The Southern California Physics GRE Bootcamp

Held at CSU Long Beach, August 23-24, 2013

Damien Martin

Big picture

TOTAL SCORE					
Raw Score	Scaled Score	%	Raw Score	Scaled Score	%
67-99	990	97	30-31	690	59
65-66	980	96	29	680	57
64	970	96	28	670	55
63	960	95	27	660	53
62	950	95	26	650	50
61	940	94	24-25	640	48
59-60	930	93	23	630	45
58	920	92	22	620	43
57	910	91	21	610	40
56	900	91	19-20	600	38
54-55	890	90	18	590	35
53	880	89	17	580	32
52	870	88	16	570	29
51	860	86	15	560	27
50	850	85	13-14	550	25
48-49	840	84	12	540	23
47	830	83	11	530	20
46	820	82	10	520	18
45	810	81	9	510	15
44	800	79	7-8	500	13
42-43	790	77	6	490	11
41	780	76	5	480	9
40	770	74	4	470	7
39	760	73	3	460	6
38	750	71	1-2	450	4
36-37	740	69	0	440	3
35	730	67			
34	720	65			
33	710	63			
32	700	61			

*Percentage scoring below the scaled score is based on the performance of 11,322 examinees who took the Physics Test between October 1, 1993, and September 30, 1996.

QUESTION Number	Answer	P+	TOTAL C	I
1	B	73		
2	B	29		
3	B	55		
4	A	34		
5	B	29		
6	B	43		
7	A	22		
8	A	37		
9	A	40		
10	B	47		
11	D	36		
12	C	36		
13	B	37		
14	D	66		
15	E	12		
16	B	20		
17	E	40		
18	C	77		
19	B	17		
20	D	20		
21	C	27		
22	C	26		
23	D	24		
24	D	70		
25	E	38		
26	C	13		
27	D	49		
28	D	40		
29	E	58		
30	C	28		
31	E	65		
32	B	41		
33	C	56		
34	D	31		
35	A	79		
36	D	46		
37	D	53		
38	D	39		
39	E	54		
40	A	21		
41	A	28		
42	B	52		
43	C	17		
44	A	48		
45	E	45		
46	C	36		
47	C	28		
48	C	30		
49	C	22		
50	C	33		

QUESTION Number	Answer	P+	TOTAL C	I
51	B	70		
52	C	15		
53	C	34		
54	B	18		
55	A	30		
56	C	42		
57	C	41		
58	E	22		
59	B	36		
60	B	9		
61	B	26		
62	C	13		
63	A	56		
64	C	26		
65	D	44		
66	E	25		
67	C	28		
68	E	61		
69	A	14		
70	D	14		
71	A	20		
72	A	29		
73	C	34		
74	B	21		
75	D	27		
76	B	52		
77	D	12		
78	E	36		
79	D	22		
80	C	31		
81	B	27		
82	E	15		
83	D	15		
84	D	20		
85	B	15		
86	B	36		
87	A	6		
88	B	57		
89	D	18		
90	**	**		
91	E	25		
92	D	15		
93	D	26		
94	D	28		
95	E	23		
96	A	28		
97	E	11		
98	D	39		
99	B	44		
100	C	51		

(35) 20-35 percentile

(18) <20 percentile

Big tips and tricks

* Multiple passes through the exam

* Dimensional analysis (which answers make sense?)
Other hint -- look at exponentials, sines, cosines, ...

* Expansions, in particular $(1+x)^n = 1 + n x + \ldots\ldots$

* Limiting cases (e.g. make parameters go to 0 or infinity)

* Special cases (e.g. looking at circles)

* Powers of ten estimation

* Know scales of things [wavelength / freq of visible light, binding energies of nuclei, mass ratios of common particles (up to muon, pion), ..., mass of stars, mass of galaxies, ...]

