Reading: Tipler and Llewellyn, Sections 6.4, 6.6 and 7.6.

Due Friday 1/19/07.

1.) Taylor expanding about the point \( x = 0 \), derive the binomial expansion

\[
(1 + x)^p = 1 + p \cdot x + [p(p - 1)/2!] \cdot x^2 + [p(p - 1)(p - 2)/3!] \cdot x^3 + \ldots
\]

(Note that this will only converge if \( |x| < 1 \).) For \( p \) a positive integer, we can instead use the laws of algebra to derive an expression with a finite number \((p + 1)\) of terms. Is this consistent with the infinite series derived above from the Taylor expansion? Why or why not?

2.) Use the binomial expansion derived above to first order in \( x \) to estimate the value of

\[
\frac{1}{\sqrt{a^2 - b^2}}
\]

for \( a = 100 \) and \( b = 10 \). By what fractional error does your estimate differ from the exact answer? (Answer: 0.01005; it differs from the true answer by only 0.004%).

3.) Taylor expanding about the point \( \theta = 0 \), show that

\[
\sin \theta = \sum_{i=0}^{i=\infty} (-1)^i \frac{\theta^{2i+1}}{(2i + 1)!}
\]

\[
\cos \theta = \sum_{i=0}^{i=\infty} (-1)^i \frac{\theta^{2i}}{(2i)!}
\]

4.) Problem 6.28 (Answers: a) \( L/2 \); b) \( 0.328L^2 \))

5.) Problem 6.31 (Answer: \( \sigma_x \sigma_p = 0.568\hbar \))

6.) Consider the simple harmonic oscillator solutions given in equation 6-58 in the text. a) Show that \( \psi_0(x) \) satisfies the time-independent harmonic oscillator Shroedinger equation. Show that the energy of this state is \((1/2)\hbar \omega\). b) Do the same for \( \psi_1(x) \), showing that its energy is \((3/2)\hbar \omega\). c) What is the energy of a photon emitted by a transition between these two states?

7.) Problem 6.41 (Answers: a) \( k/\sqrt{2} \); b) 0.0294; c) 0.971; d) 9.71 \times 10^5)
8.) Problem 6.46 (Answer: $6.5 \times 10^{-5}$)

9.) Problem 6.47 (Answer: a) 0.111; b) 0.111)

10.) Consider a potential barrier of height $V_0$ that has the form of a step function at $x=0$.

a) Show that, for a particle with energy $0 < E < V_0$ arriving at the barrier from the left, the reflection coefficient is precisely 1 (i.e., all particles incident upon the barrier from the left will be reflected back to the right). Do not use reflection and transmission coefficients provided in the text; instead, start with the wavefunction solutions to the Schrödinger equation and apply matching conditions at the position of the barrier.

b) For such a particle, the wavefunction will penetrate the barrier for some distance. In terms of $V_0$ and $E$, what is the mean (expectation) value of this penetration? In other words, if you had a detector that extended to the right from $x = 0$, which could detect the position of particles penetrating the barrier, what would be the mean value of the position detected in this instrument?

11.) Problem 7.43 (Answers: a) 7.14 eV; b) 0.00712 eV)