PHYSICS 101A FALL 2008

FINAL EXAM SAMPLE QUESTIONS

PUT YOUR NAME ON THE EXAM RIGHT AWAY!

PLEASE SHOW ALL OF YOUR WORK. You may use the back of the page if necessary. Please clearly mark all problems for which you have information on the back of the page that you would like to be considered during the grading of the exam.

EQUATIONS AND FORMULAE

$x' = x - vt;\ y' = y;\ z' = z$

$ct' = \gamma(ct - \beta x);\ x' = \gamma(x - \beta ct)$

$\beta = v/c$

$\gamma = 1/\sqrt{1-\beta^2}$

$\Delta t = \gamma \Delta t'$

$L = L_P/\gamma$

$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$

$\Delta\tau = \Delta s/c$

$u'_x = (u_x - v)/[1 - (vu_x/c^2)]$

$u'_y = u_y/\gamma[1 - (vu_x/c^2)]$

$u'_z = u_z/\gamma[1 - (vu_x/c^2)]$

$\vec{p} = \gamma m \vec{v}$

$E = \gamma mc^2$

$E'/c = \gamma(E/c - \beta p_x);\ p'_x = \gamma(p_x - \beta E/c)$

$E = mc^2 + T$

$p = (E/c, p_x, p_y, p_z)$

$M = |p|/c;\ |p|^2 = (E/c)^2 - p_x^2 - p_y^2 - p_z^2$

$E^2 = (mc^2)^2 + (pc)^2$

$pc = \beta E$

$\nu = c/\lambda$

$E = h\nu$

$R = \sigma T^4;\quad \sigma = 5.67 \times 10^{-8}\ \text{W/m}^2\text{-K}^4$

$\lambda_{max} T = 2.898 \times 10^{-3}$ m-K

$n(\lambda) = 8\pi\lambda^{-4}$

$\overline{E} = kT$

$u(\lambda) = \frac{8\pi hc\lambda^{-5}}{\exp(hc/\lambda kT) - 1}$

$\overline{E} = \frac{h\nu}{\exp(hc/\lambda kT) - 1}$

$eV_{stop} = h\nu - \phi$

$\lambda_2 - \lambda_1 = \lambda_c(1 - \cos\theta)$

$\lambda_c = 2.43 \times 10^{-12}$ m

$b = \frac{kzZe^2}{2E_k} \cot\frac{\theta}{2}$

$\Delta N = \frac{I_0 A_{sc} nt}{r^2} \frac{kzZe^2}{4E_k}^2 \frac{1}{\sin^4(\theta/2)}$

$d\Omega = \sin\theta\, d\theta\, d\phi$

$\frac{d\sigma}{d\Omega} = \frac{kzZe^2}{4E_k}^2 \frac{1}{\sin^4(\theta/2)}$

$r_d = \frac{kzZe^2}{E_k}$

$U_E = \frac{kq_1q_2}{r}$

$E_n = -hcR\frac{Z^2}{n^2}$

$\frac{1}{\lambda_{Nn}} = Z^2 R\left(\frac{1}{N^2} - \frac{1}{n^2}\right)$

$\mu = \frac{mM}{m+M}$

$L = n\hbar$

$\sqrt{\nu} = A_n(Z - b)$

$\lambda = h/p$

$\omega = 2\pi\nu = E/\hbar$

$n\lambda = D\sin\theta$

$\lambda = h/\sqrt{2mE_k}$

$p = \hbar k$

$E = p^2/2m$

$v = \omega/k$

$v_g = d\omega/dk$

$\Delta x \Delta p_x \geq \bar{h}/2$

$\Delta t \Delta E \geq \bar{h}/2$

$\tau \Delta E = \bar{h}$

$\psi^2(x) = e^{-t/\tau}$

$P(x) = |\psi(x,t)|^2$

$\int P(x) dx = 1$

$(\Delta x)^2 = \sum n_i^2 (x_i - \bar{x})^2 / N$

$\phi(t) = \exp(-i\omega t)$

$$-\frac{\bar{h}^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x,t)\psi(x,t) = i\bar{h}\frac{\partial\psi(x,t)}{\partial t}$$