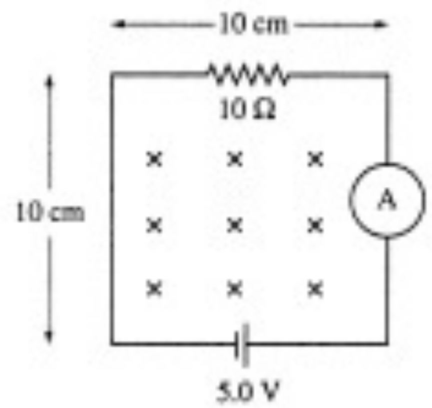
Okay to specialize on scales

Big tips and tricks -- material

* Know your "first year" general physics really well
  - Newtonian mechanics in particular

* Worth going through
  Griffiths: Intro to electromagnetism
  Griffiths: Intro to quantum mechanics
  (Concentrate on harmonic osc, infinite square well, spin systems)
  Schroeder: Thermal physics

* Look at the archive of monthly problems in *The Physics Teacher* (if you have access to a university library)



2. The circuit shown above is in a uniform magnetic field that is into the page and is decreasing in magnitude at the rate of 150 tesla/second. The ammeter reads

(A) 0.15 A
(B) 0.35 A
(C) 0.50 A
(D) 0.65 A
(E) 0.80 A

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GO ON TO THE NEXT PAGE.

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Evaluate whether question is "special" or "first year"

67. A black hole is an object whose gravitational field is so strong that even light cannot escape. To what approximate radius would Earth (mass $= 5.98 \times 10^{24}$ kilograms) have to be compressed in order to become a black hole?

(A) 1 nm
(B) 1 μm
(C) 1 cm
(D) 100 m
(E) 10 km

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6

$$C = 3kN_A\left(\frac{hv}{kT}\right)^2 \frac{e^{hv/kT}}{(e^{hv/kT} - 1)^2}$$

65. Einstein's formula for the molar heat capacity C of solids is given above. At high temperatures, C approaches which of the following?

(A) 0

(B) $3kN_A\left(\frac{hv}{kT}\right)$

(C) $3kN_A hv$

(D) $3kN_A$

(E) $N_A hv$

Dimensions here

$$C = 3kN_A\left(\frac{hv}{kT}\right)^2 \frac{e^{hv/kT}}{(e^{hv/kT}-1)^2}$$

This quantity is dimensionless

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~~(C) $3kN_A hv$~~

(D) $3kN_A$

~~(E) $N_A hv$~~

Call $x = \frac{hv}{kT}$

$$\begin{aligned} C &= 3kN_A x^2 \frac{e^x}{(e^x-1)^2} \\ &= 3kN_a x^2 \left[\frac{1+x+\ldots}{((1+x+\ldots-1)^2}\right] \\ &= 3kN_a x^2 \left[\frac{1}{x^2}+\ldots\right] \\ &= 3kN_a + \ldots \end{aligned}$$

A distant galaxy is observed to have its H-beta line shifted to a wavelength of 480nm from its laboratory value of 434nm. Which is the best approximation to the velocity of the galaxy? (Note: 480/434 ~ 1.1)

a) 0.01c
b) 0.05c
c) 0.1c
d) 0.32c
e) 0.5c

$$\lambda_{\text{obs}} = \lambda_{\text{emit}} \sqrt{\frac{c+v}{c-v}}$$

$$\frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} = \sqrt{\frac{1+(v/c)}{1-(v/c)}} \approx \sqrt{(1+(v/c))^2}$$

$$v \approx \left(\frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} - 1\right) c$$

8. A particle of mass m undergoes harmonic oscillation with period T_0. A force f proportional to the speed v of the particle, $f = -bv$, is introduced. If the particle continues to oscillate, the period with f acting is

(A) larger than T_0
(B) smaller than T_0
(C) independent of b
(D) dependent linearly on b
(E) constantly changing

37

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37

Generalize lessons!

$$\boxed{F = m\ddot{x} = -kx - b\dot{x}} \Rightarrow \boxed{\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0}$$

Can solve exactly -- but wrong approach.

Different competing effects -- which wins?

Appeal to (J)WKB, approximation schemes, etc...

