows x$, but remains stable.



e.g.



BASIC PROBLEM: Liouville's Thm: volume remains same

Here, we have only considered one (horizontal) of two possible ~~degrees of~~ transverse degrees of freedom. In fact, a simple application of Maxwell's equations for static fields shows that

restoring (focussing) force in x

$\Longleftrightarrow$

defocussing force in y of same strength and vice-versa.

(*) Show this (use $\nabla \times B = 0$)

How to deal with this?

WEAK FOCUSSING

Oliphant, Gooden, Hyde, Proc Royal Soc, 59, 666 (1947)

Showed that a <u>weak</u> defocussing gradient in x (radial direction) is overcome by centrifugal forces, so, horizontal stability is maintained a little into the normally defocussing gradient region.

⇒ Taper of radial field can be used to generate weak focussing ~~fields in both~~ forces in both x and y.

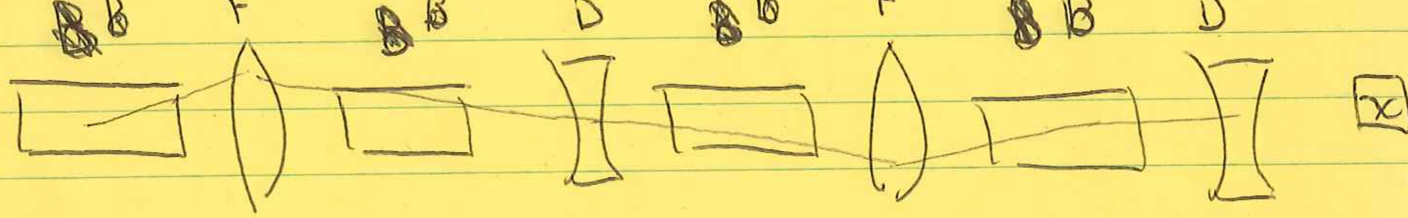
However, the operating region for these gradients is ~~very~~ close to $\emptyset$ and very narrow ⇒ machining tolerances yield limit on ~~effective~~ how far you can extend method.

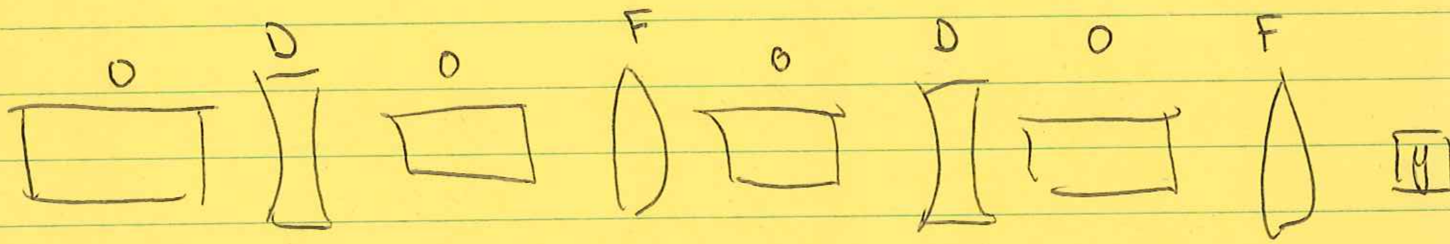
~10 GeV

STRONG FOCUSSING

Christofilos (1950; unpublished)
Courant, Livingston, Snyder, Phys Rev 88, 1190-6 (1952)

Introduce single-function quadrupole magnets which alternately focus x/defocus y and





defocus x/focus y. This can be lead to transverse stability if, roughly speaking, particles are on axis (focused in position space) at the point at which they pass through the defocussing optic, since, e.g.,

$$\cancel{B_x} \quad \cancel{B_{x,y} = -Gx} \quad B_y = Gx$$

is very small for ~~x ≈ 0~~ an ~~on-orbit partic~~ a particle w/ $x \approx \emptyset$.

The analysis of such lattices for stability conditions will be ~~a one major~~ focus of these lectures.
a major

(NOTE) that in large synchrotrons, the bend per dipole is small ($< 1°$), and so this can be ignored at first order, and re-introduced later as a perturbation in the x-coordinate lattice. Thus, lattices we consider will be combinations of uninstrumented drift spaces interrupted by ~~dipoles~~, quadrupoles.

