PHYSICS 101B – HOMEWORK SET 2

Reading: Tipler and Llewellyn, Sections 7.1-7.4

Due Friday 2/2/07. WARNING: This is a long and computationally intensive problem set. DO NOT put it off to the last minute!

1.) Consider the three-dimensional infinite cubic well of section 7.1 with sides of length L. Find the energy levels and degeneracy (number of different states with the same energy) for the six lowest allowable energies. (Answer: only first two provided for pedagogical reasons; (111) and (211) with degeneracies of 1 and 3)

2.) 7.8. Do this for the three lowest energies of the configuration (note that, due to degeneracies, there will be more than three states for you to catalogue).

3.) Let $\vec{C} = \vec{A} \times \vec{B}$. The formal definition of the cross product is given by

$$C_x = A_y B_z - A_z B_y$$

$$C_y = A_z B_x - A_x B_z$$

$$C_z = A_x B_y - A_y B_x$$

Let $\vec{A} = A\hat{x}$ and $\vec{B} = B\cos\theta\hat{x} + B\sin\theta\hat{y}$. Show that, as expected,

$$|\vec{A} \times \vec{B}| = AB\sin\theta$$

and that the direction of $\vec{A} \times \vec{B}$ is that given by the right-hand rule.

4.) Consider the time-dependent 3-d Schroedinger equation in Cartesian coordinates:

$$\frac{\hbar^2}{2\mu}\nabla^2\Psi(x,y,z,t) + V(x,y,z,t)\Psi(x,y,z,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,y,z,t),$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

Show that if $V(x,y,z)$ is not an explicit function of time, we can write

$$\Psi(x,y,z,t) = \psi(x,y,z)\phi(t)$$

where

$$\phi(t) = \exp -(iEt)/\hbar$$

and $\psi(x,y,z)$ satisfies the time-independent SE

$$\frac{\hbar^2}{2\mu}\nabla^2\psi(x,y,z) + V(x,y,z)\psi(x,y,z) = E\psi(x,y,z).$$

5.) Recall that the angular wavefunction $Y_{lm}(\theta,\phi) = f_{lm}(\theta)g_m(\phi)$ has the explicit θ dependence

$$f_{l0}(\theta) = P_l(\theta) = [\frac{d}{d\cos\theta}]^l(\cos^2\theta - 1)^l;$$

$$f_{lm}(\theta) = (\sin\theta)^{|m|}[\frac{d}{d\cos\theta}]^{|m|} f_{l0}(\theta).$$

Derive, up to an overall constant, the Y_{lm} functions for all allowable values of m for the case $l = 2$. Recall that $g(\phi) = e^{im\phi}$.

6.) The angular momentum operator L_{op}^2 is closely related to the angular part of the ∇^2 operator of equation 7.9 in the text. Use the classical relation between angular momentum and kinetic energy to extract the exact form 7.20 of L_{op}^2 from equation 7.9. Operate on the function Y_{21} derived above with both this operator and

$$L_z^{op} = -i\hbar\frac{\partial}{\partial\phi}$$

to confirm that Y_{21} is an eigenfunction of both, with the expected values of l and m.

7.) Consider the combination

$$F(\theta,\phi) = \frac{1}{\sqrt{2}}[Y_{1,1}(\theta,\phi) + Y_{1,-1}(\theta,\phi)].$$

Is $F(\theta,\phi)$ an eigenfunction of the z component of the angular momentum operator? Why or why not? Please cast your argument in two ways: first, by using the fact that the individual $Y_{l,m}$ are known to be eigenfunctions of L_z^{op}, so that $L_z^{op}[Y_{l,m}] = mY_{l,m}$; and second, by using the form of L_z^{op} from the problem above and applying it to the explicit form of $F(\theta,\phi)$ given by the expressions for the $Y_{l,m}$ found in Table 7.1 in the text.

8.) Problem 7.70; you need not find the constant A. Recall that the Bohr raduis a_0 is given by

$$a_0 = \frac{\hbar^2}{\mu k e^2}.$$

Just to make a point, argue that if this wave function is changed only very slightly, say by changing the argument of the exponent from $-r/(2a_0)$ to $-r/2.1a_0$, the TISE 7.9 is NOT satisfied, for ANY value of the total energy E.

9.) Problem 7.9

10.) Problem 7.27

11.) Problem 7.19. Note that the *radial* probability density for a given state is that function $P(r)$ satisfying the expression

$$\int_0^\infty P(r)dr = 1.$$

Recall also that the volume element in spherical coordinates is $dV = r^2 \sin\theta d\theta d\phi$. (Answers: a) $(2e^{-1})/\sqrt{4\pi a_0^3}$; b) $e^{-2}/(\pi a_0^3)$; c) $(4e^{-2})/a_0$

12.) 7.20 (Answers: a) 0.0162; b) 0.0088)

13.) 7.22. You'll probably need to use integration by parts in order to solve this problem.

14.) 7.26

15.) 7.63; again use integration by parts.