Things to know (they always seem to come up)

1) Elastic collision formula

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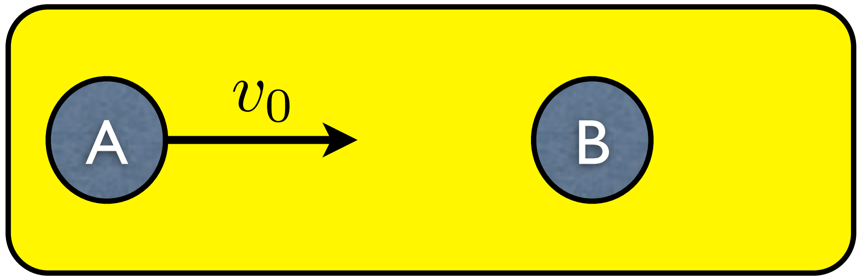
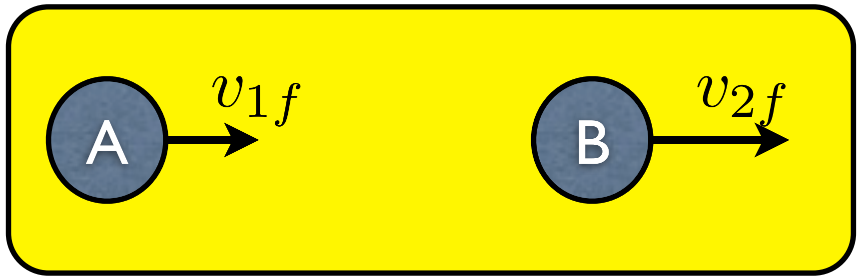


Before collision



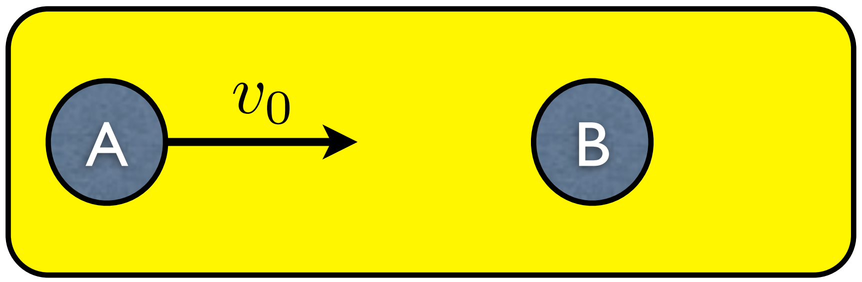
After collision

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_0$$

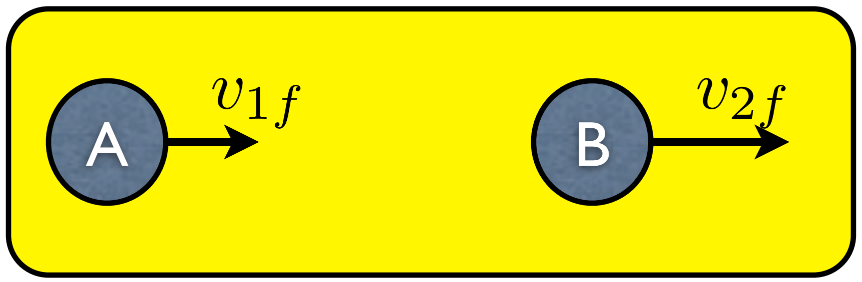
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_0$$

Things to know (they always seem to come up)

1) Elastic collision formula



Before collision



After collision

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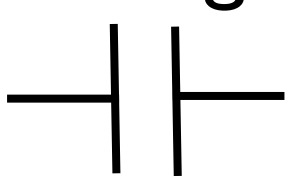
2) The limiting behavior of capacitors and inductors in DC

 acts like (while uncharged)

 (while fully charged)

(e.g. high pass filter question)

3) Virial theorem (and the quick way to get it)

m is reduced mass!