$z \leftrightarrow s$ ← bends slowly w/ nominal orbit.

are interchangeable.

STORAGE RINGS

Direct particle beam of energy E onto fixed target of mass M $(E \gg m)$

$$E_{cms} = \sqrt{2ME}$$ $\boxed{\circledast \text{ Show this}}$

Collide two beams of equal energy E

$$E_{cms} = 2E \gg \sqrt{2mE}$$

⇒ colliding beams "cheap" way to get big gain in cms energy.

But how to get interaction rates approaching that of beams impinging on fixed targets w/ material density?

⇒ Colliders. Counter-circulating beams pass through one another ~10^5 times/second.

Some History:

1961	ADA (Frascati)	2×250 MeV	e^+e^-
	Princeton-Stanford	2×500 MeV	e^+e^-
1972	SPEAR (SLAC)	2×4000 MeV	e^+e^-
	DORIS (DESY)	" "	e^+e^-

major physics discoveries

⋮

~1985	TEVATRON	FNAL	2×800 GeV	$p\bar{p}$
1996	LEP-II	CERN	2×90 GeV	e^+e^-
~1992	HERA	DESY	26GeV×820GeV	ep
200?	LHC	CERN	2×7000 GeV	pp

What limits scale of strong focussing synchrotrons?

(1) Magnetic field technology

$$B^{\text{dipole}}_{\text{max}} \sim 10\,T \quad (\text{LHC})$$

Recall for particle w/ unit charge $p = \rho B$, with 300 MeV = 1 T-m. Thus, a 10 TeV particle curls w/ a radius of about 3 km. Since practically only ~½ ring can be instrumented w/ dipoles,

$$\text{circumference} \sim (2)(2\pi)(3\,\text{km}) \doteq 40\,\text{km}.$$

⇒ Limited by \$\$\$ to ~10 TeV (100 TeV?... "pipetron")

(2) Synchrotron radiation

For fixed orbital radius ρ,

$$\text{Energy loss / revolution} = \frac{4\pi e^2}{3\rho}\gamma^4 \doteq \frac{4\pi e^2}{3\rho}\left(\frac{E}{m}\right)^4$$

goes up as $\left(\frac{E}{m}\right)^4$!! (see, e.g. Jackson)

~~Fractional~~ Energy loss / turn for coasting beam

		Energy loss / turn	Power Loss
LEP II	90 GeV e^-	1.4 GeV	3 MW ⇒ 100 MW of RF power! (efficiency is few %)
LHC	7 TeV p^+	4.4 keV	4 kW
~~Tevatron~~			

$\Rightarrow$ synchrotron radiation (due to acceleration of particle necessary to keep it in UC motion) is an issue for electrons!

(3) Beam-beam interaction, for storage rings. Will discuss later. (Actually, this limits luminosity).

It's clear that building a linear collider avoids ~~(1) and (2), and, as well as, (3) also~~! all of these!, What limits this?

(1) Accelerating gradient (Current best: SLAC LINAC, ~ 20 MeV/m. (~NLC $\lesssim$ 100 MeV/m)

(2) Crossing rate. Must re-generate beams for every pulse. ~100 Hz. (really, limited by damping time!))

SLC - first linear collider 46×46 GeV, 120 Hz

Beam spot @ focus ~ 2μm × 0.7μm. Luminosity (conv rate) ~ ⅒ that of LEP I. (storage ring collider @ same energy)

$\boxed{\text{LIVINGSTON PLOT}}$ - DOUBLING OF E_{cm} every 2-3 years P. 63 of Bjorken's article.

