THREE SAMPLE QUESTIONS FOR MIDTERM I

PLEASE SHOW ALL OF YOUR WORK. You may use the back of the page if necessary. Please clearly mark all problems for which you have information on the back of the page that you would like to be considered during the grading of the exam.

EQUATIONS AND FORMULAE

\[ x' = x - vt; \quad y' = y; \quad z' = z \]
\[ \beta = v/c \]
\[ \Delta t = \gamma \Delta t' \]
\[ (\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 \]
\[ u_x' = (u_x - v)/[1 - (vu_x/c^2)] \]
\[ u_z' = u_z/\gamma[1 - (vu_x/c^2)] \]
\[ E = \gamma mc^2 \]
\[ E'/c = \gamma(E/c - \beta p_x) \]
\[ p = (E/c, p_x, p_y, p_z) \]
\[ E^2 = (mc^2)^2 + (pc)^2 \]
\[ ct' = \gamma(ct - \beta x); \quad x' = \gamma(x - \beta ct) \]
\[ \gamma = 1/\sqrt{1 - \beta^2} \]
\[ L = L_P/\gamma \]
\[ \Delta \tau = \Delta s/c \]
\[ u_y' = u_y/\gamma[1 - (vu_x/c^2)] \]
\[ \vec{p} = \gamma \vec{m} \vec{v} \]
\[ E = mc^2 + T \]
\[ p_x' = \gamma(p_x - \beta E/c) \]
\[ M = |p|/c; \quad |p|^2 = (E/c)^2 - p_x^2 - p_y^2 - p_z^2 \]
\[ pc = \beta E \]
PROBLEM 1

A pion (rest mass 140 MeV/c$^2$) has a momentum of 500 MeV/c in the $+\hat{x}$ direction in the rest frame of a laboratory experiment.

a) What is the energy of this pion in the laboratory frame?

b) Using the properties of the Lorentz Transformation, calculate the momentum and energy of this pion when viewed from a rest frame moving with a velocity $\beta = 0.8$ in the $+\hat{x}$ direction relative to the lab frame.

c) Use these values of momentum and energy to calculate the invariant mass of the pion in this rest frame. Comment on your result.
PROBLEM 2
Consider a universe in which the speed of light is exactly

\[ c = 1 \text{ m/s} \]

(so that the following calculations will be easier).

A pulsing light with a period of two seconds (in its rest frame) passes by an observer. Relative to the observer, the pulser has a velocity \( \vec{v} = (\frac{1}{2} \text{ m/s}) \hat{x} \). The observer first observes the light flash at the spacetime point \( (ct, x) = (1.2 \text{ m}, 4.1 \text{ m}) \) in her rest frame.

a) Estimate (or calculate) the spacetime location of the next light flash in the observer’s frame.

As it turns out, this light is on the far (larger value of x coordinate) end of a two meter long stick that lies along the x axis.

b) What does the observer measure as the x coordinate of the near (smaller value of x coordinate) of the stick at the instant of the first flash?
PROBLEM 3

Each of a sample of neutral Kaons (rest mass 500 MeV/c^2) decays at rest in the lab frame into two pions (each of rest mass 140 MeV/c^2) which, according to momentum conservation, fly off with equal and opposite momenta.

a) Calculate the momentum of the decays pions (in the lab frame).

b) What is the observed mean lifetime of this sample of pions? (The mean lifetime of a pion in its rest frame is \(2.6 \times 10^{-8}\) s.)