PHYSICS 101A – HOMEWORK SET 4

Due in class Wednesday 11/12/08.

Reading: Tipler and Llewllyn, Chapter 4.

1.) An *isotropic* differential cross section is one in which the scattering probability into a unit element of solid angle is independent of scattering angles, i.e.,

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_0}{4\pi}$$

Integrate this differential cross section over all angles to show that the total cross section, for any scattering angle θ or ϕ, is just σ_0. Recall that the infinitesimal solid angle element is given by

$$d\Omega = \sin\theta d\theta d\phi$$

for θ, ϕ, in radians, and that the maximum value of the angles θ and ϕ are 180° (π rad) and 360° (2π rad), respectively.

2.) For the differential cross section of Problem 1, integrate over the appropriate ranges in θ and ϕ to find the total cross section for scattering through an angle of θ or greater, for any angle ϕ, for a) angles θ greater than 170° or 2.97 rad; b) angles θ greater than 90° or $\pi/2$ rad; c) angles θ greater than 10° or 0.175 rad. Why is it that for a), given that the scattering is isotropic, the result is much less than 1/18 of σ_0, even though 10° represents 1/18 of the range of possible scattering angles θ? (Answer to a): 0.00760 σ_0)

3.) Given the differential cross section

$$\frac{d\sigma}{d\Omega} = \left(\frac{kzZe^2}{4E_k}\right)^2 \frac{1}{\sin^4(\theta/2)}$$

derive equation 4.6 in the text, for a detector of area A_{sc} mounted at a distance r from the target. As noted in the text, ΔN is the number of α particles scattered *per second* into the detector, I_0 the number of α's per second incident on the foil, n the number of density of nuclei (number per m^3), and t the foil thickness.

To think about (you need not turn in an answer): Why does your answer not depend upon the size (width) of the incoming beam of α particles?

4.) Integrate the differential cross section from Problem 3 over the appropriate ranges in θ and ϕ to find the total cross section for scattering through an angle of θ or greater, for any angle ϕ. Argue that the θ dependence of the result is consistent with the combination of equations 4.5 and 4.3 in the text. Answer:

$$4\pi\left(\frac{kzZe^2}{4E_k}\right)^2 \cot^2\frac{\theta}{2}$$

(Hints: You may find the following relations helpful: $\sin\theta d\theta = d(-\cos\theta); sin(\theta/2) = \sqrt{(1-\cos\theta)/2}$.)

5.) Find the total cross section (in m^2) for Rutherford scattering of 7.7 MeV α ($z = 2$) particles from gold ($Z = 79$) nuclei through angles θ a) greater than 170° or 2.97 rad; b) greater than 90° or $\pi/2$ rad; c) greater than 10° or 0.175 rad; for any angle ϕ. (Answer to b): 6.84×10^{-28} m^2).

6.) Problem 4.16 (Answers: $\simeq 3 \times 10^{74}$, 0.25×10^{-40} J; $\Delta r \simeq 5.0 \times 10^{-64}$ m)

7.) Light of wavelength 410.7 nm is observed in emission from a hydrogen source. (a) What transition between hydrogen Bohr orbits is responsible for this radiation? (b) To what series does this transition belong? (Answers: a) $n = 6$ to $n = 2$; b) Balmer)

8.) Problem 4.26 (Answers: a) 0.0610 nm and 0.0578 nm; b) 0.0542 nm)

9.) What is the minimum potential that must be applied across an x-ray tube in order to observe the K_α line of tungsten? What is the λ_{min} of the continuous spectrum? (Answers: 72.5 kV; 0.017 nm)

10.) Problem 4.34 (Answers: a) The 3rd Lyman line; b) The first 3 Lyman lines, the first 2 Balmer lines, and the first Paschen line).

11.) Problem 4.40 (Answer: 29.5 fm)

12.) Problem 4.45 (Answers: a) $n = 6$ to $n = 3$ and $n = 9$ to $n = 3$; b) $\Delta\lambda = 0.056$ nm)

13.) Problem 4.47 (Answers: L: 1.94 keV; M: 2.28 keV; n: 2.31 keV)