phase stability w/o long phase space

longitudinal phase space

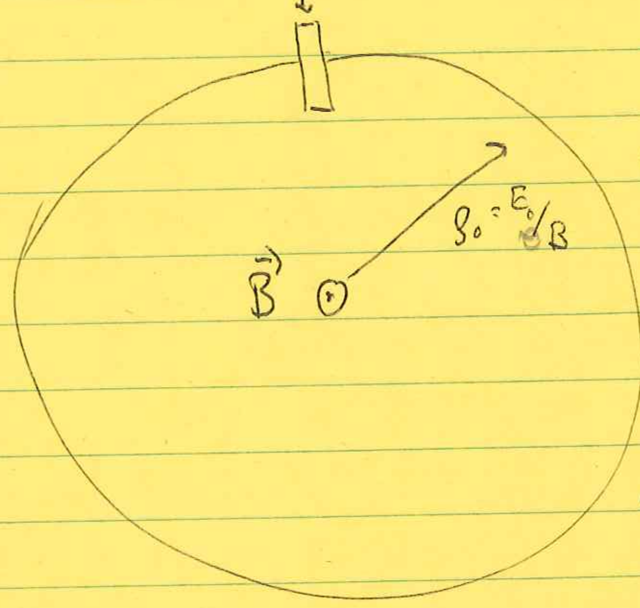
dispersion / two phase stability

MORE DETAILED LOOK AT SYNCHROTRONS + STORAGE RINGS

(I) Longitudinal phase space ⇒ phase stability + acceleration for coasting beam

For a first pass, consider a relativistic on-energy, on-orbit particle executing a circular path around an accelerator w/ uniform magnetic field B, and a single RF cavity.

$V(t)$ =



For this particle, the orbit is of radius, in appropriate units

$$\rho_0 = \frac{E_0}{eB}$$

0.3 GeV = 1 T-m

and it executes orbit w/ angular frequency (for $\beta = 1$)

$$\omega_0 = \frac{2\pi c}{\rho_0}$$

The 'RF voltage' is just the 1/e times the energy kick the particle gets in going through the RF cavity, and depends on the phase of the ~~particle~~ electric field in the cavity at the time in which it's traversed by the particle:

NOTE ON "TRANSITION" (won't have time to discuss): For $\gamma = 1$, non-rel particle,

$\rho = \frac{p}{eB} = \frac{mv}{eB}$, and $\tau = \frac{2\pi\rho}{v} = \frac{2\pi m v}{v\, eB} = \frac{2\pi m}{eB}$ $\Rightarrow$ τ indep of v $\Rightarrow$ no phase stability!

But if $\gamma > 1$, then ρ does not increase so much, and so higher energy $\Rightarrow$ faster v $\Rightarrow$ phase advance, rather than retardation for $\gamma \to \infty$. So, at some γ depending on ρ, you get transition. Ignore $\gamma \neq \infty$ in these lectures!

(*) Homework problem??

$$V(t) = V_0 \sin(h\omega_0 t + \phi_0)$$

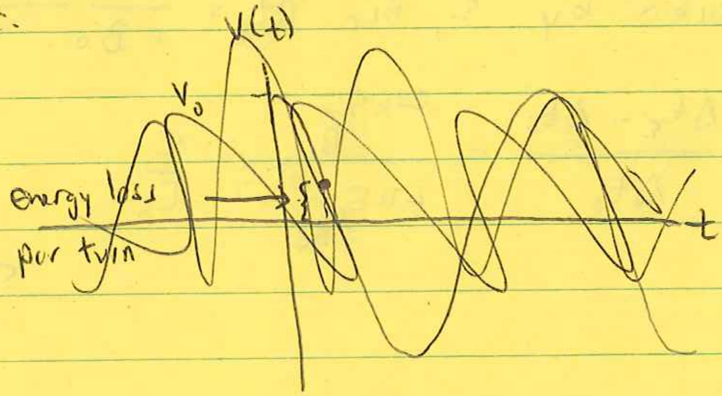
~~Clearly~~

The RF frequency $h\omega_0$ must be precisely an integer multiple h of the frequency ω_0 of the motion of the particle around the accelerator. (or more precisely, the nominal orbit; leave some ability to tune h, since $h\omega_0$ will define nominal orbit ω_0!)

