# PHYSICS 221A – HOMEWORK SET 4

Due in class Monday, November 29, 2004

Background: B. J. Bjorken, A Thousand TeV in the Center of Mass: Introduction to High Energy Storage Rings, FERMILAB–CONF-82/55-THY.

## Problem 1

Calculate the ratio of cms energies for a 100 GeV positron beam colliding with a stationary electron target *vs.* an electron in a counter-circulating 100 GeV electron beam.

## Problem 2

Consider the magnetic field of a quadrupole magnet which is focussing in $\hat{x}$ for positive particles travelling in the $+\hat{z}$ direction, i.e.

$$B_y = Gx.$$

Show that, for this same particle, the quadrupole is defocussing in $\hat{y}$.

## Problem 3

Consider two cylindrical beams of radius $1\mu m$ and uniform area density which pass through each other head-on at a rate of 120 Hz. Assuming each beam contains $5 \times 10^{10}$ particles, calculate the luminosity of the collision in units of $\text{cm}^{-2}/\text{s}$, i.e., the numbers of expected collisions per second for a process with cross section $\sigma = 1\ \text{cm}^2$. If the accelerator is an electron-positron collider running at the peak of the $Z^0$ cross section, calculate the corresponding rate of $Z^0$ production, given a peak cross section of 45 *nb*. What would you get for the luminosity if one (and only one) of the beams was focussed to a cylinder with radius $0.5\mu m$ at the collision point?

## Problem 4

Consider a ring composed of an uninterupted thin-lens FODO lattice, i.e., an exact interger number of cells consisting of the elements focus-drift-defocus-drift, with the focussing and defocussing quads having the same focal length $f$. By symmetry, the $\beta$-function of the lattice must have the periodicity of the lattice, and so transport through exactly one cell of the lattice will leave you with the same value of $\beta$ that you started with, but with an arbitrary phase advance $\mu$. Given the thin-lens transport matrices derived in class, find the value of this phase advance as a function of the drift length $l$ and the quadrupole focal length $f$. Find the resulting all-important condition on $l$ and $f$ for transverse stability.

**Problem 5**

Again exploiting the symmetry of the ring to restrict your calculation to the transport through a single cell, find the ratio of the maximum and minimum of the $\beta$ function in the lattice as a function of $l$ and $f$. (Hint: just solve for $\beta$ at the two points where your intuition tells you it will be largest and smallest.) What is the ratio of the maximum to minimum *beam size* for a phase advance of 90° per cell – fairly typical for high-energy storage rings?

**Problem 6**

Consider a singly charged particle in a high energy beam of momentum $p$ travelling at a small angle $y' = dy/dz$ with respect to the $z$ (beam) axis. Show that the angular kick $dy'$ suffered by this particle after traversing a length dz in a magnetic field with x component $B_x$ is

$$dy' = \frac{0.3B_x}{pc}dz,$$

where $B_x$ is in Tesla, $pc$ is in GeV, and $dz$ is in meters. Show that such a particle will undergo sinusoidal motion in the field of a long focussing quadrupole magnet. Find the period of this sinusoidal motion as a function of the quadrupole gradient $G$, where $B_x = Gy$. Armed with this result, derive the transfer matrix $M(z_0 + l, z_0)$ for a 'thick' quadrupole of lenth $l$. Show that this transfer matrix reduces to the appropriate 'thin-lens' form in the limit that $l \to 0$.