PHYSICS 101A – HOMEWORK SET 0

Due in class Wednesday 10/1/08. This is just a warmup to familiarize you with some common expansions which are of utmost utility throughout the field of physics.

1.) Taylor expanding about the point $x = 0$, derive the binomial expansion

$$(1 + x)^p = 1 + p \cdot x + [p(p-1)/2!] \cdot x^2 + [p(p-1)(p-2)/3!] \cdot x^3 + ...$$

(Note that this will only converge if $|x| < 1$.) For p a positive integer, we can instead use the laws of algebra to derive an expression with a finite number $(p + 1)$ of terms. Is this consistent with the infinite series derived above from the Taylor expansion? Why or why not?

2.) Use the binomial expansion derived above to first order in x to estimate the value of

$$\frac{1}{\sqrt{a^2 - b^2}}$$

for $a = 100$ and $b = 10$. By what fractional error does your estimate differ from the exact answer? (Answer: 0.01005; it differs from the true answer by only 0.004%).

3.) Taylor expanding about the point $\theta = 0$, show that

$$\sin\theta = \sum_{i=0}^{i=\infty} (-1)^i \frac{\theta^{2i+1}}{(2i+1)!}$$

$$\cos\theta = \sum_{i=0}^{i=\infty} (-1)^i \frac{\theta^{2i}}{(2i)!}$$