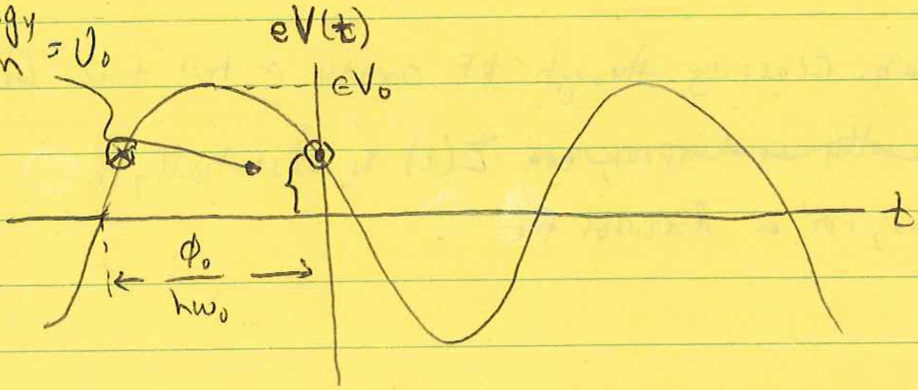
$h \equiv$ "harmonic number"

Tevatron: $h \sim 1{,}000$

We define ϕ_0 so that for our on-energy, on-orbit particle, $t = 0$, modulo 2π ($\omega_0 \equiv \tau$) and $eV_0 \sin(\phi_0) = U_0$ is just the energy loss incurred by the particle in a single orbit.



Note: Solution for ϕ_0 could also be at ⊗ rather than ⊙. This won't work - we'll see why soon...



LEAVE UP!

Note that the particle gets precisely the kick every time since h is precisely integer!

AP16

Clearly, having an entirely on-energy beam is unrealistic. For example, particles in the front of the beam will get accelerated to higher energy, and those in back will be decelerated to lower energy. What happens then?

Let ε be absolute energy difference of some particle relative to nominal energy E_0

$$\varepsilon = E - E_0$$

and can be positive or negative. Similarly, let τ = phase lag be the difference in arrival time at the RF cavity of the particle relative to the on-energy stability point t_0, which we have defined to be at $t_0 = 0$.

Now, the relativistic off-energy particle, with say $\varepsilon > 0$, travelling at c, will have a slightly longer orbit in ($\rho \propto p = E/c$) and so will slip slightly in phase. (develop $\tau > 0$!), In fact

$$\frac{d\tau}{dt} = \frac{\varepsilon}{E_0}$$

is the phase slip per unit time. But also,

$= \frac{\omega_0}{2\pi}\{eV(\tau) - U_0\}$ = extra energy per kick

Since by def'n $\tau = 0$ for an energy stability pt, i.e., $eV_0 \sin(\phi_0)$ is just energy lost in 1 revolution.

$$\frac{d\varepsilon}{dt} = \underbrace{\frac{\omega_0}{2\pi}}_{\text{number of kicks per second}}\left\{\underbrace{eV(\tau)}_{\text{size of kick, but is just } U_0 = \text{energy lost per turn for } \tau = 0!} - \underbrace{eV(\phi)}_{U_0 \text{ energy lost back to synch rad in going around ring to get next kick}}\right\} = \frac{\omega_0 eV_0}{2\pi}\left\{\sin(h\omega_0\tau + \phi_0) - \sin(\phi_0)\right\}$$

$\boxed{\text{Look at } \tau = 0 \text{ limit}}$

Thus,

$$\frac{d^2\tau}{dt^2} = \frac{1}{E_0}\frac{d\varepsilon}{dt} = \frac{\omega_0 eV_0}{2\pi E_0}\left\{\sin(h\omega_0\tau + \phi_0) - \sin(\phi_0)\right\}$$

To build intuition about this equation, consider limit in which energy loss around ring $\rightarrow \phi$ ~~proton machine~~ (all current proton machines have this quality). Then,

$\sin(\phi_0) = 0 \Rightarrow \phi_0 = 0, \pi, 2\pi, \ldots$ and

$$\frac{d^2\tau}{dt^2} = \pm\frac{\omega_0 eV_0}{2\pi E_0}\sin(h\omega_0\tau)$$

where the $\pm$ corresponds to the choice of ϕ_0

$$\begin{cases} + \Rightarrow \phi_0 = 0, 2\pi, 4\pi \\ - \Rightarrow \phi_0 = \pi, 3\pi, 5\pi \end{cases}$$

⊛ Show this
Include dispersion!