(To get levels for e.g. positronium, same formula but use reduced mass for that system)

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$$F(r) = Ar^{+n} \Rightarrow V = \frac{A}{1+n} r^{1+n} = \frac{F(r)}{1+n} r$$

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4) The Bohr formula (or know how to get it quickly)

$$\boxed{E = -\frac{Z^2(ke^2)^2 m}{2\hbar^2 n^2}}$$

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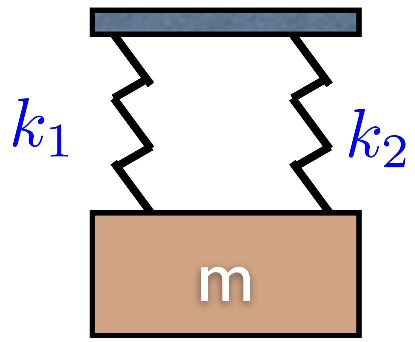
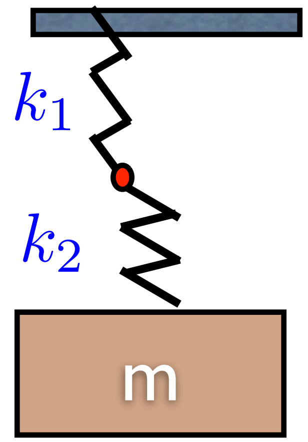
5) Combining masses, springs, capacitors, resistors





Can you find k_{equiv} ?

Frequency of oscillation?

Know reduced mass!

49. The infinite xy-plane is a nonconducting surface, with surface charge density σ, as measured by an observer at rest on the surface. A second observer moves with velocity $v\hat{\mathbf{x}}$ relative to the surface, at height h above it. Which of the following expressions gives the electric field measured by this second observer?

(A) $\dfrac{\sigma}{2\epsilon_0}\,\hat{\mathbf{z}}$

(B) $\dfrac{\sigma}{2\epsilon_0}\sqrt{1 - v^2/c^2}\;\hat{\mathbf{z}}$

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(C) $\dfrac{\sigma}{2\epsilon_0\sqrt{1 - v^2/c^2}}\;\hat{\mathbf{z}}$

(D) $\dfrac{\sigma}{2\epsilon_0}\left(\sqrt{1 - v^2/c^2}\;\hat{\mathbf{z}} + v/c\;\hat{\mathbf{x}}\right)$

(E) $\dfrac{\sigma}{2\epsilon_0}\left(\sqrt{1 - v^2/c^2}\;\hat{\mathbf{z}} - v/c\;\hat{\mathbf{y}}\right)$

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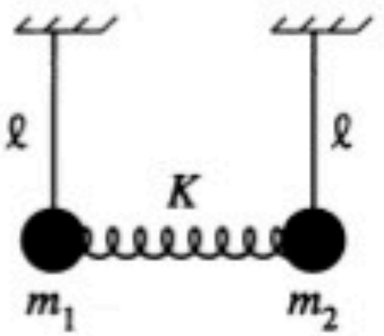
Multiple approaches:

- Know how **E** transforms (from upper div E&M, F_ab transforms as rank-2 tensor)
- Know how **A** transforms (4-vector, messy to get E)
- Have a picture of rest frame! (Last preferred)



84. Two pendulums are attached to a massless spring, as shown above. The arms of the pendulums are of identical lengths ℓ, but the pendulum balls have unequal masses m_1 and m_2. The initial distance between the masses is the equilibrium length of the spring, which has spring constant K. What is the highest normal mode frequency of this system?

(A) $\sqrt{g/\ell}$

(B) $\sqrt{\dfrac{K}{m_1 + m_2}}$

(C) $\sqrt{\dfrac{K}{m_1} + \dfrac{K}{m_2}}$

(D) $\sqrt{\dfrac{g}{\ell} + \dfrac{K}{m_1} + \dfrac{K}{m_2}}$

(E) $\sqrt{\dfrac{2g}{\ell} + \dfrac{K}{m_1 + m_2}}$

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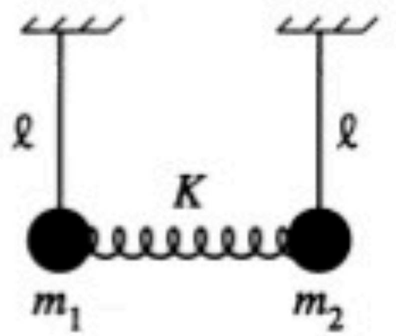

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$$f_{\text{high}} \xrightarrow{K \text{ large}} ??$$

$$f_{\text{high}} \xrightarrow{K \text{ small}} ??$$

Reduced mass

What distance from the center of the Earth does a geosynchronous satellite travel at?