$$-\frac{\bar{h}^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

$E_n = (h^2 n^2)/(8mL^2)$

$\psi_n(x) = \sqrt{2/L}\sin(n\pi x/L)$

$\mathrm{c} = 2.998 \times 10^8$ m/s

$\bar{h} = h/2\pi$

$\bar{h} = 1.054 \times 10^{-34}$ J-s $= 6.582 \times 10^{-16}$ eV-s

$h = 6.626 \times 10^{-34}$ J-s

$hc = 1240$ eV-nm

$k = 1.381 \times 10^{-23}$ J/K $= 8.617 \times 10^{-5}$ eV/K

$e = 1.602 \times 10^{-19}$ C

There are 1.602×10^{-19} J per eV

$m_e = 5.110 \times 10^5$ eV/c^2 $= 9.109 \times 10^{-31}$ kg

$a_0 = \bar{h}^2/mke^2 = 0.0529$ nm

$R_\infty = 1.097 \times 10^7$ m^{-1}

$k = 8.99 \times 10^9$ N-m^2/C^2 (electrostatic constant)

$G(x;\sigma) = A * \exp[1/2(x^2/\sigma^2)]$

$A = 1/(\sqrt{2\pi}\sigma)$

PROBLEM 1 [25 PTS]

An electron is confined to be within a one-dimensional crystal lattice of length 10 Angstroms, with an inter-atomic spacing of 2 Angstroms (there are only 5 atoms in this crystal). Assume that the electron must reside precisely at the center of each atom, i.e., that it can only be in one of five positions, each of which is separated by 2 Angstroms from its nearest neighbor. Also, assume that it's impossible to determine which of the five positions the electron occupies at any given time.

a) Estimate the minimum uncertainty on the momentum of the electron.

b) Estimate the minimum (zero-point) energy that such an electron can have.

PROBLEM 2 [25 PTS]

An electron confined inside an infinite square well extending from $x = 0$ to $x = 1$ nm (10^{-9} m) has an energy of about 3.4 eV, while the energy of the bottom of the well is 0.

a) Write down the time-independent wavefunction of the electron in the region between $x = 0$ to $x = 1$ nm. Make sure you specify the value of all constants that you use.

b) What is the probability of finding the particle in the middle half of the square well, i.e., between 0.25 and 0.75 nm?

c) Finally, write down the full time-dependent wavefunction of the electron.

Hint: The following identity may prove useful:

$$sin^2\theta = \frac{1 - \cos 2\theta}{2}.$$

PROBLEM 3 [20 PTS]

An electron beam maintains a gas of hydrogen atoms in an excited state, so that the gas glows green-yellow. Assume that this glow is dominated by a single spectral line, i.e., a single Hydrogen transition.

a) Assuming that the green-yellow portion of the spectrum extends from about 475 to 500 nm, what transition is dominating the Hydrogen gas glow? You make express your answer in terms of the quantum numbers of the initial and final state, or in terms of the established nomenclature (Lyman-α, Paschen-γ, etc.).

A portion of the light from this spectrum illuminates a photodiode, which is a device that puts out a voltage proportional to the intensity of light that falls on it. This voltage signal is monitored by an oscilloscope, so that quick changes in its output can be monitored. Two ns (2×10^{-9}s) after the electron beam is switched off, the voltage output of the photodiode is only 1% of what it was the instant before the electron beam was switched off.

b) What is the rms energy width of the observed yellow-green spectral line?

PROBLEM 4 [25 PTS]

An electron (mass 0.511 Mev/c^2) is represented by the wavefunction

$$\psi(x,t) = A_0 \exp[i(kx - \omega t)]$$

with $k = 2\pi \times 10^9$ m^{-1}.

a) If the electron has a total energy of 5 eV, what is the potential function $V(x)$ in which it moves?

b) Show that the spatial part $\psi(x)$ of the electron's wavefunction satisfies the time-independent Schroedinger equation for this form of the potential.

c) Compare the probability of finding this particle at the origin with the probability of finding it 10^{-10} meters from the origin.

d) What would your answer to c) be if the electron were composed of equal parts of the wavefunctions $A_0 \exp[i(kx - \omega t)]$ and $A_0 \exp[-i(kx + \omega t)]$?