Thus, the choice $\phi_0 = \pi$ yields a "restoring force" proportional to $\sin(h\omega_0 \tau)$ {a "restoring force" is the phase τ relative to ϕ_0?)

$\Rightarrow \tau$ has the equation of motion of a pendulum

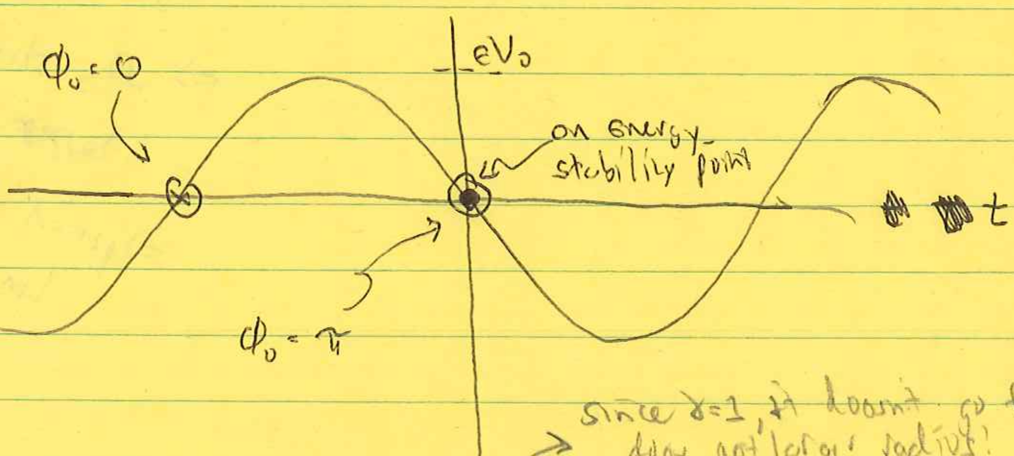
and will oscillate back and forth ~~with~~ about ~~[illegible]~~ ϕ with ~~period~~ frequency

$$\Omega_s = \omega_0 \sqrt{\frac{h e V_0}{2\pi E_0}}$$, known as the synchrotron frequency

(Typically, $\mathcal{O}(1\ s^{-1})$!) ← ~$10s^{-1}$ w/out dispersion
~$2s^{-1}$ w/ dispersion

In any case, we see that the longitudinal phase space is stable as long as $\varepsilon \lesssim eV_0$ (roughly).

What is really going on here? For $\phi_0 = \pi$



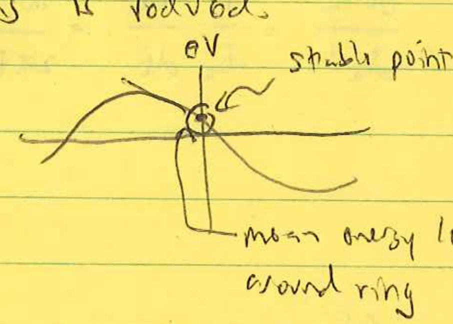
since $\gamma \approx 1$, it doesn't go faster, but it does get larger radius!

A particle that ~~gets~~ drifts towards higher energy (say, in the front of the bunch) arrives w/ a later phase, and gets decelerated by the RF, forcing it back on energy. Note that for the choice $\phi_0 = 0$ this same particle would get further <u>accelerated</u>, driving it further off energy $\Rightarrow$ INSTABILITY

Now, relaxing the limit that energy loss around ring $= \phi$, the same qualitative argument holds, except

(1) ~~...~~ Expression for Ω_s ~~changes by a factor~~ gets slightly smaller, due to change in derivative of sine for $\phi \neq \pi$

(2) Range of stable energy excursions is reduced.



But, basically it's the same thing...

"PHASE STABILITY!!"

(presumably McMillan + Veksler, 1945)

Next: a) ACCELERATION
b) DISPERSION

Note: RF timing must be accurate enough ~~so that an energy beam slips or gains phase by << 1 radian~~ in order to capture beam, so the nominal orbit defined by $h\omega$ is close enough to design orbit!