What is the emission energy from a photon going from $n = 3$ to $n = 1$ in *positronium* (one electron and one positron orbiting one another)?

What distance from the center of the Earth does a geosynchronous satellite travel at?

$$\frac{GM_E m_s}{R_s^2} = m_s \omega^2 R_s \qquad g\frac{R_E^2}{R_s^2} = \omega^2 R_s$$

$$\frac{GM_E}{R_s^2} = \omega^2 R_s \qquad R_s^3 = \frac{gR_E^2}{\omega^2}$$

$$\frac{GM_R}{R_E^2}\frac{R_E^2}{R_s^2} = \omega^2 R_s$$

What is the emission energy from a photon going from n = 3 to n = 1 in *positronium* (one electron and one positron orbiting one another)?

Rule for Hydrogen like atoms: $E_n = \frac{-13.6 \text{ eV}}{n^2} Z^2$

But 13.6 is proportional to the *reduced* mass $m = m_{\text{electron}}$ in Hydrogen

In positronium $m = m_e/2$, so we have to halve the 13.6

$$\boxed{E_n = -\frac{6.8 \text{ eV}}{n^2}}, \qquad \text{(positronium energy levels)}$$

$$E_{\text{photon}} = E_3 - E_1 = 6.8 \text{ eV}\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 6.8 \text{ eV} \times \frac{8}{9} \approx 6 \text{ eV}$$

Situation 1)   Situation 2)

Two different ways of connecting a mass *m* to two *identical springs* with spring constant k are shown above. If we denote the frequency of oscillation in situation 1 by f_1 and the frequency of oscillation in situation 2 by f_2 then f_1/f_2 is:

a) 4
b) 2
c) 1/2
d) 1/4
e) depends on m and / or k

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(Hint: can pretend k_1 and k_2 are not the same to take limits to determine formula for k_{eff})

$$\frac{f_1}{f_2} = \frac{\omega_1}{\omega_2} = \sqrt{\frac{k_{\text{eff},1}/m}{k_{\text{eff},2}/m}} = \sqrt{\frac{k_{\text{eff},1}}{k_{\text{eff},2}}} = \sqrt{\frac{2k}{k/2}} = 2$$

A particle sits in a periodic potential

$$V(x) = d\sin(kx)$$

What is its oscillation frequency about the minimum?

A particle sits in a periodic potential

$$V(x) = d\sin(kx)$$

What is its oscillation frequency about the minimum?

Let y be the distance from the minimum. Expanding about the minimum we have:

$$V(y) = V_{\min} + \boxed{0y} + \frac{1}{2}\boxed{\frac{d^2V}{dy^2}|_{\min}}\, y^2 + \ldots$$

0 (because min)

Just a number, not a function

Force is

$$F = -\frac{dV}{dy} = -\frac{d^2V}{dy^2}|_{\min} y + \ldots$$

SHM with "spring constant" $k = d^2V/dy^2$ evaluated at min!

$$\text{spring constant} = -dk^2\sin(kx) = +dk^2 \text{ evaluated at min}$$

$$f = 2\pi\sqrt{\frac{\text{spring const.}}{m}} = 2\pi\sqrt{\frac{dk^2}{m}}$$

6) Making problems look like a harmonic oscillator

$$\omega^2 = \frac{(d^2V/dx^2)|_{\rm min}}{m}$$

7) Remember spectroscopic notation (ugh)

$${}^{2s+1}(\text{orbital angular momentum symbol})_j$$

and the selection rules for an electric dipole

8) Know the *pattern* of spherical harmonics

Y_ℓ^m

$$Y_0^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{1}{\pi}} \qquad (\ell = 0)$$

$$Y_1^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\,\sin\theta\, e^{-i\varphi}$$

$$Y_1^{0}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{\pi}}\,\cos\theta \qquad (\ell = 1)$$

