PROBLEM 1 [25 PTS]

a) Given the expressions for $f_{lm}(\theta)$ and $g_m(\phi)$ from the front sheet, derive the spherical harmonic $Y_{1,-1}(\theta, \phi)$. Do this up to an overall constant (i.e., don't worry about the normalization for part a).

b) Find the constant $C_{1,-1}$ required to normalize $Y_{1,-1}$.

PROBLEM 2 [25 PTS]

Consider a particle of mass m and total energy $E = 3V_0$ in motion in one dimension in the region of a *downward* potential step of magnitude V_0. Specifically, the potential is uniformly equal to V_0 for $x < 0$, and makes a step down to to 0 for $x > 0$. As we have seen, the solutions to the time-independent Schrödinger Equation for such a potential take the form

$$\psi(x) = \alpha e^{ikx} + \beta e^{-ikx}$$

a) In terms of V_0 and m, what are the values of k associated with the motion in the two regions $x < 0$ and $x > 0$?

Now consider the specific case that a steady flux of such particles is incident upon this potential step from the left (i.e., moving in the direction of positive x).

b) What fraction of the incident particles, if any, are reflected back towards $x < 0$ by this potential barrier? Work from the expressions for the wavefuctions; remembering the value of 'R' form the book and plugging in will get only minimal credit, even if you can somehow figure out how to apply iy for a downward potential step!

PROBLEM 3 [25 POINTS]

We know that the Hydrogen atom wavefunctions take the form

$$\psi_{nlm}(r,\theta,\phi) = R_{nl}(r)Y_{lm}(\theta,\phi)$$

where the Y_{lm} are the 'spherical harmonic' functions of legend and song. Were you to look in the book (which, by the way, you may not until you've finished this exam), you could find explicit expressions of the R's and Y's for small values of n and l. However, to find the answers to this problem, you won't need to know those expressions.

Consider the following set of particular R's and Y's: $R_{10}(r)$, $R_{20}(r)$, $R_{31}(r)$, $R_{32}(r)$, and $Y_{00}(\theta,\phi)$, $Y_{10}(\theta,\phi)$, $Y_{1-1}(\theta,\phi)$, $Y_{21}(\theta,\phi)$, $Y_{2-2}(\theta,\phi)$.

a) Write down all possible combinations $\psi_{nlm}(r,\theta,\phi) = R_{nl}(r)Y_{lm}(\theta,\phi)$ of Hydrogen atom wavefunctions that can be formed from the R's and Y's. Your answers should be expressed in terms of the R's and Y's written just above, i.e., you don't need to figure out the r,θ, and ϕ dependence of the R's and Y's.

b) Write down the energy (in eV) for each of these states.

c) Write down the total and z-projection of the angular momentum (in units of $\hbar$ for each of these states.

d) For which of these states do you expect $\psi_{nlm}(0,\theta,\phi) = 0$? Why? Please be clear and explicit in your reasoning.

PROBLEM 4 [25 POINTS]

Consider a parrticle of mass m in motion inside a one-dimensional infinite square well of width L.

a) Write down the normalized wavefunction for this particle, assuming that it is in the first excited state (i.e., the state with the next-to-lowest energy).

b) Calculate the expectation value for the momentum ($\langle p \rangle$) of this particle. Comment on its value.

c) Calculate the expetation value for the squared momentum ($\langle p^2 \rangle$) for this particle. Comment on its value.