ACCELERATION

uniform ramp

What happens if some operator begins to turn up current in magnets for a formerly coasting beam? This will change the energy E_0 which is associated w/ the reference orbit (recall that the reference orbit was that orbit which always returns w/ the same phase to the RF cavity, ie, for particles moving at c, the radius of the orbit which takes exactly $2\pi h/\omega_0$ seconds to travel around (h = harmonic number).

So, our previous ~~assump~~ expression

$$\frac{d^2\tau}{dt^2} = \frac{d}{dt}\left(\frac{\varepsilon}{E_0}\right) = \frac{1}{E_0}\frac{d\varepsilon}{dt}$$

must be replaced w/

extra piece!

$$\frac{d}{dt}\left(\frac{\varepsilon}{E_0}\right) = \frac{1}{E_0}\frac{d\overset{E-E_0}{\varepsilon}}{dt} - \overset{0}{\cancel{\frac{\varepsilon}{E_0^2}\left(\frac{dE_0}{dt}\right)}} = \frac{1}{E_0}\left[\frac{dE}{dt} - \frac{dE_0}{dt}\right]$$

so apparently no change, except that ~~now~~ since $\varepsilon = E - E_0$, dE_0/dt comes in; ie, the energy of the orbit w/ exactly integer harmonic no!

$$\frac{d\varepsilon}{dt} = \frac{dE}{dt} - \frac{dE_0}{dt} =$$

$$\frac{d\mathcal{E}}{dt} \rightarrow \overbrace{\frac{\omega_0}{2\pi}\left\{ eV(\tau) - \overset{U_0}{\cancel{eV(\phi_0)}} \right\}}^{dE/dt} - \overset{\frac{dE_0}{dt}}{\frac{dE_0}{dt}}$$

$$= \frac{\omega_0}{2\pi}\left\{ eV(\tau) - \left[\overset{U_0}{\cancel{eV(\phi_0)}} + \frac{2\pi}{\omega_0}\frac{dE_0}{dt} \right] \right\}$$

So, simply ~~redefine~~ while before we defined $\tau = 0$ so that ~~$V_0 \sin\phi_0$~~ $eV(0) = eV_0 \sin\phi_0 \equiv U_0$, we now define

$$eV(0) = eV_0 \sin\phi_0' = U_0 + \frac{2\pi}{\omega_0}\frac{dE_0}{dt}$$

and again

this defines which orbit is the stable point! (the value of ω_0 such that $h\omega_0$ is exactly integer!)

$$\frac{d^2\tau}{dt^2} = \frac{1}{E_0}\frac{d\mathcal{E}}{dt} = \frac{\omega_0 eV_0}{2\pi E_0}\left\{ \sin(h\omega_0\tau + \phi_0') - \sin(\phi_0') \right\}$$

which again provides a restoring force as long as ϕ_0' is chosen so that

$$\left.\frac{d}{d\tau}\sin(h\omega_0\tau + \phi_0')\right|_{\tau=0} < 0$$

{note: all I mean here is that you have to pick the correct of the 2 soln's for ϕ

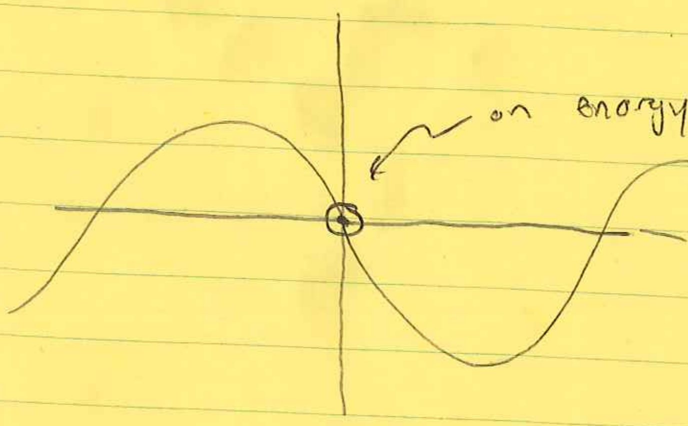
~~(e.g., $\phi_0 = \pi$ for $U_0 + \frac{2\pi}{\omega_0}\frac{dE_0}{dt} = 0$)~~

i.e., $\pi/2 < \phi_0' < 3\pi/2$.