$$Y_1^{1}(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{3}{2\pi}}\,\sin\theta\, e^{i\varphi}$$

$$Y_2^{-2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\,\sin^2\theta\, e^{-2i\varphi}$$

$$Y_2^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{15}{2\pi}}\,\sin\theta\,\cos\theta\, e^{-i\varphi}$$

$$Y_2^{0}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{5}{\pi}}\,(3\cos^2\theta - 1) \qquad (\ell = 2)$$

$$Y_2^{1}(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{15}{2\pi}}\,\sin\theta\,\cos\theta\, e^{i\varphi}$$

$$Y_2^{2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\,\sin^2\theta\, e^{2i\varphi}$$

Too detailed ...... !

(But if you can remember these, congratulations)

8) Know the *pattern* of spherical harmonics

$$Y_\ell^m$$

m – magnetic quantum number $(-\ell, -\ell+1, \ldots, \ell)$

ℓ – orbital quantum number $(0, 1, 2, \ldots)$

Y_ℓ^m contains φ dependence of the form $e^{im\phi}$

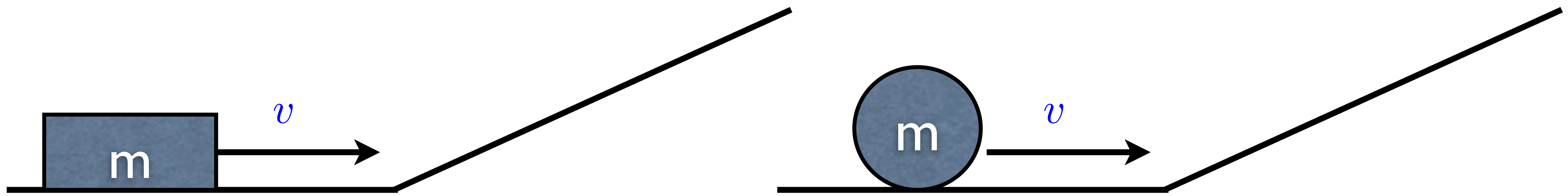
Y_ℓ^m contains ℓ dependence of the form $\sin^\ell \theta, \quad \sin^{\ell-1}\theta\cos\theta, \ldots$

(i.e. can write as ℓ sines or cosines mulitpled, or as $\sin(\ell\theta)$, $\cos(\ell\theta)$.)

Compare these rules to the spherical harmonics listed one slide ago.

Random mechanics problem:

A ball and a block of mass m are moving at the same speed *v*. When they hit the ramp they both travel up it. The block slides up with (approximately) no friction, the ball experiences just enough friction to roll without slipping. Which goes higher?



a) the ball goes higher
b) the black goes higher
c) they go the same height
d) Impossible to tell from information given

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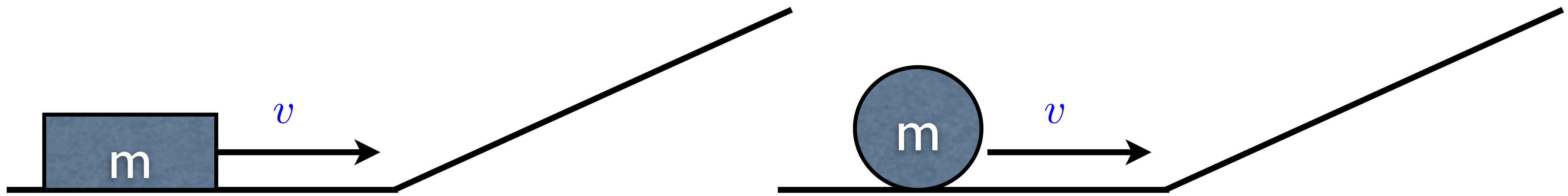

a) the ball goes higher
b) the black goes higher
c) they go the same height
d) Impossible to tell from information given

The ball has both translational kinetic energy (equal to that of the block) *and* rotational kinetic energy. Therefore

$$KE_{ball,initial} > KE_{block,initial}$$

The ball converts all this energy into potential energy, and therefore goes higher.