PHYSICS 101B SPRING 2007 SAMPLE FINAL EXAM

PROBLEM 1 [18 POINTS]

Consider an object of mass m in motion in a one-dimensional region that is free of forces (V(x) = 0).

a) Write down the *time-independent* Schrodinger Equation that governs the motion of such an object.

b) Write down any solution $\psi(x)$ for this system and prove that your chosen solution satisfies the Schrödinger Equation that you wrote down in part a).

Instead, assume that the object is in motion in a harmonic oscillator potential $V(x) = \frac{1}{2}kx^2$.

c) Write down the time-independent Schrodinger Equation that governs the motion of the object in this potential.

d) Show that, for the right value of α , the wavefunction

$$\psi(x) = e^{-\alpha x^2}$$

provides a solution to this new wave equation (this solution may not be normalized, but don't worry about that). What is this correct value of α ?

e) Which state of the harmonic oscillator is this (ground, first excited, second excited, etc.)? Why? It may help to remember that $\omega = \sqrt{k/m}$ for the harmonic oscillator.

PROBLEM 2 [15 POINTS]

A circuit contains a battery, an AC generator, a diode, and an ammeter to measure the current through the diode as a function of the sum of the known voltages from the battery and generator (see diagram). The battery is chosen to forward bias the diode with a bias voltage of $V_b = 0.05$ V. The generator puts out an oscillating voltage, with a time dependence given by

$$V_q = V_0 \cos(\omega t)$$

with $V_0 = 0.001V$. The circuit is at room temperature (T = 300° K).

a) Find the ratio of the current excursion (difference between the minimum and maximum current) to the average current in the circuit.

b) Would you expect this ratio to increase or decrease as the bias voltage V_b is lowered towards 0? Why?

PROBLEM 3 [17 POINTS]

The Z and N dependence of ground-state nuclear masses is well represented by the Bethe-Weizsacker Semiempirical Mass Formula

$$\Gamma(A,Z) = a_1 - \frac{a_2}{A^{1/3}} - a_3 \frac{Z^2}{A^{4/3}} - a_4 \frac{(Z^{5/3} + N^{5/3})}{A^{5/3}}$$

where $\Gamma(A, Z)$ is the binding energy *per nucleon* for a nucleus of A total nucleons, composed of Z protons and N=A-Z neutrons. The values for the constants, determined by fits to the masses of many different nuclei, are 57.5, 16.8, 0.72, and 66.6 MeV for a_1 through a_4 , respectively. The proton and neutron masses are on the sheet of equations.

Consider an isotope of Oxygen (Z=8) with only five neutrons (^{13}O) .

a) Write down the full list of decay products that this isotope would produce if it underwent α decay. If you don't know the name of any element resulting from this decay, just specify it in terms of Z and A.

b) Show that, in fact, it is *not* possible for this isotope to undergo α decay. Justify your answer quantitatively. (You may make use of the fact that the mass of an α particle is 3727.4 MeV/c².)

c) Nevertheless, this Oxygen isotope is unstable. How does it decay? Write down a full list of decay products for this decay.

PROBLEM 4 [17 POINTS]

Here are some wavefunctions that may or may not be of use in the problem which follows:

$$R_{10}(r) = \frac{2}{\sqrt{a_0^3}} e^{-r/a_0}$$

$$R_{20}(r) = \frac{1}{\sqrt{2a_0^3}} (1 - \frac{r}{2a_0}) e^{-r/2a_0}$$

$$R_{21}(r) = \frac{1}{2\sqrt{6a_0^3}} \frac{r}{a_0} e^{-r/2a_0}$$

$$R_{30}(r) = \frac{2}{3\sqrt{3a_0^3}} (1 - \frac{2r}{3a_0} + \frac{2r^2}{27a_0^2}) e^{-r/3a_0}$$

$$R_{31}(r) = \frac{8}{27\sqrt{6a_0^3}} (1 - \frac{r}{6a_0}) \frac{r}{a_0} e^{-r/3a_0}$$

$$R_{32}(r) = \frac{1}{2\sqrt{30a_0^3}} \frac{r^2}{a_0^2} e^{-r/3a_0}$$

$$Y_{00} = \frac{1}{\sqrt{4\pi}}$$

$$Y_{11} = -\sqrt{3/8\pi} \sin \theta e^{i\phi}$$

$$Y_{10} = \sqrt{3/4\pi} \cos \theta$$

$$Y_{1-1} = \sqrt{3/8\pi} \sin \theta e^{-i\phi}$$

$$Y_{22} = \sqrt{\frac{15}{32\pi}} \sin^2 \theta e^{2i\phi}$$

$$Y_{21} = -\sqrt{\frac{15}{8\pi}} \sin \theta \cos \theta e^{i\phi}$$

$$Y_{20} = \sqrt{\frac{5}{16\pi}} (3\cos^2 \theta - 1)$$

$$Y_{2-1} = \sqrt{\frac{15}{8\pi}} \sin \theta \cos \theta e^{-i\phi}$$

$$Y_{2-2} = \sqrt{\frac{15}{32\pi}} \sin^2 \theta e^{-2i\phi}$$

a) IGNORING electron spin, what is the degeneracy of the first excited (n=2) state of hydrogen?

b) Write down the wavefunction for *each* of these excited states. These should be explicit functions of r, θ , and ϕ . Again, ignore electron spin. Also, don't worry about normalization.

c) Calculate the radial expectation value $\langle r \rangle$ for any one of these states of your choosing. Hint: what can you say immediately about $\int Y_{lm}^*(\theta, \phi) Y_{lm}(\theta, \phi) d\Omega$ for any values of l and m? Another Hint:

$$\int_{0}^{\infty} x^{n} e^{-x} dx = n!$$

d) If you had calculated $\langle r \rangle$ for all of these states, you would have seen that one of them is smaller than all the rest. Which one, and why? You may want to take a stab at this even if you didn't get part c) (not that c) is all that hard!).

PROBLEM 5 [15 POINTS]

In the following problem, please refer to the expressions for the Y_{lm} from the previous problem. Use exactly two separate Y_{lm} 's to answer each of the following questions. Do not concern yourself with normalization. You need not write out the explicit forms of the Y_{lm} 's in terms of θ and ϕ , nor do you need to prove that your linear combinations are the eigenstates that you claim they are.

- a) Exhibit a linear combination of Y_{lm} 's that is an eigenstate of L_z but not of L^2 .
- b) Exhibit a linear combination of Y_{lm} 's that is an eigenstate of both L_z and L^2 .
- c) Exhibit a linear combination of Y_{lm} 's that is an eigenstate of L^2 but not of L_z .
- d) For your answer to part c), what is the expectation value of L_z ?

PROBLEM 6

In the following problem, approximate the Fermi-Dirac distribution function by increasing the energy of all particles within kT of the Fermi energy by an amount kT (as we did in one of the homework problems).

A one-dimensional infinite square well that contains electrons has a ground-state energy of 10^{-6} eV. The well is filled with 2×10^3 electrons.

a) What is the Fermi energy of the system?

b) Precisely what fraction of electrons are above the Fermi energy at room temperature (T = 300K)?

c) What is the total energy in the system at room temperature?

d) What is the molar heat capacity of this system at room temperature?