Note that this is still a condition for phase stability about the nominal orbit (e.g., at a fixed radius (the radius set by the requiring that $h\omega_0$ is exactly an int. mult. of ω_0 - it no, slightly new ω_0 is defined!)

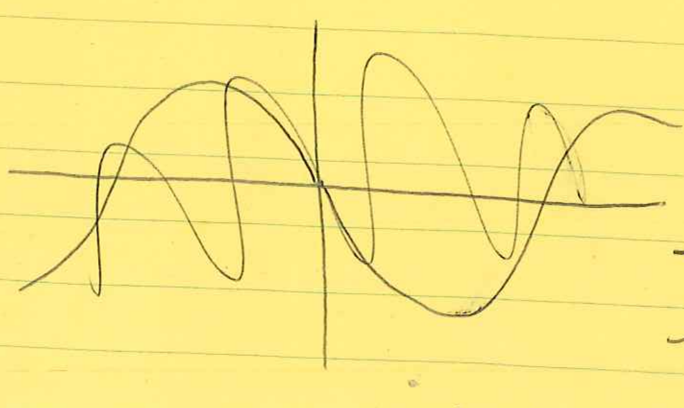
$\Rightarrow$ the energy of the beam is increasing at the rate dE_0/dt at which the reference orbit energy increases.

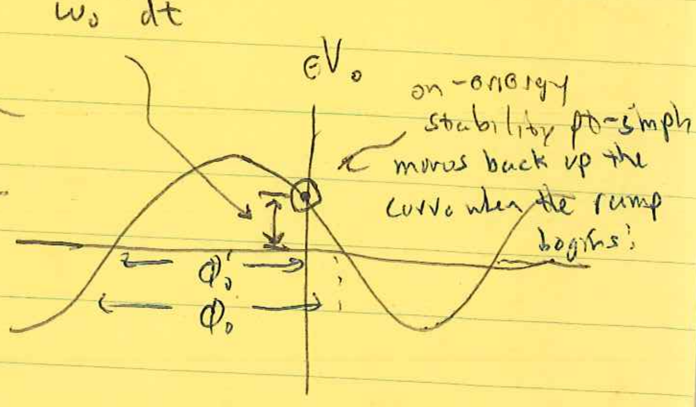
Again, what is going on here? For simplicity, assume $U_0 = \phi$ (e.g. proton ring)



no accel

acel





~~In both cases~~

We have shown that on-energy stability point simply moves back so that beam gets extra kick needed for acceleration!

$\boxed{AP23}$

In both cases, particle at, e.g., slightly higher energy arrives slightly late @ the RF cavity + receives a smaller kick. However, in the accelerating case, the phase of the stable point moves back so that extra energy is imparted by the RF to supply the necessary rate of energy increase (dE_0/dt) to keep the particle on orbit.

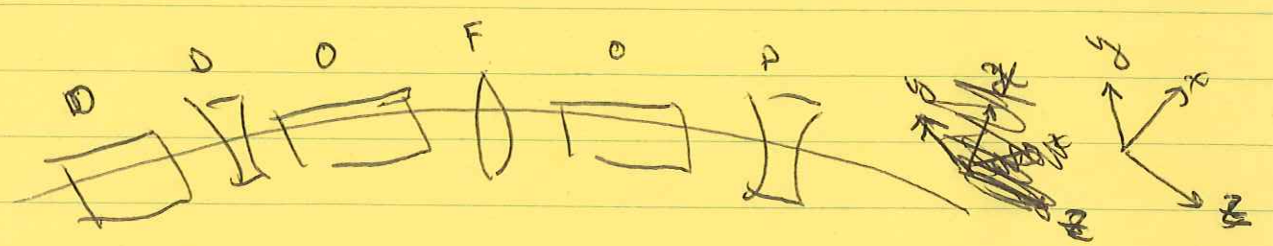
TO ACCELERATE PARTICLES IN A SYCHROTRON, YOU SIMPLY RAMP THE B FIELD - THE PRINCIPLE OF PHASE STABILITY WILL KEEP THE PARTICLES NATURALLY ON ORBIT!!

As long the ramp is slow enough, (or RF is powerful enough) the principle of phase stability works out the details for you!

