PHYSICS 101B – HOMEWORK SET 1

Reading: Tipler and Llewellyn, Sections 6.4, 6.6 and 7.6.

Due Friday 1/19/07.

1.) Taylor expanding about the point $x = 0$, derive the binomial expansion

$$(1 + x)^p = 1 + p \cdot x + [p(p-1)/2!] \cdot x^2 + [p(p-1)(p-2)/3!] \cdot x^3 + ...$$

(Note that this will only converge if $|x| < 1$.) For p a positive integer, we can instead use the laws of algebra to derive an expression with a finite number $(p + 1)$ of terms. Is this consistent with the infinite series derived above from the Taylor expansion? Why or why not?

2.) Use the binomial expansion derived above to first order in x to estimate the value of

$$\frac{1}{\sqrt{a^2 - b^2}}$$

for $a = 100$ and $b = 10$. By what fractional error does your estimate differ from the exact answer? (Answer: 0.01005; it differs from the true answer by only 0.004%).

3.) Taylor expanding about the point $\theta = 0$, show that

$$\sin\theta = \sum_{i=0}^{i=\infty} (-1)^i \frac{\theta^{2i+1}}{(2i+1)!}$$

$$\cos\theta = \sum_{i=0}^{i=\infty} (-1)^i \frac{\theta^{2i}}{(2i)!}$$

4.) Problem 6.28 (Answers: a) L/2; b)$0.328L^2$)

5.) Problem 6.31 (Answer: $\sigma_x\sigma_p = 0.568\bar{h}$)

6.) Consider the simple harmonic oscillator solutions given in equation 6-58 in the text. a) Show that $\psi_0(x)$ satisfies the time-independent harmonic oscillator Shroedinger equation. Show that the energy of this state is $(1/2)\bar{h}\omega$. b) Do the same for $\psi_1(x)$, showing that its energy is $(3/2)\bar{h}\omega$. c) What is the energy of a photon emitted by a transition between these two states?

7.) Problem 6.41 (Answers: a) $k/\sqrt{2}$; b) 0.0294; c) 0.971; d) 9.71×10^5)

8.) Problem 6.46 (Answer: 6.5×10^{-5})

9.) Problem 6.47 (Answer: a) 0.111; b) 0.111)

10.)Consider a potential barrier of height V_0 that has the form of a step function at x=0.

a) Show that, for a particle with energy $0 < E < V_0$ arriving at the barrier from the left, the reflection coefficient is precisely 1 (i.e., all particles incident upon the barrier from the left will be reflected back to the right). Do *not* use reflection and transmission coefficients provided in the text; instead, start with the wafefunction solutions to the Schroedinger equation and apply matching conditions at the position of the barrier.

b) For such a particle, the wavefunction will penetrate the barrier for some distance. In terms of V_0 and E, what is the mean (expectation) value of this penetration? In other words, if you had a detector that extended to the right from x = 0, which could detect the position of particles penetrating the barrier, what would be the mean value of the position detected in this instrument?

11.) Problem 7.43 (Answers: a) 7.14 eV; b) 0.00712 eV)