DISPERSION

There is a small conceptual, but large quantitative, correction to the discussion of longitudinal phase stability due to the coupling of the transverse strong focussing into the longitudinal phase space. This is easily handled via the introduction of dispersion.

Recall that transverse orbit fluctuations (particles displaced in x, y or p_x, p_y from nominal orbit, but w/ nominal energy E_0) are controlled by



a "lattice" of alternating focussing + defocussing magnets.

However, ~~a particle off energy will also~~ as we've seen, a particle which creeps off energy will also creep off orbit; now, w/ quadrupole magnets, it will sample ~~a different field~~ off-axis quadrupole fields, and thus will not have a simple relation between radius and energy, i.e.,

$$\rho = \frac{P}{Be}$$

no longer holds for off-energy particles.

~~Note: In reality, $\eta(s)$ doesn't depend upon initial~~ ρ by convention! See page on dispersion, Bjorken notes on 10/25

Note: Is $\eta(s)$ truly a ring property, or does it depend on each individual particle's initial conditions? I think the former, but I'm not yet sure why.

If we define our "synchrotron coordinates" (x, y, z) so that ~~$x = \rho - \rho_0$~~ and $\hat{x}$ always points in the direction of increasing radius, and $x = \rho - \rho_0$, then for a ring with no focussing elements, i.e., one for which $\rho = p/Be$ holds,

For no strong focussing,

$$x = \Delta\rho = \frac{\Delta p}{eB} = \rho_0 \frac{\Delta p}{p_0}$$

x = direction away from center of bend in bend plane (+ ⇒ further from center).

for a particle $\frac{\Delta p}{p_0}$ ~~off-energy~~. off-energy

$$\frac{\Delta p}{p_0} = \frac{E - E_0}{E_0} = \frac{\varepsilon}{E_0}$$

Thus, we introduce the dispersion function $\eta(s)$ via

$$x = \eta(s) \frac{\Delta p}{p_0}$$

and note that for a ring w/ no focussing, $\eta(s) = \rho_0$ independent of s.

It turns out that $\eta(s)$ is useful for any accelerator lattice, whether a circular storage ring, a linac, or anything in between. At a point of large dispersion, for instance, you can monitor the beams energy spread, truncate off-energy particles, etc. At SLAC, "large dispersion" ~~is~~ means a point s in the lattice for which $\eta(s) \sim 100$ mm, i.e., a .1% off-energy particle is off-orbit by $(10^{-3})(100\text{mm}) = 100\mu\text{m}$.

Back to synchrotrons, we can define the "momentum compaction factor"

$$\eta = \frac{\langle \eta(s) \rangle}{\rho_0} = 1 \text{ for no strong focussing.}$$

Thus, η is the extent to which the strong focussing ~~tends to~~ mitigates the tendency for off-energy particles to execute orbits w/ larger or smaller radii. $\eta \approx 10^{-2-4}$ are typical for modern storage rings.

Note that this significantly modifies the relation between ΔE and the phase-~~advance~~ slip $\Delta\tau$, but this modification is easily incorporated, e.g., our longitudinal ~~restoring force equation restoring force equation of motion becomes~~

$$\cancel{\frac{d^2\tau}{dt^2} = \pm \frac{\omega_0 \varepsilon \theta_0}{}} \quad \frac{d\tau}{dt} = \frac{\varepsilon}{E_0} \rightarrow \frac{\eta\varepsilon}{E_0},$$

and our longitudinal restoring-force equation of motion becomes (for $V_0 = \emptyset$ eq)

~~Ω_s~~ synch freq $\Omega_s \rightarrow \sqrt{\eta}\,\Omega_s \rightarrow \sim 1\,\text{sec}$.

$$\frac{d^2\tau}{dt^2} = -\frac{\eta\,\omega_0 e V_0}{2\pi E_0} \sin(h\omega_0\tau)$$ ← frequency decreases by ~ order magnitude.

pg. 6 for see $\boxed{\text{AP5}}$

We'll come back to dispersion, briefly after we learn a bit more about transverse stability. NOPE - NO TIME. TREATMENT NOT DIFFICULT. SEE

$\boxed{\text{AP27}}$ BJORKEN, 10/25 lecture.